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MODEL ARITHMETIC.

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No.

BY HENRY H. BELFIELD, A.M., PH.D.,
DIRECTOR OF
THE CHICAGO MANUAL TRAINING SCHOOL.

CHICAGO:
SCOTT, FORESMAN & CO.
1896

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WORKS BY SAME AUTHOR.

Little Folks' Arithmetic,
Graded Examples for Primary School,
Revised Model Elementary Arithmetic.

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PREFACE.

Recent changes in the amount and character of Arithmetical teaching have led to the preparation of this volume, which is not a revision of THE MODEL ARITHMETIC published in 1875, but is, as its name indicates, essentially a new book.

An effort has been made to retain the features of the former work which commended themselves to thoughtful educators; to abandon that which is now considered irrelevant, and to treat in the best possible method every subject presented.

In this book, as in its predecessor, will be found the combination of oral and written exercises — in which the first Model Arithmetic was the pioneer—and a large number of well graded examples.

In addition to the examples and problems, to which answers are given at the close of the volume, will be found a large number of tabulated exercises, answers to which will be furnished to teachers by the publishers without cost. The advantage of such examples in assisting in the development of independent thought is apparent.

The repetitions rendered necessary by the Summaries will not be criticised by the experienced teacher.

The greatest departure from the usual methods is found in the presentation of Decimal Fractions as an integral part of the decimal system ; which, it is confidently believed, will be appreciated by those who use the book.

CHICAGO, February 18, 1891.

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FIGURES :	0	1	2	3	4	5	6	7	8	9
NAMES :	*Zero.*	*One.*	*Two.*	*Three.*	*Four.*	*Five.*	*Six.*	*Seven.*	*Eight.*	*Nine.*

2. The figures 1, 2, 3, etc., represent one unit, two units, three units, etc.

The figures from 1 to 9 inclusive are called *digits.*

The figure 0, named *zero, naught,* or *cipher,* denotes absence of number, or *no number.*

3. A number greater than nine is expressed by two or more figures, as—

10	11	12	13	14	15	16	etc.
Ten.	*Eleven.*	*Twelve.*	*Thirteen.*	*Fourteen.*	*Fifteen.*	*Sixteen.*	

4. *Ten* is composed of ten units, just as a dime is equal in value to ten cents.

Ten units, or one ten, are written 10. The figure 1 here occupies *tens' place,* that is the second place, counting from the right. The figure 0 stands in *units' place,* that is, the first place, counting from the right, and denotes *no number* in units' place.

5. The number *eleven*, 11, is composed of 1 ten and 1 unit: 10+1=11.

The number *twelve*, 12, is composed of 1 ten and 2 units: 10+2=12.

Of what are the numbers 12, 13, 14, 15, etc., respectively composed?

6. The number *twenty*, 20, is composed of two tens: 10+10 =20. Thirty, 30, is three tens, 10+10+10=30.

Of what is *forty* composed? *Fifty?* *Sixty?* etc.

7. Units may be arranged in groups of ten each, and may be counted by *tens*, thus:

20	30	40	50	60	70	80	90
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Of how many tens and units is each of the following numbers composed? 28? 31? 34? 39? 53? 63? 69? 70? 80? 89? 98?

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10. Sixty-nine.
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12. Ninety-six.

11. Ten tens are *one hundred*, 100; that is, one hundred units. The figure 1, in 100, occupies *hundreds' place*, that is, the *third* place counting from the right: the two zeros, 00, show that there are no units in tens' place and in units' place. The hundred units are considered as grouped into one group, or body, called a

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13. The number *one hundred one*, 101, is composed of 1 hundred, no tens, 1 unit: 100+00+1=101.

The number *one hundred ten*, 110, is composed of 1 hundred, 1 ten, no units: 100+10+0=110.

The number *one hundred twenty-five*, 125, is composed of 1 hundred, 2 tens, 5 units: 100+20+5=125.

Of how many hundreds, tens and units, is each of the following numbers composed: 153? 246? 379? 600? 709? 840?

14. Read the following numbers:

116, 149, 270, 309, 465, 555, 679, 843, 989.

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2. One hundred two.
3. Two hundred twenty-one.
4. Three hundred sixty-three
5. Five hundred ninety-six.
6. Seven hundred seven.
7. Nine hundred nine.
8. Nine hundred ninety.

16. These three places, or orders, that is, units, tens, and hundreds, constitute the *period of units*, or, the *units' period.*

What is the largest number, that is, *whole* number, that can be expressed in units' place? What is the largest number that can be expressed in tens' and units' places? In the period of units?

17. Ten hundreds compose the number *one thousand*, 1000.

Units may be counted by *thousands*, as one thousand, two thousand, three thousand, etc.

18. The *period of thousands*, like the period of units, has three places, or orders, called *units of thousands*, *tens of thousands*, *hundreds of thousands*.

In the number 1000, the figure 1 occupies the *thousands'* place, or, belongs to the order of thousands, and represents one group composed of one thousand units.

In the number 2000, the figure 2 represents 2 groups, each composed of one thousand units.

Ten thousand is written 10,000 ; *one hundred thousand* is written 100,000.

19. Read the following numbers :

1. 1003.
2. 1300.
3. 1030.
4. 1330.
5. 1033.
6. 10840.
7. 17902.
8. 35679.
9. 40900.
10. 40090.
11. 50248.
12. 172456.
13. 206070.
14. 530059.
15. 805706.

20. In like manner are found the higher orders of Millions, Billions, Trillions, Quadrillions, Quintillions, Sextillions, etc., each period, when complete, consisting of three figures, representing the units, the tens, and the hundreds of that period.

The *number* of units is represented by the *figure*, but the value of the unit is determined by the order of the figure.

Thus, 1 in the first order represents *one;* but 1 in the fourth order represents *one thousand;* 1 in the seventh order represents *one million*, etc.

21. Each figure, therefore, has two values :

1. Its value independent of its position.
2. Its value as determined by the order which it occupies.

The former is called its *absolute* value ; the latter is called its *local* value.

22. Periods.	Trillions.			Billions.			Millions.			Thousands.			Units.		
	Hundred Trillions.	Ten Trillions.	Trillions.	Hundred Billions.	Ten Billions.	Billions.	Hundred Millions.	Ten Millions.	Millions.	Hundred Thousands.	Ten Thousands.	Thousands.	Hundreds.	Tens.	Units.
Orders of Units,	15th.	14th.	13th.	12th.	11th.	10th.	9th.	8th.	7th.	6th.	5th.	4th.	3rd.	2nd.	1st.
	5	4	3	2	1	0	9	8	7	6	5	4	3	2	1

The above number is read *Five hundred forty-three trillions, two hundred ten billions, nine hundred eighty-seven millions, six hundred fifty-four thousands, three hundred twenty-one.*

In writing large numbers, the comma is generally used to separate periods.

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1. Two thousand fifty.
2. Sixteen thousand three hundred four.
3. Twenty thousand sixty.
4. Three hundred thousand sixteen.

5. Two hundred fifty-six thousand three hundred ninety-four.
6. Seven million forty-two thousand and thirty-three.
7. Fifteen million nine hundred four thousand sixty.
8. Eighty billion ninety million forty thousand twenty.
9. One hundred ten billion five million.
10. Sixteen trillion thirty-two billion two hundred twenty-nine million eight hundred thousand ten.

SUMMARY.

25. A **Unit** is a single thing, or one of any kind.

26. A **Number** is a unit, or a collection of units.

27. **Figures** are characters used to represent numbers.

28. **Notation** is an expression of number by figures or letters.

29. The **Arabic system** of notation employs ten figures, viz.:

0 1 2 3 4 5 6 7 8 9

The figures from 1 to 9 inclusive are called **Digits.**

By means of these ten figures, any number may be expressed.

30. The Arabic system of notation is also called the **Decimal** system, since ten units of any order are equal to one unit of the next higher order.

United States money is a *decimal* money for the same reason.

31. Since all numbers can be expressed by the nine digits and the zero, it is necessary that each digit should have different values in different places, or orders.

32. The same digit always expresses the same number of units.

Thus, 5 always expresses five units of some order.

33. The value of any digit depends on the order which the digit occupies.

If 5 is in the order of tens, it represents 5 tens, each ten consisting of 10 units; it therefore represents 50 units; if 5 is in the order of hundreds, it represents 5 hundreds, each hundred consisting of 100 units; it therefore represents 500 units.

34. The zero, while representing no number, shows to what order the digit on its left belongs.

Thus, in 7000, the use of the three 0's is to show that the digit 7 here stands in the order of thousands.

35. Orders of figures are numbered first, second, third, etc., beginning at the right.

In 3452, 2 is in the first order, 5 is in the second order, 4 in the third order, etc.

36. A period, when complete, consists of three figures, the first of which (beginning at the right) is units, the second is tens, the third is hundreds.

In 37,542,000, the third period, millions, contains only two figures, 37.

37. **Numeration** is naming or reading numbers expressed in figures or letters.

38. To read a number written in figures:

Separate the number into periods of three figures each, beginning on the right. Then, beginning at the left, read the successive orders of figures, giving the name of each period, except the period of units.

NOTATION OF DECIMAL FRACTIONS.

39. The numbers already considered are *whole numbers*, or *integers*. The notation of decimal fractions follows the same law, viz., that a unit of any order equals in value ten units of the next lower order.

40. The value of any digit depends on the order which it occupies. (Article 33.) Orders decrease in value

from left to right. Thus, in 11, the right hand figure, 1, represents a value one tenth of the value represented by the left hand figure 1.

This is still true, if the *decimal point* be inserted between the two 1's, thus, 1.1.

The 1 on the left of the decimal point, which in 11 represented 1 ten, now, in 1.1, represents 1 unit, since the decimal point, when written, is always placed on the right of the unit order.

Therefore, the 1 on the right of the decimal point, in 1.1, represents *one tenth* of *one unit.*

41. In like manner, in .11, the right hand 1 represents *one tenth* of *one tenth*, that is, *one hundredth.* And so, indefinitely, each succeeding order represents a value one tenth of the order on its left.

42. Orders less than units, called *decimal fractions*, are thus named:

	Decimal Point.	Tenths.	Hundredths.	Thousandths.	Ten Thousandths.	Hundred Thousandths.	Millionths.	Ten Millionths.	Hundred Millionths.	Billionths.
Periods.		Thousandths.			Millionths.			Billionths.		
Orders of Units:		1st.	2nd.	3rd.	4th.	5th.	6th.	7th.	8th.	8th.
	.	1	2	3	4	5	6	7	8	9

43. United States money illustrates decimal fractions. The dollar is the unit of which dimes are tenths, cents are hundredths, and mills are thousandths. Thus, $1.375, usually read one dollar thirty seven cents five mills, or, one dollar thirty-seven and one-half cents, may also be read *one dollar and 375 thousandths of a dollar.*

44. Decimal fractions are read as integers, with the addition of the name of the lowest order in the fraction read. Thus,

.1 is read *one tenth.*
.02 is read *two hundredths.*
.34 is read *thirty-four hundredths.*
.003 is read *three thousandths.*
.045 is read *forty-five thousandths.*
.245 is read *two hundred forty-five thousandths.*
.0004 is read *four ten thousandths.*
.8000 is read *eight thousand ten thousandths.*

45. Read the following numbers:

1. .5
2. .05
3. .50
4. .25
5. .09
6. .005
7. .875
8. .049
9. .0007
10. .0029
11. 25.25
12. 81.384
13. 13.033
14. 65.849
15. 52.9812

ROMAN NOTATION.

46. The Roman Notation employs the seven capital letters I, X, C, M, V, L, D.

I=1. X=10. C=100. M=1000.
V=5. L=50. D=500.

Other numbers are represented by repeating and combining these letters in accordance with the following:

PRINCIPLES.

1. Repeating a letter repeats its value.

   Thus, I=1; II=2; III=3; X=10; XX=20; XXX=30.

   Only I, X, C, and M are repeated.

2. If a letter is placed *before* a letter of greater valu the *difference* of the two values is the value of th combination.

   Thus, IV=4; IX=9; XL=40.

3. If a letter, or combination of letters, is placed *after* letter of greater value, the *sum* of the two values i the value of the combination.

   Thus, VI=6; XI=11; LX=60; LXXX=80; XXXI =33 XXXVIII = 38; XIV = 14.

   Observe that *several* letters may *follow* a letter or letters of *great* value. Only *one* letter may *precede* a letter of greater valu In XIV, the combination IV follows X. So XIX=19; LIX=5 here the IX follows the X and L.

4. A dash (—) placed over a letter, or combination o letters, gives it a thousand-fold value.

   Thus, $\overline{I}=1000$; $\overline{X}=10{,}000$; $\overline{XL}=40{,}000$.

47. 1. Write in Roman numerals, the numbers from 1 to 2

2. Write in Roman numerals, the numbers from 20 to 3
3. Write in Roman numerals, the numbers from [illegible] to 5
4. Construct a table, in four columns, as begun belo showing the Roman method of writing units, fro to 10: the tens from 10 to 100: the hundreds f one hundred to one thousand: and the thousands f one thousand to ten thousand. There will be numbers in each column.

Thousands.	Hundreds.	Tens.	Units.
M	C	X	I
MM	CC	XX	II
etc.	etc.	etc.	etc.

48. Read the following :

1. IV.
2. VI.
3. IX.
4. XI.
5. XL.
6. LX.
7. LXXXIX.
8. XCIX.
9. CCCLIII.
10. CDLIX.
11. DXLXXII.
12. CDLXVI.
13. MDC.
14. MDCCCXC.
15. $\overline{\text{DIX}}$.
16. $\overline{\text{MD}}$.
17. $\overline{\text{MIX}}$.
18. $\overline{\text{MLX}}$.

SUMMARY.

49. An **Integer** or **whole number,** is a number that expresses whole units.

50. A **Fraction** expresses one or more of the equal parts of a unit.

51. A **Decimal Fraction** is a fraction in which the unit is divided into tenths, hundredths, thousandths, etc.

52. Decimal fractions are written as integers are written, and follow the same general laws.

53. The **Decimal Point** separates the order of units from the order of tenths.

CHAPTER II.

ADDITION.

Article 54.

1. Count by 1's from 0 to 100.
   This is adding 1 to 0, then to 1, then to 2, etc.
2. Count by 10's from 0 to 100.
   This is the adding of 10 to 0, then to 10, etc.
3. Count by 5's from 0 to 100.
4. Count by 2's from 0 to 100.
5. Count by 2's from 1 to 101.
6. Count by 3's from 0 to 102.
7. Count by 3's from 1 to 100.
8. Count by 3's from 2 to 101.
9. Count by 4's from 0 to 100.
10. Count by 4's from 2 to 102.

55. 1. Count by 10's from 1 to 101 ; thus, 1, 21, 31, etc.
2. Count by 10's from 7 to 107.
3. Count by 20's from 6 to 106.
4. Count by 30's from 9 to 129.
5. Count by 6's from 0 to 102.

56. Accuracy and rapidity in addition can generally be secured only by practice. A few minutes given each day for several months to systematic drill is valuable ; but time may easily be wasted in such work.

The exercises in the two preceding articles are merely suggestive ; the skillful teacher will invent others.

57. Six units and four units are ten units. This may be more briefly expressed thus: $6+4=10$, which is read, 6 *plus* 4 *equals* 10. Ten is the *sum* of 6 and 4. The $+$ is the sign of addition, and is called *plus*. It may read *and*. The two parallel lines, $=$, are called the *sign of equality*, and are read *equals*, or *is equal to*, or *are*. Thus, $6+4=10$, may be read, *6 and 4 are 10.*

58. Find the sums :

1. $3+7$.
2. $6+5$.
3. $8+4$.
4. $9+6$.
5. $7+9$.
6. $3+4+5$.
7. $6+3+8$.
8. $7+5+6$.
9. $8+9+3$.
10. $5+8+7$.
11. $8+12+6$.
12. $15+3+9$.
13. $7+11+12$.
14. $24+8+5$.
15. $36+4+12$.

59. 1. A boy paid 20 cents for a book and 15 cents for a slate. How many cents did he pay for both ?

2. I sold 35 acres of land to one man, 17 acres to another, and 9 acres to a third. How many acres did I sell ?

3. If I travel 25 miles on Monday, 35 miles on Tuesday, and 48 miles on Wednesday, how many miles do I travel in those three days?

4. A man paid $48 for a suit, $7 for a pair of shoes, and $5 for a hat. How many dollars did he pay for all?

5. If a family burn 12 tons of coal in one winter, 13 tons in another, 17 tons in a third, and 9 tons in a fourth, how many tons do they burn in all?

The teacher will make and use such examples and problems, if necessary.

60. 1. Find the sum of 284+8725+7643.

PROCESS.

284
8725
7643

16652

Analysis. Write the numbers to be added so that figures of the same order shall be in the same column.

Add the figures in the column of units; write the unit figure of the sum under the column of units, and reserve the tens to be added to the column of tens.

So proceed, column by column, writing the sum of the highest column in its proper place.

Name *results* only, thus, in adding column of units, say, 3, 8, 12; in adding column of tens, say, 5, 7, 21, etc.

Find the sum in each of the following examples:

2. 375+896+6748+8496+9987+4876.
3. 18924+6768+375+4967+84969+56.
4. 758692+367+692+8764+69+68728+924.

61. 1. Find the sum of 84.25+1.379+.8+.006.

PROCESS.

84.25
1.379
.8
.006

86.435

Analysis. Write as before, placing figures of the same order in the same column. The decimal points will thus be in the same column. Add as before.

Find the sum in each of the following examples:

2. 18.75+9.64+8.625+20.+.875+.666+.0056.
3. 9.45+.8475+.0098+.15+.5+5.+.75+29.834.
4. 368.492+3.756+.00768+2.49+.3974+50.092+.009.

62. The correctness of an addition may be tested in various ways, such as the following:

1. Add the columns in the opposite direction from that in which they were added at first.
2. If the columns are long, separate the example into two or more parts; find the sum of each part; then add these several sums.
3. Add two columns at once.

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4. Find the sum of each column, or of two columns, separately. Write these sums in their proper places, or orders; then find the sum of these sums. Do this beginning with the right hand column, then with the left hand column.

1.

	Sums of single columns.		Sums of two columns.	
3784	18	25		
5962	28	26		
8473	26	28	298	276
9679	25	18	276	298
27898	27898	27898	27898	27898

Find the sum of each of the following examples, and test accuracy of the addition by one or more of the methods given above.

2. 8469+9698+36745+67923+78469+87627.

3. 59764+76925+66784+664+6892+78925+6984.

4. 84.96+765.2+7.865+73.92+99.81+7.642+873.2.

5. 72.49+83.76+99.99+81.35+73.62+49.37+82.96+45.64.

6. $45.32+$95.63+$435.+$735.67+$5.345+$78.465.

PROBLEMS.

63. 1. The area in square miles of the six New England states is as follows: Maine 33040; N. H. 9305; Vt. 9565; Mass. 8315; Conn. 4990; R. I. 1250. Find their total area.

2. The area in square miles of the following States and territories of the U. S. is as follows: Idaho 84800; Wyoming 97890; Utah 84970; Arizona 113020; New Mexico 122580. Find their total area.

3. In the year 1882, the amount of corn raised in the U. S. was as follows, expressed in thousands of bushels. What was the total amount? N. E. States 6,376; Middle States, 78,586; Southern, 394,695; Western, 1,127,935; Pacific, 2,921; Nevada, Colorado, and territories, 6,512.

4. The ordinary receipts of the treasury of the U. S. for the fiscal year ending June 30, 1886, were: From customs, $192,905,023; from internal revenue, $116,805,937; from direct taxes, $108,240; from sale of public lands, $5,630,999; miscellaneous, $20,989,528. What was the total revenue?

5. The pensions paid by the U. S. Government for the year ending June 30, 1887, were: Army, $49,376,800; Navy, $834,576; War of 1812, $1,807,056; Mexican war, $806,208. What was the total amount paid in pensions?

SUMMARY.

64. The **Sum,** or **Amount,** of two or more numbers, is the number that contains exactly as many units as the given numbers.

65. **Addition** is a process of finding the sum of two or more numbers of the same kind.

66. Only **like** numbers can be added.

Eight books and 4 books are 12 books; but 8 books and 4 knives are neither 12 books nor 12 knives.

67. **Like** numbers are said to have the *same unit value.*

68. The **Sign of Addition** is +, *plus.*

69. The **Sign of Equality** is =, *Equals.*

70. Examples in Addition, to be used at the discretion of the teacher.

Examples 1 to 26, each one line across the page.

Examples 27 to 41, eight numbers in each example of the first two sections; ten in each, of the third section.

Ex.	A.	B.	C.	D.	E.
	27	28	29	30	31
1	84965	78329	33574	95549	46752
2	77349	45783	27431	82325	73834
3	84670	49672	58962	95476	15673
4	99846	38536	98834	33893	38546
5	67084	54327	57436	49324	97433
6	98576	59864	19824	76465	82475
7	69546	35753	66723	16537	91438
8	77944	47321	73725	39564	53729
	32	33	34	35	36
9	95275	54724	39876	38436	48354
10	39452	98436	43739	91456	73689
11	84693	72984	89543	83741	35468
12	29670	54676	79736	54639	92739
13	59450	84379	84329	38457	57842
14	43729	14563	59873	83275	45678
15	80407	39600	45987	15843	54632
16	14509	48900	37654	72916	29587
	37	38	39	40	41
17	37203	49300	99321	45374	66495
18	59804	48900	48345	56736	31237
19	60406	72500	76678	98437	98359
20	59307	49800	39457	15958	17486
21	34906	37400	84329	20436	99573
22	58496	32495	69804	38945	36984
23	67374	83927	57609	48763	59673
24	59686	52639	63807	79649	67684
25	91546	48247	46705	84965	59647
26	89654	39645	59309	37649	38245

CHAPTER III.

SUBTRACTION.

Article 71.

1. Count by 1's from 10 to 0.

This is subtracting 1 from 10, then from 9, etc.

2. Count by 2's from 20 to 0.

This is the subtraction of 2 from 20, then from 18 etc.

3. Count by 2's from 21 to 1.
4. Count by 3's from 30 to 0 ; from 29 to 2 ; from 28 to 1.
5. Subtract 4 from 48 ; then from 44, and so on till nothing remains.
6. Count by 5's from 60 to 0.
7. Count by 10's from 100 to 0 ; from 88 to 8.
8. Subtract 4 from every number between 7 and 15.
9. Subtract 5 from every number between 40 and 30.

Practice should be given in the subtraction of 6, of 7, of 8, of 9, etc.

72. Since 6 units and 4 units are 10 units, 10 units less 6 units are 4 units ; or, expressed briefly, 10—6=4, which is read, 10 *minus* 6 *equals* 4.

In this example, 10 is the *minuend*, the number from which 6, the *subtrahend*, is subtracted ; 4 is the *difference* of 10 and 6, or the *remainder* when 6 has been subtracted from 10. The sign —, in 10—6, is called *minus*, and shows that the number following it is to be subtracted from the number preceding it.

73. Find the differences :

1. 12—5.
2. 15—9.
3. 13—8.
4. 21—17.
5. 23—19.
6. 45—18.
7. 35-16.
8. 42-27.

9. Having 50 dollars, I spent 17 dollars. How many dollars had I left ?

10. A journey of 82 miles was to be made in two days. If 45 miles were traveled the first day, how many remained for the second ?

Additional examples and problems to be supplied by the teacher, if needed.

74. 1. From 1578 subtract 423.

PROCESS.
1578
426
———
1152

ANALYSIS. Write the subtrahend under the minuend, figures of same order in same column. When each figure in the subtrahend is less than the corresponding figure in the minuend, find their difference and write it below in same column.

2. From 2435 subtract 1649.

PROCESS.
2435
1649
———
786

ANALYSIS. Write the numbers as above.

5 being less than 9, take 1 ten of the 3 tens, and add to the 5 units, making 15 units. 15 units less 9 units are 6 units.

The three tens of the minuend are now reduced to 2 tens, and are less than the 4 tens of the subtrahend. Take 1 hundred, that is 10 tens, of the 4 hundreds, and add to the remaining 2 tens, making 12 tens, from which take the 4 tens, leaving 8 tens. So proceed whenever necessary.

Perform the following subtractions :

3. 8456—3243.
4. 5913—2936.
5. 8097—989.
6. 5604—856.
7. 2433—1048.
8. 54809—29646.
9. 89003—20908.
10. 56824—857.
11. 40046—36759.
12. 38672—19956.

13. From 26.5 take 9.375.

PROCESS.
26.500
9.375
———
17.125

ANALYSIS. Write, as directed before, figures of the same order in same column. Since 5 tenths are equal in value to 500 thousandths, two zeros may be annexed. Subtract as in integers, placing the decimal point in the remainder between the units and the tenths.

Find the remainders in the following examples:

14. 187.59—74.65.
15. 22.024—11.56.
16. 37.56—8.6754.
17 84.—1.875.
18. 9.—.625.
19. 10—.8266.
20. $50.—$3.75.
21. $8.—$5.048.

75. Of what two numbers is the minuend the sum? Illustrate by an example.

If the subtrahend and the difference are given, how may the minuend be found?

If the minuend and the difference are given, how may the subtrahend be found?

76. To test the correctness of a subtraction, either

Find the sum of the subtrahend and the difference; this sum should equal the minuend; or,

From the minuend subtract the difference; this remainder should equal the subtrahend.

Perform the last eight examples, and test the correctness of your work by both methods.

PROBLEMS.

77. 1. The area of the Chinese Empire is 4,695,334 square miles; the area of the United States is 3,578,392, What is the excess of the Chinese Empire?

2. In the year 1885, the population of Europe was 311,694,029; that of North America was 54,566,936. What was the difference?

3. The expenditure of the Post Office Department of the U. S. was in 1790, $32,140; in 1886, $51,004,744. What was the increase?

4. The receipts of the W. U. Telegraph Co. were, for 1867, $6,568,925; for 1887, $17,191,910. What was the increase?

5. In 1790, the population of the U. S. was 3,929,214; in 1880, it was 50,155,783. What was the increase?

SUMMARY.

78. **Subtraction** is a process of finding the difference of two numbers of the same kind.

79. **Subtraction** is also a process of taking away a part of a number to find what remains.

80. The **Minuend** is the number to be diminished; or, the number with which another number is to be compared.

81. The **Subtrahend** is the number to be taken out of the Minuend; or, the number to be compared with the minuend.

82. The **Remainder,** or **Difference,** is the result obtained by subtraction. When a part is taken from a whole, this result is called the **Remainder**; when two numbers are compared, the excess of the greater is called the **Difference.**

83. The *Minuend* is the sum of the *Subtrahend* and the *Remainder*, or *Difference.*

84. The **Sign of Subtraction** is —, called *Minus.*

85. For additional examples in Subtraction, find difference of corresponding numbers in adjoining columns on page 23. Thus: 84965—78329; etc.

CHAPTER IV.

MULTIPLICATION.

Article 86.

Count by 3's to 12.

This is the addition of four 3's, and may be expressed thus : 3+3+3+3=12 ; or, thus : *four 3's are 12 ;* or, *four times 3 are 12 ;* or, *3 multiplied by 4=12.*

The last three methods are different forms of the same process, which is called *Multiplication.* It is a short process of adding *equal* numbers ; or of repeating the same number a certain number of times. The number to be taken several or more times, or to be repeated, is called the *Multiplicand.*

The number which shows how many times the multiplicand is to be taken, or repeated, is called the *Multiplier.*

The result of multiplication is called the *Product.*

In the example 4 times 3 are 12, 3 is the *multiplicand* ; 4 is the *multiplier* ; 12 is the *product.*

87. Instead of the word *times*, or of the words *multiplied by*, the sign of multiplication, ×, may be used. 4 times 3 are 12, may be expressed thus : 4×3=12 ; 4×3 cents=12 cents, is read 4 times 3 cents are 12 cents; 3 cents×4=12 cents, is read, 3 cents multiplied by 4 are 12 cents.

88. The multiplicand and multipliers are called factors (that is, *makers,*) of the product.

Make four rows of stars, three stars in each row, and show that the product of 4 and 3 is the same, whichever factor is used as the multiplicand.

89. MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

The above table should be learned perfectly. Each figure in the upper row may be regarded as either multiplicand or multiplier ; and each figure in the left hand column as the other factor. The product of any two factors is found under one factor and opposite the other. Thus, 15 is under 3 and opposite 5. 15 is also under 5 and opposite 3. The table should be learned in a manner that each factor is, in turn, multiplier and multiplicand : that 8×3=24, and 3×8=24, etc.

90. Find 5 times each of the following numbers : 308, 604, 91, 607, 906, 1205.

91. Multiply 406 by 3, by 5, by 7, by 12, by 9, by 6.

92. To multiply an integer by 10, annex to the integer one cipher, thus : 7×10=70. To multiply an integer by 100, annex two ciphers, etc.

1. Multiply by 10 each of the following numbers : 86, 95, 196, 204, 692.
2. Multiply by 100 : 9, 25, 62, 73, 824, 506.
3. Multiply by 1000 : 5, 50, 75, 498, 6730.

93. Where is the decimal point always placed ? (Art. 53.) If, in the number 8.75, the decimal point be moved one place to the right, the number becomes 87.5. Is it evident that 87.5 is *ten times* 8.75 ? If it is, what is the effect on a number of moving the decimal point one place to the right ? How may a number which is, wholly or in part, a decimal fraction, be multiplied by 10 ? How may it be multiplied by 100 ? How by 1000 ?

1. Multiply by 10 by removing the decimal point : 2.5, 1.25, .5, .75, 37.5.
2. Multiply by 100 by removing the decimal point : .25, 1.25, .625, 4.89, 5.26.
3. Multiply by 100, annexing a zero when necessary, 1.25, 7.5, 2.34, 45.6, 89.2, 67.8.

94. My hat cost 3 dollars, my coat cost six times as many dollars ; how many dollars did my coat cost ?

Observe that the product, 18 dollars, is the same kind, or *denomination*, as the multiplicand. So, 4 times 7 pints are 28 pints. This is a general truth. 3 dollars, 28 pints, etc., are *concrete* numbers; they represent objects, or substance.

Observe, also, that a multiplier never represents objects or substance. Such a number is called an *abstract* number.

95. 1. Multiply 586 by 4.

PROCESS.

586 Multiplicand.
4 Multiplier.
———
2344 Product.

ANALYSIS. For convenience, write the multiplier under the multiplicand. Multiply each figure of the multiplicand by the multiplier. 4 times 6 units are 24; write the 4 units and reserve the 2 tens to be added to the product of 4 times 8, making 34 tens; write the 4 tens, reserving the 3 hundreds to be added to the product of 4 times 5 hundreds, making 23 hundreds, which write.

2. Multiply 809 by 7.

3. Multiply 345 by 90.
Multiply by 9, and annex the 0.

4. Multiply 8456 by 30; by 70; by 90.

5. Multiply 5706 by 40; by 60; by 80.

6. Multiply 4856 by 50; by 70; by 90.

7. Multiply 7983 by 800; by 600; by 500.

8. Multiply 82.5 by 3.

When the multiplier is an integer, there are as many decimal places in the product as there are in the multiplicand.

9. Multiply $36.25 by 3; by 7; by 8.

10. Multiply 84.56 by 5; by 6; by 9.

96. 1. Multiply 738 by 146.

PROCESS.

738	Multiplicand.
146	Multiplier.
4428	Partial product, 738× 6.
2952	Partial product, 738× 40.
738	Partial product, 738×100.
107748	Entire product, 738×146.

ANALYSIS. 146=100+40+6. 738 is to be multiplied by 100, by 40, and by 6; the sum of these three partial products is the entire product. In multiplying by 40 and by 100, it is not necessary to write the 0's, since they disappear in the addition of the partial products.

Perform the following examples:

2. 8462×34.
3. 4509×56.
4. 6734×78.
5. 5973×92.
6. 459,6×327.
7. 59.63×824.
8. 7.369×526.
9. .4863×492.
10. 3695×506.
11. 4833×209.
12. 2507×704.
13. 8096×608.

14. Multiply 8400×2500.

Find product of the digits, and to this product annex as many zeros as there are zeros in both factors. 84×25=2100; 8400×2500 =21,000,000.

15. 7900×370.
16. 9200×150.
17. 280×3000.
18. 500×4820.
19. 54×68×96.
20. 62×57×82.
21. 98×59×76·
22. 35×84×69.
23. 58×37×96.
24. 375×40.
25. 375×4×10.
26. 375×8×5.
27. 375×4×2×5.
28. 375×2×2×2×5.

Why are the products in examples 24 to 28 inclusive the same number?

29. 296×64.

30. (296×41)+(296×23).

First perform the operations indicated within the parentheses.

31. 347×59.

32. (347×25)+(347×34).

Why is the product in example 31 equal to the sum of the products in example 32?

33. 483×39.

34. (483×19)+(483×20).

The correctness of a multiplication may be tested in several ways, among others,

1. By changing the order of the factors.
2. When the multiplier is a composite number (Art. 221), by multiplying the multiplicand by one of the factors of the multiplier; then multiplying the product thus obtained by a second factor of the multiplier, and so on. Examples 24 to 28, above.
3. By finding the sum of the separate products obtained by multiplying the multiplicand by numbers whose sum is the multiplier. Examples 29 to 34 above.
4. By Division. (Art. 131.)

PROBLEMS.

97. 1. Light travels about 186000 miles in one second. How many miles does it travel in 8 minutes and 18 seconds?

2. The average rate of motion of the earth in its orbit is 19 miles a second. How many miles does the earth move in 24 hours?

3. Each of the 34 columns of Girard College contains 1553 cubic feet. How many cubic inches in the 34 columns? (1728 cubic inches in one cubic foot.)

4. At freezing temperature, sound moves 1090 feet in one second. How many feet does it move in 3 hours?

5. Find the cube of 356 by using this number 3 times as a factor.

SUMMARY.

98. **Multiplication** is a short process of finding the sum of two or more equal numbers: or

Multiplication is a process of taking one number as often as there are units in another, or such part of it as another is part of unity.

99. The **Multiplicand** is one of the equal numbers to be added: or

The Multiplicand is the number to be operated on.

100. The **Multiplier** is the number which shows how many equal numbers are to be added: or

The **Multiplier** is the number which shows how often the multiplicand or what part of it is to be taken.

101. The **Product** is the result obtained by multiplication.

102. The Multiplicand and Multiplier are **Factors** of the Product.

103. The **Sign of Multiplication** is ×.

104. Annexing a zero to an integer, or, moving the decimal point one place to the right, multiplies a number by ten.

105. A **Concrete Number** is a number which represents objects or substance.

106. An **Abstract Number** is a number which does not represent any object or substance.

107. A Multiplicand may be an abstract or a concrete number.

108. A Multiplier is always an abstract number.

109. A Product has the same denomination as the Multiplicand.

110. **Examples in Multiplication, to be used at the discretion of the teacher.**

Multiply each number in column A, by the corresponding number in column B, etc. Thus : 25596×46 ; etc.

Ex.	A.	B.	C.	D.	E.	F.
1	25596	46	375482	246	483729	596
2	34689	82	459620	357	695736	679
3	49727	37	533847	483	843934	806
4	56983	95	467964	929	836719	922
5	39090	76	837656	506	547386	317
6	57692	15	390547	698	729729	198
7	84321	24	843982	404	840673	224
8	75633	67	600900	695	392866	395
9	47329	33	509080	834	246890	487
10	96694	84	630092	921	327926	336
11	38729	97	960400	367	391834	900
12	59698	98	890490	509	984692	980
13	54327	32	504900	407	375849	600
14	84692	67	820048	822	496844	460
15	37846	75	629834	374	929656	306
16	98989	39	847527	829	733784	409
17	75624	74	933824	691	954623	960
18	54698	47	675485	675	784964	490
19	39724	66	302467	834	327561	629
20	66840	82	329866	596	486489	666
21	45969	94	926481	324	726333	849
22	398452	55	89456	714	729573	375
23	560071	69	92457	527	847309	483
24	837194	84	83729	384	767513	991
25	438006	37	45674	962	983124	507
26	396451	43	56789	436	534737	832
27	889008	72	37595	793	689114	963
28	909095	91	84637	495	727353	729
29	456709	37	92646	675	546784	842
30	837245	96	82457	406	375949	965

CHAPTER V.

DIVISION.

III. 1. What is the product of 3 and 5?

2. Of what two factors is 15 the product?

3. 15 is the product of 3 and what other factor?

4. 24 is the product of 6 and what other factor? Of 8 and what other factor? Of 12 and what other factor?

Questions such as the above may be worded thus: How many times is 6 contained in 24? How many times is 8 contained in 24?, etc.

When the product of two factors and one of the factors are given, as in the above examples, the process of finding the other factor is called *Division*. The given product is called the *Dividend;* the given factor is the *Divisor;* the factor sought is the *Quotient*. In the question: How many times is 8 contained in 24?, 24 is the *dividend*, 8 is the *divisor*, the number sought, 3, is the *quotient*.

5. How many times is 9 contained in 27? In 36? In 45? In 54? In 63?

May a multiplication table, like that on page 29, be called a *Division* table? Why?

6. How many times is 8 contained in 56? In 64? In 72? In 96?

7. A dividend is 42; its divisor is 7; what is the quotient?

8. A dividend is 72; its divisor is 12; what is the quotient?

112. The **Sign of Division** is ÷. $8 \div 2 = 4$ may be read: 8 *divided* by 2 *is* 4. Division may also be indicated thus: $\frac{8}{2} = 4$, and thus: $8 : 4 = 2$.

Perform the following divisions:

1. $72 \div 6$.
2. $63 \div 9$.
3. $96 \div 7$.
4. $108 \div 9$.
5. $121 \div 11$.
6. $144 \div 12$.
7. $81 \div 9$.
8. $63 \div 7$.
9. $84 \div 7$.

113. In the foregoing examples, the dividends have been *multiples* (Art. 225) of their respective integral divisors; consequently each division has been exact; that is, each quotient has been an integer, and there has been no remainder. But in the example: Divide 7 by 2, 7 is not a multiple of 2, and the quotient is $3\frac{1}{2}$, since 7 is the product of $2 \times 3\frac{1}{2}$. So $17 \div 5 = 3\frac{2}{5}$; $19 \div 4 = 4\frac{3}{4}$, etc. Or, the division may be left incomplete, thus: $7 \div 2 = 3$, and 1 remainder; $17 \div 5 = 3$, and 2 remainder, $19 \div 4 = 4$, and 3 remainder; etc.

Name the quotient and the remainder in each of the following examples:

1. $18 \div 5$.
2. $29 \div 6$.
3. $35 \div 8$.
4. $42 \div 5$.
5. $84 \div 9$.
6. $58 \div 7$.
7. $63 \div 6$.
8. $75 \div 8$.
9. $67 \div 12$.
10. $89 \div 11$.
11. $98 \div 8$.
12. $79 \div 9$.

State the complete quotient in each of the following examples: When the dividend is not a multiple of the divisor, express the quotient as a *mixed number*. (Art. 256,) thus: $23 \div 5 = 4\frac{3}{5}$.

1. $29 \div 5$.
2. $36 \div 6$.
3. $47 \div 8$.
4. $59 \div 7$.
5. $65 \div 9$.
6. $87 \div 7$.
7. $54 \div 8$.
8. $78 \div 9$.
9. $65 \div 8$.
10. $97 \div 8$.
11. $87 \div 10$.
12. $74 \div 9$.

LONG DIVISION.

114. 1. Divide 15789 by 24.

PROCESS.

24)15789(600
14400 50
7
1389 $\frac{21}{24}$
1200 657$\frac{21}{24}$
189
168
21

ANALYSIS. Write the divisor and dividend as shown. Begin the division at the left, thus: 24 is contained in 15789, 600 times (24 ×600=14400) with 1389 remainder.

24 is contained in 1389, 50 times, (24×50=1200) with 189 remainder.

24 is contained in 189, 7 times, (24×7=168) with 21 remainder.

21 divided by 24=$\frac{21}{24}$.

The total quotient is the sum of the partial quotient, that is, 657 $\frac{21}{24}$.

It is not necessary to write the zeros, as is done above, to show the order of the significant figures. The following is the brief method usually employed:

115. PROCESS.

657$\frac{21}{24}$
24)15789
144
138
120
189
168
21

ANALYSIS. Write the divisor on the left of the dividend, as before. Select from the left of the dividend the smallest number divisible by the divisor, in this example 157. 24 is contained in 157, 6 times; write the 6 as the first figure of the quotient, over the 7, the lowest order of the number divided. Subtract 144, the product of 24×6, from 157; to the remainder, 13, annex the next figure, 8, of the dividend, making 138, the new partial dividend. So proceed till all the figures of the dividend have been thus used.

Perform the following divisions, writing the remainders as common fractions:

1. 8456÷36.
2. 9546÷42.
3. 7398÷35.
4. 6433÷59.
5. 8965÷48.
6. 15264÷75.
7. 24687÷84.
8. 31062÷69.
9. 25674÷87.
10. 40960÷92.
11. 843729÷327.
12. 956098÷452.
13. 676995÷759.
14. 985692÷675.
15. 829009÷578.

```
     657.875
 24)15789.000
    144
   ----
     138
     120
    ----
      189
      168
     ----
       210
       192
      ----
        189
        168
       ----
         120
         120
        ----
```

One advantage of writing the quotient as written above, rather than on the right of the dividend, (as in Art. 114,) is seen when the division is continued into decimals. Thus: the $\frac{21}{24}$ is found to be .875: the decimal point in the quotient being placed directly above the decimal point in the dividend.

Perform the following divisions, continuing the division to thousandths, when necessary:

16. 4964÷32.
17. 5982÷16.
18. 2598÷48.
19. 3384÷25.
20. 5970÷24.
21. 9682÷72.
22. 5874÷49.
23. 3075÷71.
24. 7842÷92.
25. 4570÷29.
26. 9009÷63.
27. 7280÷56.
28. 98027÷39.
29. 56724÷83.
30. 70029÷45.
31. 15876÷92.
32. 50600÷47.
33. 38225÷73.
34. 72929÷59.
35. 30917÷86.
36. 46511÷26.
37. 38409÷51.
38. 22497÷87.
39. 59062÷99.
40. 596784÷496.
41. 370567÷573.
42. 298405÷675.
43. 709009÷409.
44. 398462÷926.
45. 507078÷730.
46. 849602÷597.
47. 596204÷662.
48. 389426÷325.
49. 846269÷536.
50. 900972÷843.
51. 567846÷729.

SHORT DIVISION.

116. When the divisor is 12 or less, only the quotient figures are written, thus :

1.
$$\begin{array}{r} 6\overline{)8375} \\ \hline 1395\frac{5}{6} \end{array}$$

The process is identical, except that the products and partial dividends are not expressed. 6 is contained in 8 once, with remainder 2, to which 3 is annexed, making 23. 6 in 23, 3 times, with remainder 5, making the next partia dividend 57. etc.

Perform the following divisions :

2. 8469÷6.
3. 3569÷8.
4. 4675÷5.
5. 3962÷7.
6. 9654÷4.
7. 12345÷3.
8. 24596÷7.
9. 49842÷6.
10. 24836÷9.
11. 59644÷8.
12. 73425÷11.
13. 89637÷12.
14. 69765÷12.
15. 36249÷11.
16. 49737÷12.

Observe the use of the zero in the following quotients :

17.
$$\begin{array}{r} 8)4832. \\ \hline 604. \end{array}$$

18.
$$\begin{array}{r} 5)25035. \\ \hline 5007. \end{array}$$

19.
$$\begin{array}{r} 12)48725. \\ \hline 4060\frac{5}{12}. \end{array}$$

20. 4249÷7.
21. 3624÷6.
22. 8172÷9.
23. 16028÷4.
24. 40064÷8.
25. 56063÷7.
26. 30453÷ 5.
27. 66887÷11.
28. 60841÷12.

117. Division is the opposite, or converse of Multiplication. A test of the accuracy of a division is, therefore, in Multiplication. The product of the divisor and quotient is the dividend. To this product the remainder, if there is any, should be added. For instance, in example 19, above.

4060 the quotient, multiplied by
12 the divisor is

48720. Add
5 the remainder : the sum then is

48725 the dividend.

Prove examples 20 to 28, inclusive.

118. Division is a "proof" of Multiplication. Perform examples 1 to 5, (Art. 94,) and test the correctness of the work by dividing the product by either factor ; the quotient should be the other factor.

PROBLEMS.

119.

	TERMS GIVEN :	TERMS TO BE FOUND.
1.	Product, 96 ; multiplier, 12.	Multiplicand.
2.	Dividend, 84 ; divisor, 7.	Quotient.
3.	Dividend, 64 ; quotient, 8.	Divisor.
4.	Divisor, 12 ; quotient, 9.	Dividend.
5.	Multiplicand, 9; product, 72.	Multiplier.
6.	Dividend, 75 ; divisor, 6.	Quotient and remainder.
7.	Dividend, 87 ; divisor, 7.	Quotient and remainder.
8.	Divisor, 7 ; quotient, 8 ; remainder, 5.	Dividend.
9.	Divisor, 11; quotient, 12; remainder, 8.	Dividend.

10. What is the multiplier when the product equals the multiplicand ? Illustrate.
11. What is the quotient when the divisor equals the dividend ?
12. When the quotient is 1, how do the divisor and dividend compare ?
13. Dividend, 179875 ; divisor, 125 ; quotient ?
14. Dividend, 887149 ; quotient, 1789 ; divisor ?

15. The product of two numbers is 6069; one of the factors is 119; what is the other?
16. The divisor is 8061, the quotient is 946, the remainder is 484; what is the dividend?
17. The product of three numbers is 265125; the product of two of them is 525; what is the third number?
18. The product of three factors is 49248; one of the factors is 48; the second is 27; what is the third?
19. Of what number is 978 both divisor and quotient?

120. In article 91 it is shown that a number is multiplied by 10 by removing the decimal point one place to the right; etc. Since division is the converse of multiplication, a number is divided by 10 when the decimal point is moved one place to the *left*.

Thus: $37.5 \div 10 = 3.75$; $98 \div 10 = 9.8$.

Removing the decimal point two places to the left divides a number by 100; removing the decimal point three places to the left, divides by 1000; etc.

Thus: $375 \div 100 = 3.75$; $375 \div 1000 = .375$.

1. Divide 258 by 10; by 100; by 1000.
2. Divide 625 by 10; by 100; by 1000.
3. Divide 483 by 10; by 100; by 1000.
4. Divide 579000 by 10; by 100; by 1000; by 10000.

121. 1. Divide 734 by 80.

SOLUTION. $80 = 10 \times 8$. Divide 734 by 10; then divide the resulting quotient by 8. $734 \div 10 = 73$, with 4 units remainder. $73 \div 8 = 9$, with 1 ten remainder; the quotient is, therefore, 9, and the remainder $10 + 4 = 14$.

Test this method by long division:

```
80)734(9.
   720
   ---
    14
```

2. Divide 8456 by 60; by 70; by 80; by 90.
3. Divide 58349 by 400; by 500; by 700.

4. Divide 7894 by 860; by 370; by 750.
5. Divide 92079 by 3600; by 8400; by 6500.
6. Divide 36,000,000 by 9,000,000.

SOLUTION. Divide each number by 1,000,000, by erasing six zeros from the right of each. The example then is, to divide 36 by 9. Since the quotient is a *ratio*, its value depends, not on the absolute values of the dividend and divisor, but on their relative values.

7. 56000÷7000. 10. 64,000,000÷8,000,000.
8. 72000÷9000. 11. 120,000,000÷12,000,000.
9. 144,000÷12000. 12. 108,000,000÷9,000,000.

122. When a number is divided into two equal parts, each part is *one half* of the original number; *one third* of a number is one of its three equal parts, etc. *One half* is written $\frac{1}{2}$; *one third*, $\frac{1}{3}$; *one fourth*, $\frac{1}{4}$, etc.

1. Find $\frac{1}{2}$ of 24; $\frac{1}{3}$ of 27; $\frac{1}{4}$ of 32; $\frac{1}{5}$ of 60; $\frac{1}{6}$ of 42.
2. Find $\frac{2}{3}$ of 9; $\frac{3}{4}$ of 16; $\frac{4}{5}$ of 20; $\frac{3}{8}$ of 24; $\frac{5}{9}$ of 36.

Observe that $\frac{1}{2}$ of 24, or $\frac{1}{2}$x24=24÷2, etc.

SOLUTION. One third of 9 is 3, two thirds are 2 times 3.

3. Find $\frac{5}{6}$ of 42; $\frac{8}{9}$ of 72; $\frac{3}{7}$ of 28; $\frac{5}{12}$ of 60; $\frac{8}{9}$ of 81.
4. Find $\frac{3}{5}$ of 25; $\frac{8}{11}$ of 99; $\frac{5}{9}$ of 108; $\frac{3}{7}$ of 63; $\frac{7}{12}$ of 96.

In each of the following examples, first perform the operations indicated within the parentheses.

5. ($\frac{1}{5}$ of 25)+($\frac{1}{8}$ of 32.) 8. ($\frac{1}{4}$ of 48)−($\frac{1}{5}$ of 35.)
6. ($\frac{1}{4}$ of 16)+($\frac{1}{9}$ of 63.) 9. ($\frac{1}{6}$ of 72)−($\frac{1}{9}$ of 63.)
7. ($\frac{1}{7}$ of 84)+($\frac{1}{6}$ of 54.) 10. ($\frac{1}{7}$ of 49)+($\frac{1}{5}$ of 60.)

---

11. ($\frac{2}{3}$×30)+($\frac{5}{9}$×27.) 14. ($\frac{4}{7}$×84)−($\frac{3}{8}$×56.)
12. ($\frac{3}{4}$×36)−($\frac{2}{7}$×21.) 15. ($\frac{8}{9}$×108)−($\frac{4}{7}$×84.)
13. ($\frac{5}{8}$×64)−($\frac{2}{9}$×63.) 16. ($\frac{5}{6}$×42)+($\frac{4}{7}$×49.)
17. ($\frac{1}{25}$×37975)+($\frac{1}{16}$×64064.)
18. ($\frac{3}{19}$×955795)+($\frac{4}{15}$×75765.)
19. ($\frac{9}{14}$×18340)+($\frac{15}{41}$×36982.)
20. ($\frac{15}{76}$×67488)−($\frac{5}{91}$×70707.)
21. ($\frac{31}{48}$×48192)−($\frac{17}{37}$×37148.)

123.

TABLE OF ALIQUOT PARTS OF 10 AND 100.

The whole	=10	100.	The whole	=10	100.
$\frac{1}{2}$	$=5$	50.			
$\frac{1}{3}$	$=3\frac{1}{3}$	$33\frac{1}{3}$.	$\frac{2}{3}=$	$6\frac{2}{3}$	$66\frac{2}{3}$.
$\frac{1}{4}$	$=2\frac{1}{2}$	25.	$\frac{3}{4}=$	$7\frac{1}{2}$	75.
$\frac{1}{5}$	$=2$	20.	$\frac{2}{5}=$	4	40.
			$\frac{3}{5}=$	6	60.
			$\frac{4}{5}=$	8	80.
$\frac{1}{6}$	$=1\frac{2}{3}$	$16\frac{2}{3}$.	$\frac{5}{6}=$	$8\frac{1}{3}$	$83\frac{1}{3}$.
$\frac{1}{8}$	$=1\frac{1}{4}$	$12\frac{1}{2}$.	$\frac{3}{8}=$	$3\frac{3}{4}$	$37\frac{1}{2}$.
$\frac{1}{12}$		$8\frac{1}{3}$.	$\frac{5}{8}=$	$6\frac{1}{4}$	$62\frac{1}{2}$.
$\frac{1}{16}$		$6\frac{1}{4}$.	$\frac{7}{8}=$	$8\frac{3}{4}$	$87\frac{1}{2}$.

The processes of multiplication and division may sometimes be shortened when the multiplier, or the divisor is an aliquot part of 10 or 100. For instance,

1. Multiply 48 by $16\frac{2}{3}$.

SOLUTION. Since $16\frac{2}{3}$ is $\frac{1}{6}$ of 100, multiply 48 by 100, and divide the product by 6. $48\times100=4800$; $4800\div6=800$. Sometimes, as in this example, the order of these operations may be reversed, thus: $48\div6=8$; $8\times100=800$.

Perform the following examples:

2. $84\times33\frac{1}{3}$.
3. $96\times12\frac{1}{2}$.
4. $72\times\ 2\frac{1}{2}$.
5. $198\times8\frac{1}{3}$.
6. $384\times6\frac{1}{4}$.
7. $474\times1\frac{2}{3}$.

8. Multiply 56 by $37\frac{1}{2}$.

SOLUTION. Since $37\frac{1}{2}$ is $\frac{3}{8}$ of 100, take $\frac{3}{8}$ of 56, which is 21, and multiply this product by 100; $21\times100=2100$; or, especially when the multiplicand is not a multiple of the denominator of the fraction, first multiply the multiplicand by the numerator of the fraction, then multiply this product by 100, and divide the resulting product by the denominator. Thus: $56\times3\times100=16800$; $16800\div8=2100$.

9. $64\times62\frac{1}{2}$.
10. $81\times66\frac{2}{3}$.
11. $75\times83\frac{1}{3}$.
12. $92\times87\frac{1}{2}$.
13. $824\times7\frac{1}{2}$.
14. $926\times66\frac{2}{3}$.
15. $480\times83\frac{1}{3}$.
16. $573\times\ 6\frac{2}{3}$.

17. Divide 62 by $8\frac{1}{3}$.

**Since $8\frac{1}{3}$ is $\frac{1}{12}$ of 100, multiply 62 by 12, and divide the product by 100.
$62\times12=744$; $744\div100=7.44$.**

Perform the following examples :

18. $846\div33\frac{1}{3}$.
19. $918\div16\frac{2}{3}$.
20. $648\div12\frac{1}{2}$.
21. $596\div\ 6\frac{1}{4}$.
22. $7842\div16\frac{2}{3}$.
23. $5468\div12\frac{1}{2}$.
24. $4972\div\ 3\frac{1}{3}$.
25. $8964\div\ 2\frac{1}{2}$.

PROBLEMS.

124. 1. In the year 1882 Pennsylvania raised 43,518,800 bushels of corn. At an average yield of 32 bushels an acre, how many acres were planted in corn ?

2. In the same year, Illinois raised 182,336,900 bushels of corn, on 7,914,042 acres. What was the average yield per acre ? Disregard fractions.

3. The population of the United States was, in 1800, 5,308,483 ; in 1880, it was 50,155,783. What was the average annual increase ? Disregard fractions.

4. The settled area of the United States in 1880 was 1,569,570 square miles ; the population was 50,155,783. What was the average number of persons to a square mile ? Disregard fractions.

5. The population of Belgium is 5,253,821 ; its area 11373 square miles. What is the average population to the square mile ?

SUMMARY.

125. **Division** is a process of finding how many times one number is contained in another; or, Division is a process of separating a number into equal parts.

126. The **Dividend** is the number to be divided.

127. The **Divisor** is the number by which the Dividend is to be divided.

128. The **Quotient** is the result obtained by Division. It shows how many times the Divisor is contained in the Dividend.

129. The **Remainder** is that part of the Dividend remaining undivided.

130. The **Sign of Division** is ÷.

131. A Dividend is the product of its Divisor and Quotient. Division is thus the converse of Multiplication. Multiplication is the process of finding the product when its two factors are given. Division is the process of finding the other factor when the product and one of the factors are given. Division is thus a "proof" of Multiplication, and Multiplication is a proof of Division.

132. The removal of the decimal point one place to the left, divides a number by ten.

133. For additional examples in Division, use Article **110.** Divide each number in column A. by the corresponding number in column B, etc. Thus, 25596÷46 ; **etc.**

CHAPTER VI.

SOME GENERAL PRINCIPLES.

Article 134.

I. The whole is equal to the sum of all its parts.

1. A boy's library consists of 12 volumes of fiction, 8 volumes of poetry, 20 volumes of history, and 25 volumes on miscellaneous subjects. How many volumes does he have in all?

2. A regiment of 900 men are quartered as follows: 200 at A, 300 at B, the others at C. How many men are at C?

II. The greater of two numbers is equal to the less plus their difference.

III. The less of two numbers is equal to the greater, minus their difference.

3. Henry walked 23 miles, a distance 7 miles greater than John walked. How many miles did John walk?

4. The difference of the ages of Mary and Ruth is 5 years. Ruth is 12 years old; how old is Mary?

IV. The greater of two numbers is equal to one half their sum plus their difference.

$$\text{Greater} = \frac{s+d,}{2.}$$

V. The less of two numbers is equal to one half their sum less their difference.

$$\text{Less} = \frac{s-d,}{2,}$$

VI. The sum of two numbers is equal to twice the less number plus their difference. $S=2\,l+d.$

5. The sum of two numbers is 12; their difference is 2; what are the numbers?

6. The sum of two numbers is 24; their difference is 8; what are the numbers?
7. The less of two numbers is 15; their difference is 6; what are the numbers?

VII. The multiplication or division of either factor of a product, by any number, multiplies or divides the product by that number.

8. Mr. A. gave 3 pears to each of 4 boys. How many pears did he give to all? If he had given 6 pears to each boy, how many pears would he have given to all? If he had given 3 pears to each of 2 boys, how many pears would he have given to all?

VIII. The multiplication or division of a dividend by any number, multiplies or divides the quotient by that number.

9. 48 dollars are to be divided among 6 men; how many dollars will each man receive? If one half as many dollars, 24, be divided among the same number of men, how many dollars will each receive?

IX. The multiplication of a divisor by any number divides the quotient by that number; the division of a divisor by any number multiplies the quotient by that number.

10. 40 bushels of potatoes were divided equally among 4 families; how many bushels did each receive? If the same number of bushels had been divided among 8 families, how many bushels would each have received? If among two families?

X. The multiplication or division of both dividend and divisor by the same number does not change the quotient.

11. 50 dollars are divided equally among 5 men; how many dollars does each man receive? 500 dollars are divided equally among 50 men; how many dollars does each man receive? What are the two dividends? How do they compare? What are the two divisors? How do they compare? What is the quotient in each?

REVIEW PROBLEMS.

135. 1. The area of Illinois is 610 square miles greater than the area of Wisconsin. Together these two states contain 112690 square miles. What is the area of each?

2. The area of Palestine was probably 9840 square miles, which is 275 square miles more than the area of Vermont. How many square miles has Vermont?

3. Of the 444,027 immigrants who arrived in the United States in the year 1889, 153,549 came from the British Isles, 279,270 from Continental Europe, 1725 from Asia, 81 from Africa, the others from other countries. How many came from other countries?

4. The sum of 3 numbers is 847960; one of the numbers is 275406; the other two are equal; what are they?

5. The quotient is 784, the remainder is 75, the divisor is 256. What is the dividend?

6. One half the sum of two numbers is 896; one half their difference is 87. What are the numbers?

7. The product is 876158; the multiplier is 562; what is the multiplicand?

8. The quotient is $\frac{1}{3}$ of the dividend which is 968127; what is the divisor?

9. A merchant bought 125 barrels of flour at $5 a barrel, and 28 barrels of sugar at $18 a barrel. How many dollars did he pay for all?

10. Of what number is 75 both divisor and quotient ?

11. What is the product of the factors 24, 63, 36 and 40?

12. The product of four numbers is 55080 ; three of the numbers are 12, 15 and 17 ; what is the fourth number ?

13. I have $18750 with which to purchase horses at $125 each. How many horses can I buy ?

14. A gallon contains 231 cubic inches. How many gallons in a cubic foot, which contains 1728 cubic inches?

15. The polar diameter of the earth is 41,708,700 feet. There are 5280 feet in one mile ; how many miles in this diameter ?

16. The average distance of the moon from the earth is 238793 miles. If a locomotive could travel to the moon at the rate of a mile a minute, for 24 hours each day, in how many days would the journey be made ?

17. The public debt of the United States decreased from $2,773,286,173 in 1866, to $1,608,595,583 in 1889. What was the total decrease ?

18. In 1889, pensions were paid by the U. S. government to the amount of $88,275,113, to 489,725 pensioners. What was the average amount paid to each pensioner?

19. In January, 1889, the number of horses in the United States was 13,663,294. Their total value was $982,194,872. What was their average value?

20. The population of Chicago was in 1837, 4170 ; in 1890 it was 1208669. What was the gain in those 23 years ? What was the average annual increase?

CHAPTER VII.

DECIMAL FRACTIONS.

Article 136.

The fundamental principle of the Decimal System is that *Ten units of any order are equal in value to one unit of the next higher order; and one unit of any order is equal in value to ten units of the next lower order.* (Art. 30.)

137. In the notation of Decimal Fractions, Articles 39 to 45 inclusive, this principle is shown to apply to orders both on the right and on the left of the Decimal Point. It is also shown that the local value of any order is determined by its position relative to the Decimal Point.

138. The orders written on the *right* of the decimal point are called *Decimal Fractions*, or simply *Decimals.*

Articles 39 to 45 should now be carefully studied again. The pupil will see that decimal integers and decimal fractions are but parts of one system, and that the mere removal of the decimal point from left to right will change a decimal fraction to a decimal integer; while the removal of the decimal point from right to left will convert a decimal integer into a decimal fraction.

139. The relation of decimal fractions to decimal integers may be shown thus :

Decimal Integers.								Decimal Fractions.					
Millions.	Hundred-Thousands.	Ten-Thousands.	Thousands.	Hundreds.	Tens.	Units.	*Decimal Point.*	Tenths.	Hundredths.	Thousandths.	Ten-Thousandths.	Hundred-Thousandths.	Millionths.
7	6	5	4	3	2	1	.	1	2	3	4	5	6

140. Observe the change in the local value of every digit in the following numbers, by the removal of the decimal point to the left :

1234. is one thousand two hundred thirty four.

123.4 is one hundred twenty-three and *four tenths.*

12.34 is twelve and *thirty-four hundredths.*

1.234 is one and *two hundred thirty-four thousandths.*

.1234 is *one thousand two hundred thirty-four ten thousandths.*

By each successive removal of the decimal point to the left, the local value of each digit has been divided by ten.

141. Observe in the preceding article, the change in the local value of each digit in the number .1234, by the removal of the decimal point to the right. As the number .1234 becomes, by each successive removal of the decimal point, 1.234, 12.34, 123.4, and 1234., the local value of each digit has been multiplied by 10.

142. This principle can be further illustrated by the use of the cipher, thus :

12340. is twelve thousand three hundred forty.

.01234 is *one thousand two hundred thirty-four hundred thousandths.*

.001234 is *one thousand two hundred thirty-four millionths.*

143. What is the effect of annexing a cipher to the right of a decimal integer?

What is the difference in the values of 5 and 50 ?

What is the effect of prefixing a cipher to the left of a decimal integer?

What is the difference in the values of .2 and .02 ? Of .8 and .008 ?

What is the effect of annexing a cipher to the right of a decimal fraction?

What is the difference in the values of .1 and .10? Of .1 and .100? .100, which is usually read 100 *thousandths*, may be considered as 1 tenth +0 hundredths +0 thousandths.

What is the effect of prefixing a cipher to the left of a decimal fraction ? (That is, between the decimal point and the left hand figure of the fraction.)

What is the difference in the values of .1 and .01? Of .01 and .001?

READING DECIMAL FRACTIONS.

144. To read a Decimal Fraction :

Read the decimal as if it were an integer, and add the name of the lowest decimal order.

Thus : .856 is read, Eight hundred fifty six *thousandths.*

In reading a number consisting of an integer and a decimal fraction, it is customary to use the word *and* between the integral and fractional parts, and to omit it elsewhere.

Thus, 100.089 is read One hundred and eighty-nine thousandths. .189 is is read One hundred eighty nine thousandths. It is frequently an assistance to make a slight pause before pronouncing the denominator, or name, of the fraction. Thus: .600 is read Six hundred —thousandths; and .00006 is read six—hundred thousandths.

145. Read the following numbers:

1. .8.
2. .08.
3. .008.
4. .800.
5. .375.
6. 0.849.
7. 0.5076.
8. 0.3901.
9. 0.00309.
10. 0.00084.
11. 1.3024.
12. 4.0509.
13. 8.005906.
14. 6.030708.
15. 9.500083.

NOTE. The cipher on the left of the decimal point, in examples 6 to 10, is merely to indicate the absence of integers.

WRITING DECIMAL FRACTIONS.

146. 1. Write in decimal notation, one hundred twenty-five thousandths.

SOLUTION. Since *thousandths* is the third order on the right of the decimal point, three orders on its right are necessary. Write the 125 as an integer, and place the decimal point so as to make three orders on its right, thus, .125.

2. Write in decimal notation, eight ten thousandths.

SOLUTION. Ten thousandths is the fourth fractional order. Write the 8 and prefix three ciphers and the decimal point, thus: .0008.

To write a decimal fraction:

Write the decimal fraction as an integer. Prefix ciphers, if necessary, until the right hand figure occupies the order required by the denomination of the fraction. Then prefix the decimal point.

Write in decimal notation:

3. Five tenths.
4. Five hundredths.
5. 56 hundred-thousandths.
6. 118 millionths.

7. Sixteen thousandths. 10. 7 ten-millionths.
8. Nine ten-thousandths. 11. 109 hundred-millionths.
9. Five thousandths. 12. 31 billionths.
13. Twenty-six and fifteen thousandths.
14. Eighty-one and one thousand nine hundred ten-thousandths.
15. Eleven and twenty thousand nine hundred six hundred-millionths.
16. Fifteen hundred and fifteen hundred-thousandths.
17. Eighty-nine thousand hundredths.

SOLUTION. 89000 hundredths=890.00=890.

18. Fifteen thousand twenty-nine thousandths.
19. One hundred thousand three hundred sixty five ten-thousandths.
20. Six hundred fifty-eight tenths.
21. Six hundred fifty and eight tenths.

ADDITION AND SUBTRACTION OF DECIMAL FRACTIONS.

147. Since Decimal Fractions are *decimals*, their addition and subtraction involve no new principles. Study articles 60 and 61.

1. Add .65, .3264, 800. 7 and 5.84306.

PROCESS.

$$\begin{array}{r} .65 \\ .3264 \\ 800.7 \\ 5.84306 \\ \hline 807.51946 \end{array}$$

ANALYSIS. Write the numbers as in integers, placing figures of the same order in the same column. The decimal points will be in a column. Add as in integers.

Examples.

148.

Oral.

1. What is the sum of .4+ .8+ .6+ .5 ?
2. What is the sum of .08+.09+.07+.05 ?
3. What is the sum of .6+.04+.5 ?
4. What is the sum of .11+.09+.8+.25+.25 ?
5. Find the sum of .15+.10+.5+.75.
6. From .50 subtract .25.
7. From .5 subtract .15.
8. From .8 subtract .75.
9. From 1 subtract .95.
10. From 3 subtract 2.5.
11. Add .005+.007+.009.
12. Add .1+ .01+.001+.0001.
13. Add .12+.012.
14. Add .4+.15+.45.
15. From the sum of .6 and .04 subtract .55.

The character and frequency of such exercises should depend on the judgment of the teacher. The above are but suggestive.

Written.

16. Add .34, 5.86, 374.596, .98754 and 324.4.
17. Add .591, .830, 72.83, 495.6, 8.4379 and 180.25.
18. Add 62.84, .00009, 3.5679, .326506, and .40026.
19. Add 1.267, 92.8463, .09678, 5.672 and .9984686.
20. From 84.6 subtract 5.36984.

Process.
84.6
5.36984
————
79.23016

Analysis. Write the numbers as in integers. 84.6=84.60000. From this number subtract 5.36984. It is not necessary to annex ciphers to the minuend in order to make the decimal places in the minuend and subtrahend equal.

21. From 39.4567 subtract 25.9056.

22. From 5.4689 subtract .54639.
23. From 375.24 subtract 1.48783.
24. From 86.9987 subtract 9.8.
25. From .540904 subtract .00675.
26. From the sum of 15.3928 and 346.81 subtract the sum of 45.8429 and 1.268434.
27. Add $37.50, $2.75, $10.375 and $.40.
28. From $298.75 subtract $146.875.

MULTIPLICATION OF DECIMAL FRACTIONS.

149. In accordance with the fundamental principle of the Decimal system (see Articles 141 and 142), the removal of the decimal point of a number one place to the right, increases the local value of each figure of that number ten fold; that is, this removal multiplies the entire number by 10.

Thus, the number 845.6 is 10 times the number 84.56, and 100 times the number 8.456; 60 is 10 times 6, and 100 times .6

Hence, to multiply a number by 10, 100, 1000, etc.:

Remove the decimal point in the multiplicand as many places to the right, annexing ciphers when necessary, as there are ciphers in the multiplier.

150. In accordance with the same principle, a number is divided by 10, 100, 1000, etc., by removing the decimal point as many places to the *left* as there are ciphers in the divisor, *prefixing* ciphers when necessary.

Thus: 57.6÷10=5.76; 57.6÷100=.576: 896.4÷1000=.8964; .5÷10=.05; .5÷100=.005.

151. To multiply a number by .1, is to take out one tenth of it; that is, to multiply by .1 is to divide by 10: to multiply by .01 is to divide by 100; etc.

Thus: 75.×.1=75÷10=7.5; 75×.01=75÷100=.75; etc.

Therefore, to multiply a number by .1, .01, etc., which is the same as to divide it by 10, 100, etc.,

Remove the decimal point of the multiplicand as many points to the left as there are ciphers in the multiplier, prefixing ciphers when necessary.

EXAMPLES.

1. 8.954×10.
2. 37.54×100.
3. 1.925×1000.
4. 3156.×1000.
5. 48.75×10000.
6. 32.4×.1.
7. 4.56×.01.
8. 7.93×.001.
9. .532×.001.
10. 1.76×.0001.

152. 11. Multiply 69.38 by 4.

PROCESS.

$$\begin{array}{r} 69.38 \\ 4 \\ \hline 277.52 \end{array}$$

ANALYSIS Multiply as in integers. 4 times 8 hundredths are 32 hundredths, equal to 3 tenths and two hundredths. Write the 2 hundredths in hundredths' column. 4 times 3 tenths are 12 tenths, equal to 1 unit and 2 tenths; to the 2 tenths add the 3 tenths of the 32 hundredths, making 5 tenths, which write in tenths' column, reserving the 1 unit to be added to 4 times 9 units. So proceed.

Observe that when the multiplier is an integer, the number of decimal orders in the product is the same as the number of decimal orders in the multiplicand.

12. Multiply 69.38 by .4.

PROCESS.

$$\begin{array}{r} 69.38 \\ .4 \\ \hline 27.752 \end{array}$$

ANALYSIS. 69.38 multiplied by 4=277.52. But 4 is ten times .4, the given multiplier, and the product is therefore ten times as large as the required product. 277.52÷10 =27.752.

ANOTHER ANALYSIS. The multiplier .4=4×.1. Therefore, multiply the multiplicand by 4, and the resulting product, 277.52, by.1, by simply moving the decimal point one place to the left, making 27.752.

13. Multiply 3.067 by .024.

PROCESS.

3.967
.024
———
15868
7934
———
.095208

ANALYSIS. The multiplier, .024=24×.001. Therefore multiply first by 24. and then multiply the resulting product, 95.208, by .001, by simply removing the decimal point three places to the left, prefixing the necessary cipher.

153. Observe that, in these three examples, the number of decimal orders in the product is equal to the number of decimal orders in both multiplicand and multiplier. This is true in the multiplication of decimals, whether integers or fractions. Therefore, in the multiplication of decimals,

Multiply as in integers, and make the number of decimal orders in the product equal to the number of decimal orders in both factors, prefixing ciphers when necessary.

EXAMPLES.

ORAL.

154. Multiply :

1. .5 by 7.
2. .5 by .7.
3. .05 by 7.
4. .05 by .7.
5. .12 by 8.
6. 1.2 by 8.
7. 1.2 by .8.
8. 12. by .8.
9. .12 by .8.
10. .012 by .8.
11. .6 by .5.
12. .8 by .05.
13. .4 by 1.1.
14. .9 by .9.
15. 1.1 by 1.1.
16. 1.2 by .012.
17. 1.5 by 3.
18. 1.5 by .3.
19. .15 by .3.
20. .15 by .03.

155. Multiply: WRITTEN.

1. 25.96 by 48.
2. 3.857 by 29.
3. 4959. by .45.
4. 873.2 by 3.9.
5. 4.873 by .84.
6. 72.9 by .326.
7. 483 by .037.
8. 926 by .009.
9. .4853 by 7.05.
10. 27.48 by 6.73.
11. 758.67 by .00025.
12. .43626 by .00076.
13. 56.046 by .04005.
14. $4956. by .08.
15. $75.625 by .04.
16. $352.50 by .06.
17. $43.75 by .09.
18. $156.25 by .05.
19. $722.50 by .075.
20. $450.00 by .0425.

DIVISION OF DECIMAL FRACTIONS.

156. Any decimal number, whether integral or fractional, is divided by 10, 100, 1000, etc., by moving the decimal point to the left as many places as there are ciphers in the divisor. (Art. 150.)

Thus: 5976.÷10=597.6; 5976.÷100=59.76, etc.

157. A dividend and divisor may both be either multiplied or divided by the same number without changing the quotient. (Art. 134, Principle X.)

Thus: 12÷4=3; multiplying both dividend and divisor by 4, 48÷16=3; or, by dividing both terms by 4, 3÷1=3.

This general principle is used in the analysis of the following examples:

158. 1. Divide 159.63 by .3.

PROCESS.

$$\begin{array}{r} 3)\underline{1596.3} \\ 532.1 \end{array}$$

ANALYSIS. Multiply both dividend and divisor by 10 by removing the decimal point. This changes the divisor .3, to the integer, 3, without changing the quotient sought. Divide as in integers: 3 is contained in 15 hundreds 5 hundreds times; 3 is contained in 9 tens 3 tens times; 3 is contained in 6 units 2 units times; 3 is contained in 3 tenths .1 tenth times; that is, in general, *when the quotient is an integer, each quotient figure is of the same order as the right hand figure of the partial dividend divided by it.*

Therefore, since the division of the unit figure of the dividend produces the unit figure of the divisor, *when the divisor is an integer, place the decimal point in the quotient as soon as the decimal point in the dividend is reached.*

When the divisor is an integer, the number of decimal orders in the quotient is always equal to the number of such orders in the dividend.

Divide:

2. 185.495 by .5.
3. 29.7256 by .4.
4. 3.79038 by .06.
5. 285.678 by .09.
6. 3.85788 by .012.
7. 360.84 by .012.
8. 2.952 by .009.
9. 37.565 by .05.
10. 36.5827 by .007.
11. 596.375 by .05.
12. 3752.1 by .003.
13. 48064. by .08.
14. 96750. by .005.
15. 88589.6 by .11.
16. 494956 by .07.
17. 608.892 by .12.

18. Divide 17.2544 by 2.56.

Process.

$$\begin{array}{r} 6.74 \\ 256)1725.44 \\ 1536 \\ \hline 1894 \\ 1792 \\ \hline 1024 \\ 1024 \\ \hline \end{array}$$

Analysis. Multiply both dividend and divisor by 100 by removing the decimal point, in order to change the divisor to an integer. Divide as in integers, placing the decimal point in the quotient when the decimal point in the dividend has been reached.

19. Divide 28.4967 by 5.84 to four decimal orders.

PROCESS.

```
         4.8795+
584)2849.67
    2336
    ----
     5136
     4672
     ----
      4647
      4088
      ----
       5590
       5256
       ----
        3340
        2920
        ----
         420
```

ANALYSIS. Proceed as before. When the right hand figure of the dividend has been used, annex a cipher to each successive remainder until the required number of decimal orders shall appear in the quotient. The + at the end of the quotient shows that the division is incomplete.

Divide to four decimal orders:

20. 83.7456 by 84.5.
21. 7.35279 by 3.76.
22. 153.738 by 7.092.
23. 375.462 by 25.68.
24. .456308 by .3675.
25. 298.624 by 45.67.
26. Divide 4.875 by 2145 to three decimal orders.

PROCESS.

```
        0.002+
  2145)4.875
       4290
       ----
        585
```

ANALYSIS. 2145 is contained in 4 units 0 units times. Write a cipher in the quotient, and place the decimal point at its right. 2145 is contained in 48, 0 tenths times; place a cipher in the quotient in tenths' place. 2145 is contained in 487, 0 hundredths times; place a cipher in the quotient in hundredths' place. 2145 is contained in 4875, 2 thousandths times. So write.

Divide to six decimal orders:

27. 5.6724 by 379.
28. 61.8756 by 8462.
29. 365.908 by 567.
30. 75.9624 by 3758.
31. .3456 by 8469.
32. .5967 by 1375.
33. .2378 by 1564.
34. .3892 by 1875.

159. In each of the examples analyzed, the divisor has been changed to an integer unless it was already an integer. This has been done in order to make the process clear to the pupil. But it is not necessary in practice.

Division is the converse of multiplication, (Art. 131.) the dividend being the product of the divisor and quotient. In Multiplication, the number of decimal orders in the product is always equal to the number of decimal orders in both factors ; (Art. 153.) and in Division, the number of decimal orders in the dividend is always equal to the number of decimal orders in both divisor and quotient, which are the two factors of the dividend. Therefore, to divide decimal fractions:

Divide as in integers. Make as many decimal orders in the quotient as the number of decimal orders in the dividend exceed the number of decimal orders in the divisor. If necessary, prefix ciphers to the digits of the quotient.

EXAMPLES.

ORAL.

160. Divide :

1. .48 by 6.
2. .48 by .6.
3. .64 by 8.
4. .64 by .08.
5. .96 by 12.
6. .96 by 1.2.
7. .96 by .12.
8. .81 by 9.
9. 8.1 by 9.
10. 8.1 by .9
11. 72 by 8.
12. 7.2 by 8.
13. 7.2 by .8.
14. 7.2 by .08.
15. .72 by .008.
16. .072 by .008.
17. 56 by .7.
18. .56 by .7.
19. 14.4 by 1.2.
20. 1.21 by 1.1.

The teacher can determine to what extent such examples are needed.

WRITTEN.

161. 1. Divide 16.8 by 4000.

PROCESS.

4).0168

.0042

ANALYSIS. Divide each term by 1000 by erasing the three ciphers from the divisor, and removing the decimal point of the dividend three places to the left. (See Art. 150.)

Divide:

2. 5.25 by 5000.
3. .735 by 700.
4. 95.4 by 3000.
5. 75.075 by 2500.
6. 5.0407 by 7000.
7. 729.036 by 1800.

Divide:

8. 783.5 by 6.25.
9. .50312 by .19.
10. 4.2 by .3125.
11. 63 by 4000.
12. 9.45 by 1125.
13. 81.2096 by 1.28.
14. 1.29 by .8256.
15. 12.82561 by 1.505.
16. 33.66431 by 1.01.
17. .010001 by .01
18. .002475 by .066.
19. 21.13212 by 9.16.
20. 240.1 by 21.4375.
21. 12.755 by 81.632.
22. 16.275 by .41664.
23. .46565 by 31.05.
24. 2.46616 by .145.
25. 374.598 by .00009.
26. 232.655 by 375.25.
27. .000177089 by .0374.

SUMMARY.

162. The Decimal System of Notation includes Decimal Integers and Decimal Fractions.

163. The local value of any digit is determined by its position relative to the decimal point.

164. The Addition, Subtraction, Multiplication and Division of Decimal Fractions are governed by the same laws as are the corresponding operations of decimal integers.

165. The number of decimal places in any product is equal to the number of decimal places in both its factors.

166. The number of decimal places in any dividend is equal to the number of decimal places in both divisor and quotient.

PROBLEMS.

167. 1. The larger of two numbers is 7346.215; their difference is 3671.146. What is the less number?

2. The less of two numbers is 3642.1438; their difference is 4361.11189. What is the larger number?

3. The larger of two numbers, 7930.316, exceeds the less by 1845.729. Find the sum of the two numbers.

4. A farmer owned 1800 acres. He sold 139.75 acres to one neighbor, 186.5 acres to another, and 116.12 acres to a third. How many acres did he still own?

5. Find the least decimal fraction which, added to the sum of 87.43 and 169.578, will make the result an integer.

6. If a man can build .375 of a rod of fence in 1 hour, how many rods can 8 men build in 4.5 days, working 8.5 hours a day?

7. A man traveled 185.6 miles in June, 295.98 in July, and in August twice as many miles as in July. How many miles did he average per month?

8. Find 4 equal numbers whose sum is 1984.756.

9. The multiplicand is 87.92; the product is .35168; what is the multiplier?

10. The product of three factors is 78.66; two of the factors are, respectively, 6.9 and 7.125. What is the third factor?

11. At $3.75 per cord, how many cords of wood may be exchanged for 9.5 tons of coal worth $4.25 per ton ?

12. The large wheel of a bicycle is 14.36 feet in circumference. How many times will it turn in going one mile (5280 feet) ?

13. If a man earns $5.35 a day, and spends $3.625 a day, what will be the amount of his savings in 120 days ?

14. At $9.375 a ton, how many tons of hay can be bought for $140.625 ?

15. The ratio of the circumference of a circle to its diameter is 3.1415926, nearly. How many tires for wheels 4 feet in diameter can be made from 4712.3889 feet of iron, allowing nothing for welding ?

16. The French unit of linear measure, the meter, is 39.37 inches. How many meters are equal to 5905.5 yards ?

17. Mr. C. has 3.75 times as many bushels of corn as Mr. B., who has 5840 bushels. How many bushels has Mr. C?

18. What is .08 of $3940 ?

19. What is .06 of $45.50?

20. Mr. D. loaned $5000 for one year, and charged .05 of it for its use. How many dollars did he receive for its use ?

21. Twenty-five hundredths of a cargo of wheat of 6000 bushels were injured in a storm. How many bushels were injured? What was the value of the uninjured part of the cargo, at $.975 a bushel ?

22. A man pays .18 of his yearly income of $2200 for rent. How many dollars does he pay for rent?

168. Examples in Multiplication and Division of Decimals, to be used at the discretion of the teacher :

1. 84.56 × 35.
2. 9.245 × 8.7.
3. 495.7 × .09.
4. .8763 × 2.3.
5. 36.75 × .87.
6. 450.6 × .008.
7. .9763 × .125.
8. 8.379 × .326.
9. .0084 × .0096.
10. .0567 × 1.084.
11. .3004 × 8.009.
12. 376.4 × .0536.
13. .975 × 84.
14. 3.259 × 7.5.
15. .084 × .092.
16. .0029 × .063.
17. .846 × 927.
18. 3650. × .091.
19. 22.66 × .303.
20. 4.509 × .006.
21. 83.75 × 2.07.
22. 4500 × .0009.
23. .00005 × .00004.
24. .0906 × 37.54.
25. .8432 × .0025.
26. 1.6032016 ÷ 8.008.
27. .20377656 ÷ 4.1418.
28. .00421875 ÷ .0625.
29. 348648 ÷ 4800.
30. 2291.328 ÷ 21.216.
31. 108. ÷ .045.
32. 563.717 ÷ .0385.
33. .1016688 ÷ 236.
34. 187.12264 ÷ 73.72.
35. .371875 ÷ 42.5.
36. 15.7768 ÷ .037.
37. 10.9536 ÷ 456.4.
38. 1.125 ÷ 937.5.
39. 84375 ÷ 3.75.
40. .0607822 ÷ .4786.
41. .3225 ÷ 750.
42. 179.5 ÷ 8975.
43. .0063 ÷ .9.
44. 30.084 ÷ 920.
45. 15.374133 ÷ 30.27.
46. 834.84 ÷ .92760.
47. 10.201 ÷ 101.
48. 737.25 ÷ .0075.
49. 10 ÷ 10000.
50. .0607822 ÷ 47.86.

CHAPTER VIII.

UNITED STATES CURRENCY.

Article 169. **United States Money** is a decimal currency, adopted by the Federal government in 1786. It is often called *Federal Money.* The unit of the system is the *dollar.*

170. TABLE OF UNITED STATES MONEY.

10 Mills, (*m.*) - =1 Cent, (*c.*).
10 Cents, - - =1 Dime, (*d.*).
10 Dimes, or 100 Cents, =1 Dollar, ($).
10 Dollars - - =1 Eagle, (E.).

171. The coin of the United States consists of gold, silver, nickel, and bronze pieces.

172. The gold coins are the two and one-half dollar, the five dollar, the ten dollar, and the twenty dollar pieces.

The one dollar and three dollar gold pieces are no longer coined.

The silver coins are the one dollar, the half dollar, the quarter dollar, and the dime.

The three cent and five cent silver pieces are no longer coined.

The nickel coins are the five cent and the three cent pieces. The bronze coin is the one cent piece.

The copper cent and half cent, and the bronze two cent pieces, are no longer coined.

173. The gold coins are 9 parts pure gold and 1 part alloy, consisting of silver and copper. The silver coins are 9 parts pure silver and 1 part copper. The nickel coins are 75 parts copper and 25 parts nickel. The bronze coins are 95 parts copper and 5 parts zinc and tin.

174. The paper money of the United States consists of Treasury Notes, (popularly called *greenbacks,*) National Bank Notes, Gold Certificates, and Silver Certificates.

175. The sign $ denotes United States money, and is written on the left of figures denoting dollars or cents.

Thus : $ 25 is read *twenty five dollars.*
$.25 is read *twenty five cents.*
$.005 is read *five mills.*

176. Cents are hundredths of a dollar, and, when written as a fraction of a dollar, occupy the two decimal places on the right of the decimal point.

Thus: $8.75, *eight dollars seventy-five cents.*
$ .50, *fifty cents.*

But when the cent is regarded as the unit, no decimal point is necessary, as 10 cents, etc. The sign $ is then not used.

177. Mills occupy the third decimal place on the right of the decimal point.

Thus : $1.125, *one dollar twelve cents five mills,*
or, *one dollar twelve and one half cents.*
$.005, *five mills,* or, *one half cent.*

EXAMPLES.

ORAL.

178. 1. How many cents in one half a dollar ? In a quarter of a dollar ? In one eighth of a dollar ?

2. How many cents in $\frac{3}{8}$ of a dollar ? In $\frac{5}{8}$ of a dollar ? In $\frac{7}{8}$ of a dollar ? In $\frac{3}{4}$ of a dollar ?

3. How many cents in one third of a dollar ? In $\frac{2}{3}$ of a dollar ? In $\frac{4}{3}$ of a dollar ?

4. How many cents in one sixth of a dollar ? In $\frac{5}{6}$ of a dollar ? In $\frac{7}{6}$ of a dollar ?

5. What part of a dollar is 75 cents ? $12\frac{1}{2}$ cents ? $37\frac{1}{2}$ cents ? $62\frac{1}{2}$ cents ? $87\frac{1}{2}$ cents ?

6. What part of a dollar is $33\frac{1}{3}$ cents? $66\frac{2}{3}$ cents? $16\frac{2}{3}$ cents? $83\frac{1}{3}$ cents?
7. How many cents in $2\frac{1}{2}$ dollars? In $7\frac{1}{2}$ dollars?
8. How many mills in 25 cents? In $3\frac{3}{4}$ dollars?
9. At $12\frac{1}{2}$ cents each, how many slates can be bought for $5?
10. At $16\frac{2}{3}$ cents each, how many books can be bought for $8?

WRITTEN.

11. Add seven dollars seven cents; ten dollars fifty cents; three dollars thirty seven cents five mills; sixty two and one half cents; seventy five cents; eight mills; two dollars twenty five cents.
12. Add 5 cents; 20 dollars; 20 cents; 82 cents; 5 mills. 90 cents.
13. \$8.+\$45.50+\$3.75+\$.80+\$1.125+\$8.625+\$.005.
14. \$.625+\$.375+\$2.50+\$75.+\$7.50+\$.75+\$.075
15. \$845.+\$959.+\$345.+\$1.96+\$8.437+\$9.333+\$85.00.
16. From $85 subtract $.85.
17. From $3.75 subtract $.375.
18. What is the difference of $8.25 and $.625?

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19. $45.90×32.
20. $867.50×56.
21. $8.459×25.
22. $56.50×89.
23. $.75×394.
24. $2.625×400.

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25. $4000.÷80.
26. $3750.÷25.
27. $52.80÷48.
28. $45.14÷37.
29. $204.80÷64.
30. $466.08÷59.
31. $588.00÷84.
32. $890.60÷73.

179. When merchandise is bought by the hundred, or by the thousand, the hundred or the thousand is regarded as the unit, and a less quantity as a decimal fraction.

1. Find the cost of 2875 tiles at $28 a hundred.
   Solution. $28.×28.75=$805.
2. Find the cost of 5250 feet lumber at $13.50 per thousand.
   Solution. $13.50×5.250=$70.875.
3. Find the cost of 4865 feet flooring at $22 a thousand.
4. Find cost of 9824 roofing slate at $6.50 per hundred.
5. Find cost of 12500 paving stones at $8.75 per hundred.
6. Find cost of 87500 shingles at $3.75 per thousand.
7. A lumber merchant paid $13.25 per thousand for fencing boards, and sold the same for $16.50 per thousand. What was his gain on 84 thousand ?
8. A builder bought lath at 40 cents a hundred. His bill was $371. How many lath did he buy ?
9. The brick in a building cost $10875, at $7.25 a thousand. How many brick were used ?

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180. When merchandise is bought by the ton of 2000 pounds, multiply the cost of one ton by the number of pounds divided by 2000.

1. Find the cost of 8759 pounds of coal at $6 a ton.
   Solution. 8759÷2000=8.759÷2=4.3795; $6×4.3795=$26.277.
2. Find the cost of 15270 pounds of coal at $3.50 per ton.
3. Find the cost of 9584 pounds of coal at $5.75 per ton.
4. Find the cost of 13250 pounds of plaster at $5.50 per ton.
5. If a ton of coal costs $2.50 at the mine, $.90 for freight and $.50 for delivery, what does a dealer gain who sells 125800 tons of coal at $5.25 per ton ?

SIMPLE BOOK-KEEPING.

For convenience, these definitions are given here in a body; but they should be studied in connection with the illustrations given on the following pages.

181. A **Debtor** is a person, or firm, owing money.

182. A **Creditor** is a person, or firm, to whom money is owing.

183. A **Debit** is a written record of money or merchandise owing.

184. A **Credit** is a written record of payment made.

185. A **Cash Book** is a book in which an account is kept of all moneys received and paid.

Cash on hand should always equal the balance as shown in the Cash Book.

186. A **Day Book** is a book in which all debits and credits are entered.

187. A **Ledger** is a book in which each person's or firm's accounts are collected under that person's or firm's name.

188. **Posting** is the transfer of debits and credits from the Day Book to the Ledger.

189. A **Bill of Goods** is a written statement, given by a seller to a buyer, of the quantity and price of each article bought, with the total amount of the purchase, and its date.

A bill is usually copied from the Day Book, or from a *Sales Book*, if such a book is kept.

190. When the buyer pays a bill, the seller ***receipts*** **it.** **It is** then called a **receipted bill.**

The following illustrates a receipted bill:

191. Chicago, Sept. 1, 1890.

Messrs. Barry & Co.

Bought of James H. Ames.

8 bbls. Patent Flour @ $6.75	54	00
364 lbs. Bacon . . @ .08	29	12
56 boxes Soap . . @ $2.	112	00
285 lbs. Cheese . . @ .06	17	10
Received Payment	212	22

James H. Ames,
Per S. J.

192. Make out the following bills, and receipt each of them, signing your name, or initials, as acting for the seller. Make the place your own city or town ; the date, to-day.

1. William Jones, bought of Henry Robins,
   32 yds. Flannel, at 70c.
   18 yds. Muslin, at 10c.
   17 spools Cotton, at 6c.
   15 gross Buttons, at 4c.

2. Walter Lane bought of John R. Doane & Co.

22 lbs. Butter, at 25c.
8 doz. Eggs, at 14c.
16 lbs. Cheese, at 9c.
10 qts. Milk, at 4c.

3. Samuel Waters bought of Andrew Simmons,

15 lbs. Sugar, at 7c.
12 lbs. Coffee, at 22c.
5 lbs. Tea, at 40c.
12 lbs. Soap, at 8c.

4. E. C. Ray & Co. bought of Carr, Smith & Co

8940 ft. Common Lumber,	@$15 per M.
3600 ft. Flooring, - -	@ 28 " "
1800 Shingles, - - -	@ 3.50 " "
2400 Lath, - - - -	@ 0.40 " C.
300 do. - - - -	@ 0.45 " "

5. Mrs. E. M. Barlow bought of The Lehigh Mining Co

3750 lbs. S. Egg, - -	@$6.25.
4840 " Range, - -	@ 6.35.
1850 " Nut, - -	@ 6.25.
5289 " Indiana Block,	@ 3.75.

6. F. J. Brown bought of A. B. Reed & Co.

40 boxes Oranges, -	@$4.75.
13 bbls. Malaga Grapes,	@ 6.20.
69 boxes Lemons, -	@ 3.90.
80 baskets Pears, - -	@ 0.70.
78 bbls. Apples, - -	@ 2.86.

193.

DAY BOOK.

Ledger Page.	January 2, 1891.				
80	S. W. Ray, Dr.				
	To 92 bush. Potatoes 70c.			64	40
	3.				
80	Nye & Ames Dr.				
	To 12 bbl. Flour $4.			48	00
	4.				
80	S. W. Ray Cr.				
	By Cash			40	00
	5.				
80	Nye & Ames, Cr.				
	By Cash,	25	00		
	" 292 lbs. Crackers 4c.	11	68	36	68
80	Ford & Hall, Dr.				
	To Cash,			85	00

DAY BOOK.

Ledger Page.	January 12, 1891.				
80	S. W. Ray, Dr.				
	To 40 bush. onions, $1.50	60	00		
	" 10 bbl. potatoes, $2.75	27	50	87	50
	15.				
80	S. W. Ray, Cr.				
	By Cash,			76	00
	16.				
80	S. W. Ray, Dr.				
	To 17 bbls. apples, $5.			85	00
	18.				
80	Henry Swift, Cr.				
	By 580 lbs. Butter, 20c.			116	00
	20.				
80	Nye & Ames, Dr.				
	To 12 bbl. Flour, $4.50			54	00
	By Cash, Cr.			65	00

DAY BOOK.

January 21, 1891.

Ledger Page.					
80	Henry Swift,	Dr.			
	To Cash,			42	00
	22.				
80	S. W. Ray,	Dr.			
	To 950 lbs. Meal,	2c.		19	00
	23.				
80	S. W. Ray,	Cr.			
	By Cash,			50	00
	24.				
80	Henry Swift,	Dr.			
	To Cash,			28	00

Dr. Cash, January, 1891.

Jan.	1	Balance, ...	265	78
	4	S. W. Ray, . .	40	00
	5	Nye & Ames, . .	25	00
	15	S. W. Ray, . .	76	00
	20	Nye & Ames, . .	65	00
	23	S. W. Ray, . .	50	00
			521	78
Feb.	1	Balance, ...	366	78

CASH BOOK.

Cash, January, 1891. Cr.

Jan.	10	Ford & Hall . .	85	00
	21	Henry Swift . .	42	00
	24	Henry Swift . .	28	00
	31	Balance, . . .	366	78
			521	78

Dr. S. W. Ray, Cr.

1891.			Day B'k Page			1891.			Day B'k Page.		
Jan.	2	To Mdse.	75	64	40	Jan.	4	By Cash	75	40	00
	12	" "	76	87	50		15	" "	76	76	00
	16	" "	76	85	00		23	" "	77	50	00
	22	" "	77	19	00		31	" Balance		89	90
				255	90					255	90
Feb.	1	To Balance		89	90						

The balance, $89.90, is found by subtracting the sum of these credit items from the sum of the four debit items.

The entry, "Feb. 1, To Balance $89.90," shows that when the account was balanced at the end of January, Mr. Ray owed $89.90.

Under what circumstances would the balance be on the other side of the account?

On which side is the balance of each of the other accounts in the Day Book?

196. Rule paper for Cash Book, Day Book, and Ledger. Enter the following transactions in the proper books. Post from Day Book to Ledger, and balance cash account.

Remember that all cash transactions, and nothing else, are entered in the Cash Book.

Let each pupil consider that he is himself the firm whose books he is keeping.

CHICAGO, Feb. 1, 1890.

1. John J. Smith bought :
   18 tons Hay, @$8.50.
   124 bu. Oats, @ .35.
   20 bu. Bran, @ .03.

2. Andrew Perkins & Co. bought :
   15 doz. Men's Kip Boots, @$28 per doz.
   18 " Women's Goat Boots, @ 30 " "
   22 " Misses' Kid Slippers, @ 15 " "

3. George S. Todd bought :
   5500 ft. Flooring, @$24. per m.
   1480 " Fencing, @ 15. " "
   3400 Shingles, @ 3.75 " "

February 2, 1890.

4. John J. Smith paid $65 on account.

5 Brown & Robinson bought :
   160 yds. Sheeting, @$ .11.
   90 " Calico, @ .06.
   170 " Gingham, @ .13.

6. I sold for cash to Hayes & Porter :
   1 Phaeton, $285.
   1 Road Wagon, $190.

7. George S. Todd paid $100 on account.

February 9, 1890.

8. Andrew Perkins & Co. bought:
 96 pairs Women's Rubbers, @$ .48.
 Paid $250 on account.

February 20, 1890.

9. Brown & Robinson paid $85 on account.

10. Wing & Co. bought:
 5 doz. Copy Books, @$ .50 per doz.
 8 " First Readers, @ 1. " "
 6 " Arithmetics, @ 6. " "

CHAPTER IX.

FACTORS AND MULTIPLES.

Article 197.

As has been stated in Article 101, a **Product** is the result obtained by the multiplication of two or more *Factors*. Thus: 24 is the product of the two factors 3 and 8.

198. A product is a **Multiple** of each of its factors. Thus: 24 is a multiple of 3, of 4, of 6, of 8, and of 12.

199. Since a product is exactly divisible by each of its factors it is a **Composite** number. Thus: 24 is a composite number. Its factor 3 is not divisible by any integer except itself and 1, and is therefore a **Prime** number. The other factor of 24, 8, is a composite number.

The number 1 is not considered a factor.

200. Which of the following numbers are prime? Why? Which are composite? Why? 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20.

Except 2 and 5, every prime number has 1, 3, 7, or 9 for its unit's figure.

Does this mean that every number ending in 1, 3, 7, or 9, is a prime number?

201. Two or more composite numbers may have no *common* factor. Thus: 8, 9, and 25, are not divisible by the same number. Such numbers are *prime to each other.*

202. The unit figure of an **even** number is 2, 4, 6, 8, or 0.

203. The unit figure of an **odd** number is 1, 3, 5, 7, or 9.

204. All *even* numbers are divisible by 2.

205. A number, the sum of whose digits is divisible by 3, is itself divisible by 3.

1. Write the multiples of 3 in their order, beginning with 12; 12, 15, 18, 21, etc. Observe that the sum of the digits of every multiple is divisible by 3.
2. Which of the following numbers are divisible by 3? 231, 459, 311, 579, 800, 95625.

206. A number, whose two right hand figures are divisible by 4, or are ciphers, is itself divisible by 4. Thus, 1$\underline{28}$, 84$\underline{56}$, are divisible by 4, since 28 and 56 are divisible by 4.

1. Which of the following numbers are divisible by 4? 724, 538, 700, 1896, 329, 482.

207. A number whose unit figure is 0 or 5 is divisible by 5.

Study the multiples of 5, viz., 10, 15, 20, etc.

208. A number which is divisible by 2 and by 3 is also divisible by 6.

That is, all *even* numbers divisible by 3 are divisible by 6.

1. Which of the following numbers are divisible by 6? 136, 846, 530, 624, 1590, 18750.

209. A number, whose three right hand figures are divisible by 8, or are ciphers, is itself divisible by 8. Thus, 14$\underline{032}$, 51$\underline{848}$, 62$\underline{120}$, 3$\underline{000}$, are each divisible by 8.

210. A number, the sum of whose digits is divisible by 9, is itself divisible by 9.

1. Write the multiples of 9 in order: 18, 27, 36, 45, etc. What is the sum of the digits in each multiple?
2. Which of the following numbers are divisible by 9? 684, 324, 405, 1827, 1609, 1521, 5272, 8469.

3. By what factors is each of the following numbers divisible? Why? 12, 24, 15, 27, 123, 132, 321, 3285, 522, 8190.

To Resolve a Number into its Prime Factors.

211. 1. What are the prime factors of 210?

Process.

2)210

3)105

5)35

7

Analysis. Since 210 is an even number, it is divisible by the prime number 2, with the quotient 105. Since the sum of the digits of 105 is divisible by 3. the quotient 105 is divisible by 3, with the quotient 35. The factors of 35 are the prime numbers 5 and 7. Hence, the prime factors of 210 are 2, 3, 5, and 7. Therefore,

To resolve a number into its prime factors:

Divide the number by any of its prime factors; divide each succeeding quotient by any of its prime factors, until a quotient is obtained which is a prime number. The several divisors and the last quotient are the prime factors of the number.

Test the truth of this rule and the correctness of your work by finding the product of the prime factors found. What should this product equal?

Find the prime factors of each of the following numbers:

2. 36.
3. 56.
4. 81.
5. 88.
6. 115.
7. 325.
8. 428.
9. 570.
10. 1416.
11. 2502.
12. 3735.
13. 4020.
14. 3224.
15. 5696.
16. 4500.
17. 2416.

What prime factors are common to the following numbers?

18. 15 and 35.
19. 49 and 84.
20. 45 and 60.
21. 77 and 88.
22. 140 and 215.
23. 108 and 132.
24. 555 and 175.
25. 297 and 303.

COMMON FACTORS.

212. 1. What factor is common to 15 and 25? To 21 and 35? To 11 and 44?

2. What is the greatest factor common to 24 and 36? To 20 and 50? To 25 and 75?

3. What integers are divisors common to 18 and 27? Of 42 and 49? Of 9 and 45?

4. What is the greatest divisor common to 63 and 81? To 56 and 84? To 96 and 108?

The greatest integer that will exactly divide two or more numbers is their *greatest common factor*, (G. C. F.) otherwise called their *greatest common divisor* (G.C.D.)

TO FIND THE GREATEST COMMON FACTOR.

213. 1. Find the greatest common factor of 18, 27 and 45.

FIRST METHOD, BY FACTORING.

$$\begin{array}{lcr} 18=2\times3\times3 & \quad\text{or}\quad & 3\underline{)18\quad 27\quad 45} \\ 27=3\times3\times3 & & 3\underline{)\;\;6\quad\;\; 9\quad 15} \\ 45=3\times3\times5 & & 2\quad\;\; 3\quad\;\; 5 \end{array}$$

The only prime factors common to these numbers are 3 and 3. Hence, 3 ×3, that is 9, is the greatest common factor. Therefore,

To find the G. C. F. of two or more numbers:

Resolve the numbers into their prime factors. The product of the common prime factors is the greatest common factor.

Find the greatest common factor of

2. 56, 70 and 28.
3. 40, 60 and 80.
4. 15, 45 and 75.
5. 48, 72 and 96.
6. 42, 56 and 84.
7. 132 and 144.
8. 108 and 156.
9. 31 and 93.
10. 57 and 190.
11. 85 and 102.

214. The above method may be used when the given numbers are easily factored. When the numbers are larger, use the

SECOND METHOD, BY CONTINUED DIVISION.

1. Find the greatest common factor of 468 and 572.

PROCESS.

```
468)572(1
    468
   ----
    104)468(4
        416
        ----
         52)104(2
            104
            ---
```

ANALYSIS. Divide the greater number, 572, by the less, 468. Divide the less by the remainder, 104. Divide this divisor, 104, by the remainder, 52. Since this division produces no remainder, the last divisor, 52, is the G. C. F.

This method depends on the three following principles:

1. Every factor of a number is also a factor of every multiple of that number.
2. Any common factor of two numbers is also a factor of their sum, and of their difference.
3. The greatest common divisor of the difference of two numbers and of the less number, is the greatest common divisor of the two numbers.

These principles are illustrated in the above example, as follows:

52, a factor of 104, is a factor of 416, a multiple of 104. (1.)

52 is also a factor of 468, which is the sum of 416 and 52. (2.)

Since 52 is a factor of 468 and of 104, it is also a factor of 572, the sum of 468 and 104. (2.)

52 is therefore *a common factor* of 572 and 468.

52 is the *greatest common factor* of 572 and 468, since it is the greatest common factor of 468, the less of the two numbers, and 104, their difference. (3.)

To find the G. C. F. of two numbers:

Divide the greater number by the less; divide the less number by the remainder, if there be one. Continue to divide each divisor by its own remainder, until there is no remainder. The last divisor is the greatest common factor.

Find the G. C. F. of the following numbers:

2. 187 and 209.
3. 243 and 297.
4. 252 and 294.
5. 119 and 187.
6. 630 and 1134.
7. 462 and 1764.
8. 714 and 1088.
9. 759 and 1155.
10. 352, 672, and 992.

Suggestion. Find the greatest common factor of two of the numbers; then find the G. C. F. of the third number and of the G. C. F. already found.

11. 105, 231 and 1001.
12. 165, 231 and 385.
13. 1274, 2002 and 2366.
14. 816, 1360 and 4080.
15. What is the circumference of a wheel that will revolve an exact number of times in moving either 3520 yards, or 125 yards, or 440 yards ?
16. How many pecks is contained in the largest measure that can be used to measure exactly 198 pecks, 126 pecks, or 252 pecks ?
17. Three tracts of land, containing respectively 870 acres, 1479 acres, and 1827 acres, are divided into farms of equal size. What is the greatest number of acres which each farm may have ?

COMMON MULTIPLES.

215. A multiple of any number is exactly divisible by that number.

Thus, 12, 18, 24, 30, etc., are multiples of 6.

216. A **Common Multiple** of several numbers is exactly divisible by each of them.

Thus, 30 is a common multiple of 2, 3, 5, 6, 10 and 15.

1. Name three multiples of 5; of 9; of 10; of 12.
2. Of what two prime numbers is 14 a multiple ?
3. Name a common multiple of 3 and 7.
4. Of what four numbers is 42 a common multiple?
5. Name three common multiples of 2 and 3.
6. Name the *least* common multiple of 2 and 3.
7. Name the least common multiple, or L. C. M., of 3 and 5 ; of 5 and 7 ; of 2, 3 and 5.
8. What is the L. C. M. of 3, 4, and 7 ?
9. What is the L. C. M. of 5, 4, and 9 ?

TO FIND THE LEAST COMMON MULTIPLE.

217. 1. Find the least common multiple of 15, 16, and 20.

FIRST METHOD, BY FACTORING.

PROCESS.

$15=3\times5$

$16=2\times2\times2\times2$

$20=2\times2\times5$

$2\times2\times2\times2\times3\times5=240$

Since the number sought is a multiple of 15, it must contain the factors of 15, 3 and 5, as factors of itself. It must also contain the factors of 16 ; that is, 2 taken four times as a factor. The factors of 20, 2, 2, and 5 are found in the factors of 15 and 16. Therefore the L. C. M. is the product of $3\times5\times2\times2\times2\times2$, which is 240.

This process is based on the principle that the least common multiple of two or more numbers is equal to the product of all the prime factors of the numbers, and of no other factors. Therefore, To find the L. C. M. of two or more numbers:

Resolve the numbers into their prime factors. Find the product of all the prime factors, using each factor the greatest number of times that it occurs in any one of the numbers.

Find the L. C. M. of

2. 14, 24 and 28.
3. 21, 33 and 42.
4. 25, 35 and 40.
5. 27, 36 and 45.
6. 18, 25 and 40.
7. 56, 35 and 63.
8. 54, 96 and 108.
9. 44, 55 and 77.

218. When the numbers given are large, the following method may be used:

SECOND METHOD.

1. Find the least common multiple of 216, 18, 175, 25, 27, and 252.

PROCESS.

$$\begin{array}{r|rrrrrr}
2) & 216 & \cancel{18} & 175 & \cancel{25} & \cancel{28} & 252 \\ \hline
2) & 108 & & 175 & & & 126 \\ \hline
7) & 54 & & 175 & & & 63 \\ \hline
 & 54 & & 25 & & & \cancel{9}
\end{array}$$

$2 \times 2 \times 7 \times 54 \times 25 = 37800$.

ANALYSIS. Write the numbers as shown. Strike out 18, since 18 is a factor of 216; strike out 25, since 25 is a factor of 175; strike out 28, since 28 is a factor of 252. Divide by any prime number which is contained in two or more of the remaining numbers. Since two of them are even numbers, divide each by 2, writing the undivided number, 175, in line with the quotients. Divide 108 and 126 by 2, writing 175 with the quotients. Divide by 7, writing 54 with the quotients. Strike out the 9, a factor of 54.

The numbers which now remain, 54 and 25, are prime to each other. Find the product of all the divisors and the remaining numbers. This product, 37800, is the L. C. M.

Find the L. C. M. of

2. 336, 576 and 720.
3. 642, 876 and 984.
4. 810, 108 and 270.
5. 48, 65, 130 and 192.
6. 78, 156, 185 and 390.
7. 96, 216, 320, 480.
8. What is the shortest distance that may be exactly measured by a 24 inch rule, and 36 inch stick, or a 60 inch tape line ?
9. What is the smallest portion of land that may be divided into an exact number of lots containing either 64 acres, 36 acres, 100 acres, or 180 acres ?
10. How many gallons in the smallest vessel that may be exactly filled by a bucket of 9 gallons, a tub of 36 gallons, or a barrel of 52 gallons ?

CANCELLATION.

219. The division of a dividend and its divisor by the same number does not change the quotient. (Art. 134, Principle X.) Thus : $12 \div 4 = 3$; and 6, the half of 12, divided by 2, the half of 4, equals 3. Time and labor are frequently saved by cancelling equal factors from both dividend and divisor, the quotient being unchanged.

1. Divide $3\times5\times8\times27$ by $3\times5\times8\times9$.

PROCESS.

$$\frac{\cancel{3}\times\cancel{5}\times\cancel{8}\times\overset{3}{\cancel{27}}}{\cancel{3}\times\cancel{5}\times\cancel{8}\times\cancel{9}}=3$$

ANALYSIS. Write the dividend above, and the divisor below, a horizontal line. Divide each term by 3, 5, and 8, by striking out, that is, *cancelling* these factors from both dividend and divisor. Divide 27, the remaining factor of the dividend, by 9, the remaining factor of the divisor.

2. Divide $24\times18\times72\times15$ by $18\times72\times24\times5$.
3. Divide $37\times19\times23\times55$ by $11\times23\times19\times37$.

4. Divide 22×49×25×13 by 11×63×5×3.

PROCESS.

$$\frac{\overset{2}{\cancel{22}}\times\overset{7}{\cancel{49}}\times\overset{5}{\cancel{25}}\times 13}{\cancel{11}\times\underset{9}{\cancel{63}}\times\cancel{5}\times 3}=\frac{910}{27}=33\tfrac{19}{27}$$

SOLUTION. Cancel as much as possible, that is, until the factors in the dividend are prime to those of the divisor. Then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.

5. Divide 24×9×10×72 by 20×12×18.
6. Divide 225×65×640 by 26×75×32×15.
7. Divide 84×96×108×90 by 42×72×54×45.
8. Divide 250×325×490 by 125×65×70×5.
9. Divide 75×35×24×80 by 25×70×96×20.
10. Divide 56×27×48×90 by 63×35×240×18.
11. Divide 25×100×$33\frac{1}{3}$ by $12\frac{1}{2}$×$16\frac{2}{3}$×$66\frac{2}{3}$.
12. Divide 15×75×$12\frac{1}{2}$ by $7\frac{1}{2}$×$37\frac{1}{2}$×50.
13. How many barrels of flour, worth $8 a barrel, are worth as much as 24 tons of hay, at $7 a ton?
14. How many packages of butter, each package containing 57 pounds, and worth 35 cents a pound, are equal in value to 14 chests of tea, each chest containing 24 pounds, and worth 95 cents a pound?
15. How many bales of cotton, each bale containing 400 pounds, worth 16 cents a pound, should be exchanged for 32 hogsheads of molasses, each hogshead containing 120 gallons, worth 60 cents a gallon?
16. How many firkins of butter, each containing 54 pounds, worth 24 cents a pound, have the same value as 756 yards of carpet, worth 96 cents a yard.
17. What is the quotient of the product of 104 times 36 times 54 times 91, divided by 28 times 39 times 36 times 234?

SUMMARY.

220. A **Prime Number** is a number divisible only by itself and by 1.

221. A **Composite Number** is a number divisible by numbers other than itself and 1.

222. A **Factor** of a number is an exact divisor of that number.

223. A **Common Factor** of two or more numbers is a factor of each of them.

224. The **Greatest Common Factor** (G. C. F.) of two or more numbers is the greatest factor common to them.

225. A **Multiple** of a number is exactly divisible by the number of which it is a multiple.

226. A **Common Multiple** of two or more numbers is a multiple of each of them.

227. The **Least Common Multiple** (L. C. M.) of two or more numbers is the least multiple common to them.

228. **Cancellation** is the rejection of equal factors from a dividend and its divisor.

229. For examples in Factoring, find the prime factors of each number given.

For examples in G. C. F., find the G. C. F. of the numbers given.

For examples in L. C. M., find the L. C. M. of the numbers given.

For examples in Cancellation, divide the product of the numbers in the G. C. F. column by the product of the numbers in the L. C. M. column, thus: $\frac{15\times34}{8\times7}$.

	G. C. F. of	L. C. M. of		G. C. F. of	L. C. M. of
1	15, 34	8, 7	30	25, 150	75, 375
2	18, 30	10, 15	31	72, 180	48, 360
3	45, 60	15, 20	32	90, 275	60, 465
4	32, 48	16, 24	33	63, 504	81, 567
5	36, 60	27, 45	34	49, 343	35, 385
6	72, 84	24, 48	35	96, 852	60, 258
7	42, 56	21, 49	36	37, 296	74, 333
8	6, 12, 30	3, 9, 10	37	51, 340	65, 170
9	15, 18, 24	5, 9, 15	38	81, 459	27, 297
10	28, 43, 63	21, 35, 30	39	33, 484	51, 653
11	36, 60, 96	24, 36, 48	40	56, 740	72, 336
12	36, 54, 90	18, 45, 27	41	32, 256	40, 360
13	21, 42, 84	63, 24, 42	42	45, 345	54, 270
14	38, 76, 57	19, 15, 57	43	64, 548	72, 168
15	25, 50, 75	30, 40, 60	44	70, 630	50, 275
16	45, 90, 75	25, 45, 50	45	93, 426	35, 516
17	84, 36, 60	12, 20, 25	46	72, 918	48, 852
18	34, 68, 51	17, 85, 21	47	27, 543	63, 441
19	39, 52, 78	65, 26, 10	48	53, 424	14, 106
20	69, 46, 92	23, 12, 18	49	71, 497	12, 355
21	29, 31, 67	12, 62, 10	50	90, 324	75, 435
22	35, 49, 91	21, 63, 40	51	96, 488	30, 726
23	93, 78, 39	16, 11, 16	52	85, 357	34, 680
24	56, 28, 16	32, 14, 24	53	41, 451	92, 440
25	63, 45, 49	27, 25, 45	54	36, 540	96, 360
26	48, 32, 60	15, 18, 60	55	84, 420	36, 840
27	63, 96, 72	28, 35, 14	56	63, 720	28, 560
28	35, 49, 77	32, 28, 42	57	64, 512	48, 240
29	75, 60, 90	25, 50, 40	58	96, 256	56, 420

SANTA CLARA CO. TEACHERS' L. No.

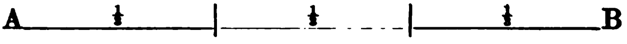
CHAPTER X.

COMMON FRACTIONS.

Article 230.

Divide anything, as the line A B, into two equal parts ; each part is *one half* of the line A B.

Divide the line A. B. into three equal



parts ; each part is *one third* of the line A B.

231. Equal parts of anything are *fractions* of that thing.
As *halves* of a dollar, *twelfths* of a foot, etc.

232. *One half* is expressed in figures thus: ½ or ½ ; *one third* thus, ⅓ or ⅓ ; *two thirds*, thus, ⅔ or ⅔.

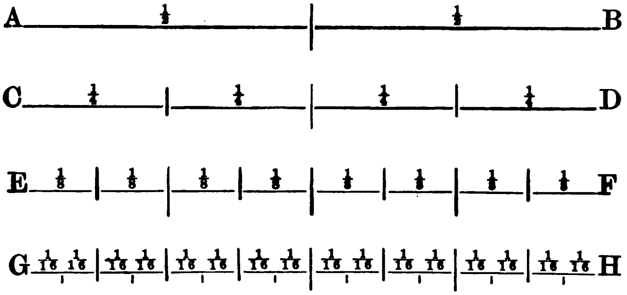
233. The figure written *under* the short line, as in ¾ or ¾, shows into how many equal parts the thing, or unit, has been divided. A figure thus written is the *denominator* of the fraction.

234. The figure written *above* the line, as in ⅓ or ⅔, shows how many of the fractional parts, such as halves, thirds, etc., compose the fraction. A figure thus written is the *numerator* of the fraction. Thus, ⅔ means *two of the three equal parts* of the same unit.

REDUCTION OF FRACTIONS.

CASE I.

235. To reduce an integer or a mixed number to an improper fraction.



1. If the line A B be divided into two equal parts, what is each part called? If a line is divided into four equal parts, what is each part called? If into eight? If into sixteen?
2. If a unit be divided into three equal parts, what is each part called? What are two of the parts called?
3. How many halves in the line A B? How many fourths in the line C D? How many eighths in the line E F? How many sixteenths in the line G H?
4. How many halves in any unit? How many thirds? How many 5ths? How many 6ths? How many 7ths?
5. How many halves in one apple? How many halves in two apples? In three? In four? In five?
6. How many thirds in two units? How many fourths, in two units? How many eighths in two units? In three units?

Such fractions as $\frac{3}{3}$, $\frac{6}{3}$, $\frac{16}{8}$, etc., are called *improper fractions.*

7. How may you find the number of eighths in *any* number? This process is called *reducing an integer to an improper fraction.*

8. State a general rule for reducing an integer to an improper fraction.

9. Reduce 4 to 4ths; to 5ths; to 6ths; to 7ths.

10. Change 8 to 3rds; 9 to 8ths; 12 to 10ths.

11. How many 5ths in 2? In $2\frac{1}{5}$? In $2\frac{3}{5}$? In $2\frac{4}{5}$?

Numbers composed of an integer and a fraction are called *mixed numbers.*

12. State a general rule for changing a mixed number to an improper fraction.

13. Reduce $8\frac{1}{3}$ to 3rds; $7\frac{3}{4}$ to 4ths; $5\frac{4}{9}$ to 9ths.

14. Reduce $6\frac{5}{12}$ to 12ths; $9\frac{7}{10}$ to 10ths; $12\frac{5}{7}$ to 7ths.

15. Change $3\frac{4}{5}$ to 5ths; $5\frac{5}{6}$ to 6ths; $4\frac{5}{8}$ to 8ths.

Reduce to improper fractions :

16. $39\frac{11}{12}$.
17. $85\frac{24}{25}$.
18. $96\frac{48}{71}$.
19. $375\frac{18}{19}$.
20. $420\frac{17}{21}$.
21. $562\frac{21}{29}$.
22. $757\frac{13}{20}$.
23. $840\frac{15}{31}$.
24. $972\frac{18}{59}$.

CASE II.

236. To reduce an improper fraction to an integer or to a mixed number.

1. How many 5ths in 1? How many units in $\frac{5}{5}$?
2. How many units in $\frac{10}{5}$? In $\frac{15}{5}$? In $\frac{20}{5}$? In $\frac{25}{5}$.
3. How many units in $\frac{6}{5}$? In $\frac{7}{5}$? In $\frac{8}{5}$? In $\frac{9}{5}$?
4. State a general rule for changing an improper fraction to an integer or to a mixed number.

5. When is the result an integer ? When is the result a mixed number ?

Reduce to integers or mixed numbers :

6. $\frac{24}{8}$; $\frac{27}{9}$; $\frac{33}{11}$.
7. $\frac{72}{12}$; $\frac{80}{7}$; $\frac{92}{10}$.
8. $\frac{54}{6}$; $\frac{39}{5}$; $\frac{62}{9}$.
9. $\frac{48}{8}$; $\frac{69}{12}$; $\frac{50}{11}$.
10. $\frac{100}{12}$; $\frac{35}{4}$; $\frac{28}{6}$.
11. $\frac{42}{5}$; $\frac{125}{12}$; $\frac{78}{11}$.
12. $\frac{66}{3}$; $\frac{108}{6}$; $\frac{90}{5}$.
13. $\frac{69}{7}$; $\frac{83}{8}$; $\frac{95}{9}$.

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14. $\frac{425}{9}$.
15. $\frac{620}{11}$.
16. $\frac{533}{17}$.
17. $\frac{489}{27}$.
18. $\frac{596}{48}$.
19. $\frac{626}{35}$.
20. $\frac{849}{40}$.
21. $\frac{758}{51}$.
22. $\frac{5780}{235}$.
23. $\frac{4872}{527}$.
24. $\frac{9800}{626}$.
25. $\frac{8463}{629}$.

CASE III.

237. To change a fraction to higher or lower terms.

1. How many 4ths in 1 ? In $\frac{1}{2}$?
2. How many 8ths in 1 ? In $\frac{1}{2}$? In $\frac{1}{4}$?
3. How many 16ths in 1 ? In $\frac{1}{2}$? In $\frac{1}{4}$? In $\frac{1}{8}$?
4. Which has the greater value, $\frac{1}{4}$ or $\frac{2}{8}$?
5. Is there any difference in the *value* of $\frac{1}{8}$ and $\frac{2}{16}$? Is there a difference in their form ?
6. How may $\frac{1}{8}$ be changed to $\frac{2}{16}$? How may $\frac{2}{16}$ be changed to $\frac{1}{8}$?

$\frac{1}{8}$ is said to be in *lower terms* than $\frac{2}{16}$, because the denominator is a smaller number. $\frac{2}{16}$ is in *higher terms* than $\frac{1}{8}$. Why ?

A fraction may be considered as an *indicated division.* Thus, $\frac{25}{5}$ means *25 divided by 5,* and may be so read. So, $\frac{1}{5}$ may be read, *1 divided by 5.*

7. In the fraction $\frac{25}{5}$, which term is the dividend? Which term is the divisor? Divide both dividend and divisor by 5 ; what is the quotient ? Does $\frac{25}{5}=\frac{5}{1}$ or 5 ?

What general principle is here illustrated ?

8. Divide each term of the fraction $\frac{8}{16}$ by 4; what is the result? Is the result equal to $\frac{8}{16}$?
9. Divide each term of the fraction $\frac{4}{8}$ by 4. Which has the greater value, $\frac{8}{16}$, $\frac{4}{8}$, or $\frac{1}{2}$?
10. Change $\frac{5}{10}$ to lower terms. Reduce $\frac{5}{15}$ to lower terms.
11. State a general rule for reducing a fraction to lower terms.
12. Reduce to lower terms $\frac{3}{12}$, $\frac{8}{24}$, $\frac{7}{28}$.
13. Reduce $\frac{7}{35}$ to 5ths; $\frac{8}{64}$ to 8ths; $\frac{9}{27}$ to 3rds.
14. How do you know what divisor to use in order to change 35ths to 5ths?
15. State a general rule for examples such as example 13.
16. Reduce $\frac{36}{48}$ to 12ths; $\frac{54}{81}$ to 9ths; $\frac{24}{60}$ to 15ths.
17. Change $\frac{49}{63}$ to 9ths; $\frac{72}{144}$ to 24ths; $\frac{25}{125}$ to 5ths.
18. Change $\frac{1}{2}$ to 10ths; to 18ths; to 24ths; to 50ths. This is reducing a fraction to higher terms. Upon what general principles does it depend?
19. State a general rule for reducing fractions to higher terms.
20. Reduce $\frac{3}{5}$ to 15ths; to 20ths; to 40ths; to 60ths.
21. Reduce $\frac{4}{9}$ to 27ths; $\frac{5}{7}$ to 56ths; $\frac{7}{12}$ to 48ths.
22. Reduce $\frac{3}{15}$ to 10ths.

    First reduce to 5ths.
23. Reduce $\frac{8}{24}$ to 15ths; $\frac{9}{36}$ to 20ths; $\frac{21}{28}$ to 16ths.

CASE IV.

238. To reduce a fraction to lowest terms.

A fraction is in lowest terms when its numerator and denominator are prime to each other.

1. Reduce $\frac{12}{27}$ to lowest terms.

   SOLUTION. Divide each term by 3: $\frac{12\div3}{27\div3}=\frac{4}{9}$.
2. Reduce $\frac{24}{36}$ to lowest terms.

SUGGESTION. Divide each term by 12, the greatest common factor of of the two terms.

3. Reduce $\frac{105}{270}$ to lowest terms.

SUGGESTION. The greatest common factor of 105 and 270 may not be evident. If so, divide by any common factor, as 3, or 5, and continue the division till the two quotients are prime to each other.

4. Why cannot $\frac{35}{81}$ be reduced to lower terms ?

239. To reduce a fraction to its lowest terms :

Divide each term of the fraction by any factor common to both; and continue the division till the terms are prime to each other.

Reduce to lowest terms :

5. $\frac{60}{72}$.
6. $\frac{81}{108}$.
7. $\frac{96}{144}$
8. $\frac{55}{75}$.
9. $\frac{49}{84}$.
10. $\frac{120}{310}$.
11. $\frac{144}{196}$.
12. $\frac{125}{225}$.
13. $\frac{81}{9}$.
14. $\frac{75}{25}$.
15. $\frac{180}{10}$.
16. $\frac{270}{90}$.
17. $\frac{210}{280}$.
18. $\frac{196}{256}$.
19. $\frac{45}{180}$.
20. $\frac{63}{315}$.

21. $\frac{12\frac{1}{2}}{100}$.
22. $\frac{16\frac{2}{3}}{100}$.
23. $\frac{33\frac{1}{3}}{100}$.
24. $\frac{12\frac{1}{2}}{50}$.
25. $\frac{66\frac{2}{3}}{100}$.
26. $\frac{16\frac{2}{3}}{50}$.
27. $\frac{33\frac{1}{3}}{66\frac{2}{3}}$.
28. $\frac{12\frac{1}{2}}{37\frac{1}{2}}$.
29. $\frac{37\frac{1}{2}}{75}$.
30. $\frac{16\frac{2}{3}}{33\frac{1}{3}}$.
31. $\frac{12\frac{1}{2}}{62\frac{1}{2}}$.
32. $\frac{37\frac{1}{2}}{100}$.
33. $\frac{87\frac{1}{2}}{100}$.
34. $\frac{12\frac{1}{2}}{87\frac{1}{2}}$.
35. $\frac{62\frac{1}{2}}{100}$.
36. $\frac{16\frac{2}{3}}{66\frac{2}{3}}$.

37. $\frac{378}{729}$.
38. $\frac{238}{336}$.
39. $\frac{654}{762}$.
40. $\frac{539}{637}$
41. $\frac{249}{612}$.
42. $\frac{321}{535}$.
43. $\frac{126}{963}$.
44. $\frac{469}{536}$.
45. $\frac{856}{936}$.
46. $\frac{374}{361}$.
47. $\frac{425}{272}$.
48. $\frac{648}{810}$.
49. $\frac{539}{1519}$.
50. $\frac{304}{1072}$.
51. $\frac{1261}{1649}$.
52. $\frac{1995}{7035}$.

240. Fractions having the same value are **equivalent** fractions.

Thus : $\frac{1}{2}$, $\frac{2}{4}$, $\frac{5}{10}$, $\frac{25}{50}$, are *equivalent* fractions.

241. Fractions having the same number for a denominator have a **Common Denominator.**

Thus, $\frac{1}{12}$, $\frac{5}{12}$, $\frac{7}{12}$, are three fractions having a *common denominator*. Such fractions are sometimes called *Similar* Fractions.

1. Change $\frac{2}{3}$ and $\frac{3}{4}$ to 12ths; to 24ths; to 36ths.

SUGGESTION To change 3rds to 12ths, multiply each term of the fraction by 4; to change 4ths to 12ths, multiply each term by 3.

2. Change $\frac{2}{5}$ and $\frac{5}{6}$ to 30ths; to 60ths.
3. Reduce $\frac{1}{2}$ and $\frac{5}{6}$ to 18ths; to 36ths; to 54ths.
4. Reduce $\frac{12}{24}$ and $\frac{1}{3}$ to 6ths; to 12ths.
5. Reduce $\frac{5}{8}$ and $\frac{16}{48}$ to 16ths; to 24ths.
6. State a general rule which will apply to the preceding examples.

CASE V.

242. To reduce several fractions to equivalent fractions having the **Least Common Denominator.**

1. Reduce $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{5}{8}$ to equivalent fractions having the least common denominator.

PROCESS.

$24 \div 3 = 8.$ $\frac{1 \times 8}{3 \times 8} = \frac{8}{24}.$

$24 \div 4 = 6.$ $\frac{3 \times 6}{4 \times 6} = \frac{18}{24}.$

$24 \div 8 = 3.$ $\frac{5 \times 3}{8 \times 3} = \frac{15}{24}.$

ANALYSIS. The least common denominator of these fractions is the L. C. M. of their denominators, which is 24. Divide this L. C. M. by the denominator of each fraction, and multiply each numerator by the quotient thus obtained.

2. Reduce $\frac{2}{5}$, $\frac{5}{6}$, $\frac{7}{10}$ to equivalent fractions having the least common denominator.
3. Reduce $\frac{15}{24}$, $\frac{12}{15}$, and $\frac{20}{25}$ to equivalent fractions having the least common denominator.

SUGGESTION. First reduce the fractions to lowest terms.

243. To reduce several fractions to equivalent fractions having the least common denominator:

Find the least common multiple of the denominators. This is the least common denominator of the fractions. Divide this denominator by the denominator of each fraction, and multiply both terms of the fraction by the quotient.

Reduce the following fractions to equivalent fractions having the least common denominator.

4. $\frac{5}{9}$, $\frac{7}{12}$.
5. $\frac{7}{8}$, $\frac{10}{14}$.
6. $\frac{3}{20}$, $\frac{4}{5}$.
7. $\frac{4}{12}$, $\frac{8}{9}$.
8. $\frac{4}{9}$, $\frac{3}{7}$.
9. $\frac{9}{16}$, $\frac{10}{12}$.
10. $\frac{3}{8}$, $\frac{1}{6}$, $\frac{5}{12}$.
11. $\frac{4}{7}$, $\frac{3}{14}$, $\frac{5}{28}$.
12. $\frac{6}{7}$, $\frac{3}{4}$, $\frac{5}{2}$.
13. $\frac{4}{5}$, $\frac{8}{15}$, $\frac{7}{30}$.
14. $\frac{3}{4}$, $\frac{8}{9}$, $\frac{2}{3}$.
15. $\frac{8}{24}$, $\frac{6}{8}$, $\frac{5}{9}$.

16. $\frac{50}{256}$, $\frac{88}{320}$.
17. $\frac{125}{180}$, $\frac{49}{280}$.
18. $\frac{29}{50}$, $\frac{31}{40}$.
19. $\frac{7}{49}$, $\frac{9}{63}$, $\frac{27}{126}$, $\frac{108}{252}$.
20. $\frac{4}{19}$, $\frac{7}{10}$, $\frac{11}{38}$, $\frac{37}{95}$.
21. $\frac{3}{17}$, $\frac{17}{51}$, $\frac{91}{255}$, $\frac{5}{3}$.

SUMMARY.

244. A **Fraction** is one or more of the equal parts of a unit, or of any number regarded as a unit.

245. The **Unit of a Fraction** is the number which has been divided into equal parts.

246. The **Fractional Unit** is one of the equal parts into which the unit of the fraction has been divided.

In "three fourths of a dollar," the *unit of the fraction* is *one dollar;* the *fractional unit* is *one fourth of a dollar.*

When no fractional unit is expressed, the unit of the fraction is the abstract unit 1.

247. A **Decimal Fraction** is a fraction in which the unit is divided into tenths, hundredths, thousandths, etc.

248. A **Common Fraction** is a fraction in which the unit is divided into any number of equal parts.

249. The **Terms** of a fraction are the *Numerator* and the *Denominator.*

250. In **Common Fractions,** the numerator is written above a short horizontal or oblique line, and the denominator below it.

251. The **Denominator** shows the number of equal parts into which the unit of the fraction is divided.

252. The **Numerator** may show how many fractional units are expressed by the fraction ; or, the **Numerator** may express the unit of the fraction.

Thus, the fraction $\frac{4}{5}$ may be considered as 4 times $\frac{1}{5}$, or, as $\frac{1}{5}$ of 4. Four fifths of one dollar are equal to one fifth of four dollars.

253. The **Value of a Fraction** is the quotient resulting from the division of the numerator by the denominator.

When the numerator equals the denominator, the value of the fraction is 1. $\frac{4}{4}=1$.

When the numerator exceeds the denominator, the value of the fraction is greater than 1. $\frac{8}{4}=2$; $\frac{9}{4}=2\frac{1}{4}$.

When the numerator is less than the denominator, the value of the fraction is less than 1. $\frac{1}{2}$ is less than 1.

254. A **Proper Fraction** is a fraction whose value is less than a unit, $\frac{1}{3}$, $\frac{15}{16}$.

255. An **Improper Fraction** is a fraction whose value is equal to, or greater than, a unit. $\frac{3}{3}$, $\frac{5}{4}$.

256. A **Mixed Number** is composed of an integer and a fraction. $8\frac{1}{3}$, 1.25.

257. A **Simple Fraction** is a fraction each of whose terms is an integer. $\frac{8}{12}$.

258. A **Complex Fraction** is a fraction having at least one of its terms fractional. $\frac{\frac{1}{2}}{5}$, $\frac{\frac{4}{2}}{3}$, $\frac{\frac{5}{6}}{\frac{7}{8}}$, $\frac{3\frac{1}{3}}{10}$, $\frac{15}{7\frac{1}{2}}$, $\frac{8\frac{1}{3}}{16\frac{2}{3}}$.

259. A **Compound Fraction** is a fraction of a fraction. $\frac{2}{3}$ of $\frac{5}{8}$; $\frac{1}{5}$ of $\frac{3}{4}$ of $\frac{2}{7}$.

260. **Equivalent Fractions** are fractions having different expression, but the same value. $\frac{2}{5}$ and $\frac{4}{10}$.

261. A **Reduction of a Fraction** is a change of its expression without a change of its value; as the change of $\frac{1}{2}$ to $\frac{3}{6}$; the change of $\frac{15}{20}$ to $\frac{3}{4}$.

262. A fraction is in its **Lowest Terms** when its terms are prime to each other, as $\frac{15}{16}$, $\frac{24}{49}$.

General Principles.

263. The terms of a fraction bear to each other the relation of dividend and divisor; that is,

The *Numerator* is the *Dividend.*

The *Denominator* is the *Divisor.*

The *Value of the Fraction,* that is the fraction itself, is the *Quotient.*

Hence, the principles of Division, (Art. 134, VIII, IX, X.) are applicable to Fractions, thus :

I. The multiplication or division of a numerator by any number, multiplies or divides the fraction by that number.

II. The multiplication of a denominator by any number divides, and the division of a denominator multiplies, the fraction by that number.

III. The multiplication or division of both terms of a fraction by the same number does not change the value of the fraction.

Addition.

264. 1. Find the sum of $\frac{2}{9}$ and $\frac{5}{9}$.

Solution. The sum of two *ninths* and 5 *ninths* is 7 *ninths.*

2. Find the sum of $\frac{4}{7}$, $\frac{5}{7}$, and $\frac{6}{7}$.

Solution. The sum of $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$, is $\frac{15}{7}$, or $2\frac{1}{7}$.

3. Find the sum of $\frac{3}{5}$, $\frac{5}{8}$, and $\frac{3}{4}$.

PROCESS.

$\frac{3}{5}=\frac{24}{40}$

$\frac{5}{8}=\frac{25}{40}$

$\frac{3}{4}=\frac{30}{40}$

ANALYSIS. In order that fractions may be added, they must be reduced to equivalent fractions having a common denominator. (Art 242.) Reducing the fractions thus, divide the sum of their numerators by the common denominator. Hence,

$\frac{24}{40}+\frac{25}{40}+\frac{30}{40}=\frac{79}{40}=1\frac{39}{40}$.

To add two or more fractions.

265. *Reduce the fractions to equivalent fractions having a common denominator; find the sum of the numerators; divide this sum by the common denominator.*

NOTE I. It is usual to reduce the fractions to their least common denominator.

NOTE II. Results should be reduced to their simplest forms.

Find the sums of the following fractions:

4. $\frac{5}{16}+\frac{9}{16}$. 9. $\frac{13}{29}+\frac{18}{29}$. 14. $\frac{1}{2}+\frac{2}{3}+\frac{5}{6}$.
5. $\frac{11}{12}+\frac{7}{12}$. 10. $\frac{4}{9}+\frac{8}{11}$. 15. $\frac{3}{8}+\frac{5}{12}+\frac{3}{4}$.
6. $\frac{8}{15}+\frac{7}{15}$. 11. $\frac{5}{8}+\frac{7}{12}$. 16. $\frac{5}{7}+\frac{3}{5}+\frac{1}{2}$.
7. $\frac{12}{17}+\frac{8}{17}$. 12. $\frac{9}{10}+\frac{5}{6}$. 17. $\frac{4}{9}+\frac{2}{3}+\frac{5}{6}$.
8. $\frac{2}{9}+\frac{5}{9}$. 13. $\frac{7}{12}+\frac{3}{5}$. 18. $\frac{4}{11}+\frac{8}{12}+\frac{2}{3}$.

Find the sums of the following mixed numbers. Add the integers and fractions separately, then find the sum of these results:

19. $8\frac{1}{2}+9\frac{2}{5}$. 21. $6\frac{1}{7}+3\frac{4}{11}$. 23. $2\frac{3}{15}+4\frac{1}{5}+1\frac{5}{6}$.
20. $7\frac{3}{8}+5\frac{4}{9}$. 22. $3\frac{4}{9}+5\frac{1}{3}+2\frac{7}{27}$. 24. $5\frac{7}{12}+2\frac{3}{4}+3\frac{11}{24}$.

WRITTEN.

25. $\frac{7}{15}+\frac{11}{12}+\frac{9}{10}+\frac{5}{6}$. 30. $19\frac{5}{17}+8\frac{13}{51}+2\frac{2}{3}$.
26. $\frac{8}{9}+\frac{19}{36}+\frac{29}{72}+\frac{11}{18}$. 31. $8\frac{14}{39}+7\frac{2}{13}+5\frac{15}{78}$.
27. $\frac{12}{27}+\frac{25}{54}+\frac{31}{108}+\frac{11}{12}$. 32. $10\frac{5}{56}+3\frac{11}{28}+4\frac{11}{63}$.
28. $\frac{21}{32}+\frac{17}{96}+\frac{7}{8}+\frac{91}{192}$. 33. $4\frac{11}{27}+6\frac{24}{45}+7\frac{13}{72}$.
29. $\frac{55}{72}+\frac{29}{144}+\frac{31}{36}+\frac{41}{48}$. 34. $5\frac{25}{66}+11\frac{15}{54}+9\frac{17}{42}$.

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35. $\frac{16\frac{2}{3}}{100}+\frac{83\frac{1}{3}}{100}+\frac{25}{100}+\frac{12\frac{1}{2}}{50}+\frac{37\frac{1}{2}}{75}$.
36. $\frac{62\frac{1}{2}}{100}+\frac{37\frac{1}{2}}{100}+\frac{66\frac{2}{3}}{100}+\frac{33\frac{1}{3}}{100}+\frac{6\frac{1}{4}}{25}+\frac{18\frac{3}{4}}{75}$.

SUBTRACTION.

266. 1. Find the difference of $\frac{8}{11}$ and $\frac{5}{11}$.

SOLUTION. The difierence of 8 *elevenths* minus 5 *elevenths*, is 3 *elevenths*.

2. Find the difference of $\frac{5}{12}$ and $\frac{2}{5}$.

SOLUTION. $\frac{5}{12}=\frac{25}{60}$, and $\frac{2}{5}=\frac{24}{60}$. $\frac{25}{60}-\frac{24}{60}=\frac{1}{60}$.

267. To find the difference of two fractions :

Reduce the fractions to equivalent fractions having a common denominator ; find the difference of their numerators ; divide this difference by the common denominator.

Find the differences :

3. $\frac{5}{6}-\frac{3}{8}$.
4. $\frac{5}{16}-\frac{7}{64}$.
5. $\frac{24}{35}-\frac{3}{10}$.
6. $\frac{11}{36}-\frac{5}{48}$.
7. $\frac{19}{24}-\frac{3}{20}$.
8. $\frac{9}{14}-\frac{1}{2}$.
9. $\frac{7}{15}-\frac{5}{12}$.
10. $\frac{9}{20}-\frac{11}{25}$.
11. $\frac{15}{16}-\frac{2}{4}$.
12. $\frac{51}{72}-\frac{4}{5}$.
13. $\frac{4}{9}-\frac{3}{7}$.
14. $\frac{15}{24}-\frac{5}{82}$.
15. From $8\frac{1}{2}$ take $5\frac{7}{9}$.

PROCESS.

$$8\tfrac{1}{2}=8\tfrac{9}{18}=7\tfrac{27}{18}$$
$$5\tfrac{7}{9}=5\tfrac{14}{18}=5\tfrac{14}{18}$$
$$\overline{2\tfrac{13}{18}}$$

ANALYSIS. Reduce the fractions to equivalent fractions having a common denominator. Since $\frac{9}{18}$ is less than $\frac{14}{18}$, take $1=\frac{18}{18}$, from the 8 of the minuend, and add it to the $\frac{9}{18}$, then from $7\frac{27}{18}$ take $5\frac{14}{18}$.

16. From $9\frac{2}{5}$ take $5\frac{11}{12}$.
17. From $12\frac{7}{10}$ take $3\frac{4}{5}$.
18. From $7\frac{3}{8}$ take $5\frac{4}{9}$.
19. From $10\frac{5}{11}$ take $3\frac{1}{10}$.
20. From $12\frac{4}{7}$ take $1\frac{5}{12}$.
21. From $13\frac{5}{6}$ take $6\frac{4}{15}$.

WRITTEN.

22. $28\frac{16}{63}-13\frac{11}{14}$.
23. $19\frac{17}{216}-15\frac{19}{54}$.
24. $84\frac{21}{23}-72\frac{15}{22}$.
25. $15\frac{48}{61}-11\frac{4}{9}$.
26. $45\frac{62}{93}-29\frac{105}{124}$.
27. $26\frac{19}{56}-16\frac{69}{168}$.

ADDITION AND SUBTRACTION.

28. $8\frac{1}{3}+5\frac{5}{6}-3\frac{7}{24}+6\frac{3}{4}-7\frac{3}{8}$.

First perform the operations indicated in the parentheses.

29. $59\frac{2}{5}-(8\frac{3}{10}+5\frac{7}{15}+18\frac{4}{5})$.

30. $40\frac{3}{7}-(5\frac{2}{3}+6\frac{4}{5}-2\frac{5}{14})$.

31. $25-(6\frac{5}{9}-2\frac{2}{3}-1\frac{6}{7})$.

32. $17\frac{5}{16}+3\frac{7}{8}-(4\frac{2}{3}+5\frac{3}{4}-2\frac{5}{6}.)$

33. $59\frac{2}{9}-(4\frac{1}{3}+7\frac{2}{3}+8\frac{2}{21})+6\frac{7}{9}$.

34. $27\frac{4}{5}-15\frac{3}{4}+5\frac{3}{8}-2\frac{7}{10}+5\frac{31}{40}$.

MULTIPLICATION.

CASE I.

268. To multiply a fraction by an integer.

1. Multiply $\frac{2}{3}$ by 5.

SOLUTION. 5 times 2 *thirds* are 10 *thirds*$=3\frac{1}{3}$; $\frac{2\times5}{3}=\frac{10}{3}=3\frac{1}{3}$. The product is always the same denomination as the multiplicand. (Art. 109)

2. Multiply $\frac{4}{15}$ by 5.

SOLUTION. By the preceding method, $\frac{4\times5}{15}=\frac{20}{15}=\frac{4}{3}=1\frac{1}{3}$. But instead of taking 5 times as many fractional units, that is 20 *fifteenths*, the same result is obtained by taking the same number of fractional units, each 5 times as large; that is, $\frac{4\times5}{15}=\frac{4}{15\div5}=\frac{4}{3}=1\frac{1}{3}$.

3. Multiply $\frac{4}{25}\times35$.

PROCESS.

$$\frac{4\times\overset{7}{\cancel{35}}}{\underset{5}{\cancel{25}}}=\frac{28}{5}=5\frac{3}{5}.$$

ANALYSIS. Indicate the multiplication of the numerator by the integer, $\frac{4\times35}{25}$, and cancel 5, the factor common to the denominator 25 and the integer 35. Then perform the indicated operations. Hence,

269. To multiply a fraction by an integer :

I. *Multiply the numerator of the fraction by the integer, and divide this product by the denominator.*

If the integer is a factor of the denominator,

II. *Divide the denominator of the fraction by the integer; then divide the numerator of the fraction by this quotient.*

If the denominator and the integer have a common factor, or factors,

III. *Cancel the factor or factors, common to the denominator of the fraction and the integer; find the product of the numerator and the remaining factor of the integer, and divide this product by the remaining factor of the denominator.*

These three rules are all included in this general

270. RULE. *Multiply the numerator of the fraction*, or, *Divide the denominator, by the integer.*

ORAL.

4. Multiply $\frac{3}{4}$ by 5; by 7; by 9; by 11; by 4.
5. Multiply $\frac{5}{9}$ by 2; by 4; by 5; by 7; by 9.
6. Multiply $\frac{7}{12}$ by 2; by 3; by 4; by 6; by 12.
7. Multiply $\frac{5}{18}$ by 2; by 3; by 6; by 9; by 18.
8. Multiply $\frac{5}{12}$ by 8; by 10; by 15; by 14; by 9.
9. Multiply $\frac{7}{24}$ by 10; by 15; by 18; by 20; by 36.
10. Multiply $\frac{9}{16}$ by 5; by 8; by 12; by 24; by 30.
11. Multiply $\frac{8}{15}$ by 4; by 5; by 30; by 35; by 40.
12. Multiply $7\frac{1}{2}$ 4; by $3\frac{1}{3}$ by 3; $8\frac{1}{3}$ by 6.

PROCESS. $7\frac{1}{2}\times4=(7\times4=28)+(\frac{1}{2}\times4=2)=30$

13. Multiply $16\frac{2}{3}$ by 6; $33\frac{1}{3}$ by 3; $12\frac{1}{2}$ by 8.
14. Multiply $37\frac{1}{2}$ by 2; $6\frac{1}{4}$ by 8; $8\frac{3}{4}$ by 8.

Find the products:

15. $5\frac{3}{4}\times6$.
16. $3\frac{5}{8}\times9$.
17. $4\frac{11}{12}\times7$.
18. $6\frac{5}{9}\times8$.
19. $7\frac{3}{10}\times9$.
20. $8\frac{5}{12}\times12$.
21. $9\frac{8}{15}\times7$.
22. $12\frac{4}{11}\times9$.
23. $15\frac{7}{16}\times10$.
24. $11\frac{3}{7}\times12$.
25. $9\frac{7}{8}\times11$.
26. $6\frac{15}{16}\times12$.
27. $8\frac{10}{21}\times5$.
28. $9\frac{12}{25}\times9$.
29. $7\frac{9}{40}\times10$.

WRITTEN.

30. $\frac{21}{40}\times29$.	34. $\frac{85}{144}\times48$.	38. $\frac{149}{360}\times144$.
31. $\frac{46}{91}\times28$.	35. $\frac{76}{125}\times75$.	39. $\frac{225}{375}\times125$.
32. $\frac{32}{51}\times42$.	36. $\frac{49}{210}\times70$.	40. $\frac{219}{213}\times71$.
33. $\frac{91}{160}\times73$.	37. $\frac{56}{108}\times54$.	41. $\frac{347}{600}\times150$.

42. Multiply $85\frac{41}{70}$ by 6.

PROCESS.

$$\begin{array}{r} 85\frac{41}{70} \\ 6 \\ \hline 70)246 \\ \hline 3\frac{36}{70} \\ 510 \\ \hline 513\frac{18}{35} \end{array}$$

ANALYSIS. Multiply 41 by 6, and divide the product by 70; this $3\frac{36}{70}$ is the product of $\frac{41}{70}\times6$. To this add the product of 85×6, reducing the $\frac{36}{70}$ to lowest terms.

Find the following products:

43. $249\frac{18}{35}\times7$.	45. $321\frac{15}{16}\times12$.	47. $315\frac{14}{61}\times41$.
44. $145\frac{27}{50}\times8$.	46. $224\frac{25}{28}\times30$.	48. $415\frac{45}{52}\times50$.

CASE II.

271. To multiply an integer by a fraction.

1. Multiply 9 by $\frac{3}{4}$.

SOLUTION. It is required to take $\frac{3}{4}$ of 9. $\frac{1}{4}$ of 9 is $9\div4$, or $2\frac{1}{4}$; $\frac{3}{4}$ of $9=2\frac{1}{4}\times3=6\frac{3}{4}$. Observe that the result is the same by Case I. $\frac{3}{4}\times9=\frac{27}{4}=6\frac{3}{4}$. Hence,

272. To multiply an integer by a fraction:

Multiply the numerator by the integer, and divide the product by the denominator of the fraction.

Since the product of two factors is the same, which ever is used as the multiplier. Cases I and II are identical, so far as the process is concerned. But since the denomination of the product is always that of the multiplicand, case II is required.

2. Multiply 96 by $\frac{11}{12}$.

Solution. $\frac{1}{12}$ of 96 is 8; $\frac{11}{12}$ of 96 are 11 times 8, which is 88.

Note. When the integer is a multiple of the denominator, this method is preferable. That is,

Divide the integer by the denominator, and multiply the quotient by the numerator of the fraction.

3. Multiply 72 by $\frac{5}{9}$; by $\frac{7}{12}$; by $\frac{3}{8}$; by $\frac{1}{24}$.
4. Find $\frac{4}{5}$ of 40; $\frac{7}{8}$ of 64; $\frac{5}{6}$ of 66; $\frac{4}{11}$ of 121.
5. Find $\frac{5}{9}$ of 108; $\frac{9}{5}$ of 45; $\frac{6}{7}$ of 49; $\frac{10}{7}$ of 63.
6. Multiply 8 by $\frac{11}{32}$; by $\frac{7}{64}$; by $\frac{5}{96}$; by $\frac{9}{16}$.
7. Multiply 5 by $9\frac{2}{5}$.
8. Multiply 8 by $6\frac{2}{3}$.
9. Multiply 5 by $12\frac{3}{7}$.
10. Multiply 9 by $8\frac{5}{11}$.

Written.

11. Multiply 372 by $\frac{7}{12}$.

Process.

$$\begin{array}{r} 12)\underline{372} \\ 31 \\ \underline{7} \\ 217 \end{array}$$

Analysis. $\frac{1}{12}$ of $372 = 372 \div 12 = 31$. $\frac{7}{12}$ of $372 = 7$ times $31 = 217$.

12. Multiply 625 by $\frac{21}{25}$.
13. Multiply 1728 by $\frac{33}{48}$.
14. Multiply 407 by $\frac{13}{39}$.
15. Multiply 1596 by $\frac{17}{133}$.
16. Multiply 1908 by $\frac{59}{106}$.
17. Multiply 2592 by $\frac{71}{144}$.
18. Multiply 754 by $\frac{11}{15}$.

Process.

$$\begin{array}{r} 754 \\ \underline{11} \\ 15)\underline{8294} \\ 552\frac{14}{15} \end{array}$$

Analysis. $\frac{11}{15}$ of 754 is equal to $\frac{1}{15}$ of 11 times 754.

19. Multiply 379 by $\frac{7}{25}$. 23. Multiply 689 by $32\frac{16}{25}$.
20. Multiply 596 by $\frac{24}{31}$. 24. Multiply 728 by $49\frac{25}{95}$.
21. Multiply 783 by $\frac{16}{29}$. 25. Multiply 975 by $63\frac{46}{67}$.
22. Multiply 849 by $\frac{25}{26}$. 26. Multiply 837 by $159\frac{17}{31}$.

CASE III.

273. To multiply a fraction by a fraction.

1. Multiply $\frac{3}{4}$ by $\frac{5}{7}$.

SOLUTION. This is equivalent to: Find $\frac{5}{7}$ of $\frac{3}{4}$. $\frac{1}{7}$ of $\frac{3}{4}$ is $\frac{3}{28}$; $\frac{5}{7}$ are 5 times $\frac{1}{7}$ of $\frac{3}{4}$, or $5\times\frac{3}{28}=\frac{15}{28}$. Hence,

274. To multiply a fraction by a fraction:

Divide the product of the numerators by the product of the denominators.

2. Multiply $\frac{2}{3}$ by $\frac{5}{11}$. 5. Multiply $\frac{6}{7}$ by $\frac{11}{12}$.
3. Multiply $\frac{5}{12}$ by $\frac{7}{9}$. 6. Multiply $\frac{5}{11}$ by $\frac{8}{9}$.
4. Multiply $\frac{8}{11}$ by $\frac{9}{10}$. 7. Multiply $\frac{11}{12}$ by $\frac{7}{10}$.
8. Multiply $\frac{10}{50}$ by $\frac{72}{96}$.

Reduce to lowest terms.

9. Multiply $\frac{25}{36}$ by $\frac{24}{75}$.

Cancel whenever possible.

10. Multiply $2\frac{1}{2}$ by $3\frac{1}{3}$.

Reduce mixed numbers to improper fractions: $2\frac{1}{2}\times3\frac{1}{3}=\frac{5}{2}\times\frac{10}{3}=\frac{50}{6}=8\frac{1}{3}$.

11. $\frac{49}{56}\times\frac{8}{24}$. 16. $\frac{33}{72}\times\frac{18}{55}$. 21. $8\frac{1}{3}\times1\frac{1}{2}$.
12. $\frac{18}{27}\times\frac{15}{35}$. 17. $\frac{24}{25}\times\frac{35}{48}$. 22. $4\frac{2}{5}\times3\frac{3}{4}$.
13. $\frac{64}{96}\times\frac{120}{144}$. 18. $\frac{27}{64}\times\frac{20}{81}$. 23. $7\frac{7}{9}\times\frac{9}{70}$.
14. $\frac{63}{81}\times\frac{12}{48}$. 19. $\frac{39}{40}\times\frac{15}{26}$. 24. $5\frac{5}{6}\times\frac{12}{35}$.
15. $\frac{72}{144}\times\frac{35}{105}$. 20. $\frac{16}{75}\times\frac{25}{64}$. 25. $\frac{18}{31}\times3\frac{4}{9}$.

COMPOUND FRACTIONS.

26. Simplify $\frac{1}{2}$ of $\frac{2}{5}$ of $\frac{3}{4}$.

This is equivalent to $\frac{1}{2}\times\frac{2}{5}\times\frac{3}{4}$ which equals $\frac{3}{20}$.

Simplify

27. $\frac{1}{3}$ of $\frac{3}{5}$ of $\frac{5}{9}$. 29. $\frac{2}{3}$ of $\frac{1}{2}$ of $2\frac{1}{2}$. 31. $\frac{5}{12}$ of $\frac{4}{5}$ of 27.
28. $\frac{3}{4}$ of $\frac{2}{7}$ of $\frac{1}{10}$. 30. $\frac{3}{10}$ of $\frac{5}{6}$ of $\frac{6}{7}$. 32. $\frac{4}{9}$ of $\frac{3}{4}$ of $\frac{24}{25}$.

WRITTEN.

Find the products :

33. $\frac{51}{105}\times\frac{35}{102}\times\frac{8}{9}$.

34. $\frac{36}{49}\times\frac{56}{144}\times\frac{15}{16}$.

35. $\frac{45}{60}\times\frac{29}{90}\times7\frac{1}{4}$.

36. $\frac{4}{5}$ of $\frac{21}{63}$ of $17\frac{2}{5}$.

37 $\frac{48}{95}$ of $\frac{19}{72}$ of $23\frac{7}{11}$.

38. $15\frac{4}{5}\times\frac{60}{158}\times3\frac{3}{4}$.

39. $\frac{16}{27}\times\frac{81}{84}\times29\frac{3}{8}$.

40. $\frac{7}{9}$ of $27\times16\frac{2}{3}$.

41. $\frac{4}{5}\times\frac{6}{7}\times\frac{15}{24}\times8\frac{5}{9}$.

42. $2\frac{2}{5}\times\frac{15}{48}\times4\frac{1}{4}\times\frac{2}{34}$.

DIVISION.

CASE I.

275. To divide a fraction by an integer.

1. Divide $\frac{5}{9}$ by 5.

SOLUTION. To divide any number by 5 is to take one fifth of that number; $\frac{1}{5}$ of $\frac{5}{9}$ is 1 ninth; therefore, $\frac{5}{9}\div5=\frac{5\div5}{9}=\frac{1}{9}$.

2. Divide $\frac{5}{9}$ by 4.

SOLUTION. By the preceding method, $\frac{5}{9}\div4=\frac{5\div4}{9}=\frac{\frac{5}{4}}{9}$. To avoid this complex fraction, $\frac{\frac{5}{4}}{9}$, instead of dividing the numerator by the integer, multiply the denominator by the integer, thus: $\frac{5}{9}\div4=\frac{5}{9\times4}=\frac{5}{36}$. The fractional unit $\frac{1}{36}$, is $\frac{1}{4}$ of the fractional unit $\frac{1}{9}$; therefore, $\frac{5}{9}\div4=\frac{5}{9}\times\frac{1}{4}=\frac{5}{36}$.

3. Divide $\frac{35}{56}$ by 15.

SOLUTION. By the preceding method, this is equivalent to $\frac{35}{56\times15}$. By cancellation $\frac{35}{56\times15}=\frac{1}{24}$. Hence,

276. To divide a fraction by an integer: If the numerator is a multiple of the integer,

I. *Divide the numerator of the fraction by the integer.*

If the numerator is not a multiple of the integer :

II. *Multiply the denominator of the fraction by the integer.*

If the numerator and the integer have a common factor, or factors,

III. *Cancel the factor or factors, common to the numerator of the fraction and the integer, divide the remaining factor of the numerator by the product of the denominator and the remaining factor of the integer.*

These three rules are all included in the following general

277. RULE. *Divide the numerator of the fraction*, or,
Multiply the denominator of the fraction, by the integer.

ORAL.

4. Divide $\frac{12}{5}$ by 2; by 3; by 4; by 6.
5. Divide $\frac{24}{7}$ by 4; by 6; by 8; by 12.
6. Divide $\frac{48}{11}$ by 6; by 8; by 12; by 16.
7. Divide $\frac{35}{12}$ by 7; $\frac{56}{9}$ by 8; $\frac{72}{13}$ by 9.
8. Divide $\frac{5}{8}$ by 4; by 6; by 7; by 8.
9. Divide $\frac{7}{3}$ by 5; by 9; by 10; by 12.
10. Divide $\frac{4}{9}$ by 3; by 5; by 7; by 9.
11. Divide $\frac{10}{11}$ by 4; by 12; by 15; by 8.
12. Divide $\frac{15}{7}$ by 9; by 10; by 12; by 20.
13. Divide $\frac{12}{5}$ by 8; by 10; by 15; by 18.
14. Divide $10\frac{5}{9}$ by 5.

SOLUTION. $\frac{1}{5}$ of 10 is 2; $\frac{1}{5}$ of $\frac{5}{9}$ is $\frac{1}{9}$; therefore, $10\frac{5}{9}\div5=2\frac{1}{9}$.

15. Divide $16\frac{4}{7}$ by 4.
16. Divide $24\frac{9}{10}$ by 3.
17. Divide $9\frac{5}{8}$ by 4.

SOLUTION I. Reduce the mixed number $9\frac{5}{8}$ to the improper fraction $\frac{77}{8}$. Then, $\frac{77}{8}\div4=\frac{77}{8\times4}=\frac{77}{32}=2\frac{13}{32}$.

SOLUTION II. $9\frac{5}{8}=8+1\frac{5}{8}=8\frac{13}{8}$. $8\div4=2$; $\frac{13}{8}\div4=\frac{13}{32}$; $2+\frac{13}{32}=2\frac{13}{32}$. That is, separate the mixed number into two parts the first of which is the greatest multiple of the divisor, contained in the mixed number. Thus, 8 is the greatest multiple of 4 found in 9. Then, divide each part of the mixed number separately, and find the sum of the quotients.

18. Divide $7\frac{4}{5}$ by 3.
19. Divide $8\frac{3}{4}$ by 5.
20. Divide $10\frac{5}{6}$ by 4.
21. Divide $12\frac{3}{7}$ by 5.
22. Divide $6\frac{7}{12}$ by 4.
23. Divide $11\frac{5}{8}$ by 5.

WRITTEN.

24. $\frac{144}{196} \div 48$.
25. $\frac{189}{221} \div 21$.
26. $\frac{175}{190} \div 25$.
27. $\frac{156}{191} \div 16$.
28. $\frac{250}{181} \div 50$.
29. $\frac{71}{45} \div 22$.
30. $\frac{56}{57} \div 19$.
31. $\frac{67}{25} \div 18$.
32. $\frac{42}{45} \div 15$.
33. $\frac{39}{40} \div 13$.
34. $35\frac{1}{5} \div 44$.
35. $49\frac{7}{8} \div 57$.
36. $65\frac{4}{7} \div 153$.
37. $58\frac{7}{16} \div 5$.
38. $125\frac{8}{9} \div 103$.

CASE II.

278. To divide an integer by a fraction.

1. Divide 5 by $\frac{1}{4}$.

SOLUTION. $5=\frac{20}{4}$; $\frac{20}{4} \div \frac{1}{4}=20 \div 1=20$.

3. Divide 5 by $\frac{3}{4}$.

SOLUTION. $5=\frac{20}{4}$; $\frac{20}{4} \div \frac{3}{4}=20 \div 3=6\frac{2}{3}$. In these examples, the result is obtained by multiplying the integer by the denominator, and dividing this product by the numerator of the fraction; that is, by multiplying the integer by the fraction *inverted*. Thus: $5 \div \frac{3}{4}=5 \times \frac{4}{3}=\frac{5 \times 4}{3}=\frac{20}{3}=6\frac{2}{3}$. Hence,

279. To divide an integer by a fraction:
Multiply the integer by the fraction inverted.

ORAL.

1. Divide 3 by $\frac{2}{5}$; by $\frac{3}{4}$; by $\frac{5}{6}$; by $\frac{3}{7}$; by $\frac{8}{11}$.
2. Divide 6 by $\frac{5}{7}$; by $\frac{8}{9}$; by $\frac{9}{10}$; by $\frac{7}{12}$; by $\frac{11}{12}$.
3. Divide 6 by $\frac{1}{2}$; by $\frac{1}{3}$; by $\frac{1}{5}$; by $\frac{1}{8}$; by $\frac{1}{10}$.
4. Divide 5 by $\frac{3}{4}$; by $\frac{3}{8}$; by $\frac{4}{9}$; by $\frac{5}{6}$; by $\frac{7}{12}$.
5. Divide 8 by $\frac{2}{7}$; by $\frac{5}{9}$; by $\frac{8}{11}$; by $\frac{7}{10}$; by $\frac{5}{12}$.
6. Divide 12 by $\frac{4}{5}$; by $\frac{7}{6}$; by $\frac{5}{7}$; by $\frac{7}{8}$; by $\frac{10}{9}$.
7. Divide 10 by $\frac{5}{9}$; by $\frac{9}{5}$; by $\frac{3}{8}$; by $\frac{8}{3}$; by $\frac{5}{6}$.

When is the quotient greater than the dividend?

When is it less than the dividend? When equal to it?

8. Divide 9 by $\frac{4}{5}$; by $\frac{5}{4}$; by $\frac{3}{8}$; by $\frac{8}{3}$.

9. Divide 11 by $\frac{4}{9}$; by $\frac{3}{7}$; by $\frac{11}{12}$; by $\frac{9}{10}$.

10. Divide 8 by $3\frac{1}{3}$.

SOLUTION I. Reduce each number to 3rds: $8=\frac{24}{3}$, $3\frac{1}{3}=\frac{10}{3}$; $\frac{24}{3}\div\frac{10}{3}=24\div10=2\frac{4}{10}=2\frac{2}{5}$.

SOLUTION II. Reduce the divisor to an improper fraction, and proceed according to Art. 278. $8\div3\frac{1}{3}=8\div\frac{10}{3}=8\times\frac{3}{10}=\frac{24}{10}=2\frac{2}{5}$.

11. Divide 2 by $1\frac{4}{5}$.
12. Divide 3 by $2\frac{1}{4}$.
13. Divide 5 by $3\frac{2}{3}$.
14. Divide 6 by $4\frac{1}{2}$.
15. Divide 9 by $2\frac{2}{5}$.
16. Divide 4 by $5\frac{1}{2}$.

WRITTEN.

17. Divide 24 by $\frac{5}{12}$.
18. Divide 30 by $\frac{7}{15}$.
19. Divide 48 by $\frac{11}{24}$.
20. Divide 37 by $\frac{8}{9}$.
21. Divide 50 by $\frac{12}{17}$.
22. Divide 176 by $\frac{16}{25}$.
23. Divide 74 by $7\frac{2}{5}$.
24. Divide 308 by $8\frac{5}{9}$.
25. Divide 264 by $9\frac{3}{7}$.
26. Divide 465 by $12\frac{11}{12}$.
27. Divide 198 by $10\frac{3}{8}$.
28. Divide 220 by $19\frac{1}{5}$.

CASE III.

280. To divide a fraction by a fraction.

1. Divide $\frac{6}{7}$ by $\frac{3}{7}$.

SOLUTION. As 3 *units* are contained in 6 *units* 2 times, so 3 *sevenths* are contained in 6 *sevenths* two times. $\frac{6}{7}\div\frac{3}{7}=6\div3=2$.

2. Divide $\frac{2}{3}$ by $\frac{3}{4}$.

SOLUTION I. Reduce the fractions to equivalent fractions having a common denominator: $\frac{2}{3}=\frac{8}{12}$; $\frac{3}{4}=\frac{9}{12}$. $\frac{8}{12}\div\frac{9}{12}=8\div9=\frac{8}{9}$.

SOLUTION II. The same result may be obtained by multiplying the dividend by the divisor inverted. Thus, $\frac{2}{3}\div\frac{3}{4}=\frac{2}{3}\times\frac{4}{3}=\frac{8}{9}$. Hence,

To divide a fraction by a fraction:

281. I. *Reduce the fractions to equivalent fractions having a common denominator. Divide the numerator of the dividend by the numerator of the divisor.* Or,

II. *Multiply the dividend by the divisor inverted.*

NOTE. Rule II is generally used. Employ cancellation whenever possible.

3. Divide $\frac{14}{27}$ by $\frac{7}{27}$.
4. Divide $\frac{9}{10}$ by $\frac{2}{5}$.
5. Divide $\frac{3}{8}$ by $\frac{2}{7}$.
6. Divide $\frac{8}{9}$ by $\frac{5}{6}$.
7. Divide $\frac{4}{5}$ by $\frac{7}{12}$.
8. Divide $\frac{4}{11}$ by $\frac{3}{4}$.
9. Divide $\frac{8}{9}$ by $\frac{6}{7}$.
10. Divide $\frac{4}{5}$ by $\frac{8}{81}$.
11. Divide $\frac{3}{10}$ by $\frac{5}{12}$.
12. Divide $\frac{11}{12}$ by $\frac{11}{13}$.
13. Divide $\frac{8}{9}$ by $\frac{8}{27}$.
14. Divide $\frac{10}{11}$ by $\frac{1}{5}$.

State a brief method for solving examples like Nos. 12 and 13, in which the numerators are equal.

State a brief method for examples like No. 14, in which the divisor is one fractional unit, and its denominator is a factor of the numerator of the dividend.

15. Divide $\frac{3}{5}$ by $3\frac{1}{3}$.

SOLUTION. Reduce the mixed number to an improper fraction: $3\frac{1}{3}=\frac{10}{3}$; then proceed as before, $\frac{3}{5}\div\frac{10}{3}=\frac{3}{5}\times\frac{3}{10}=\frac{9}{50}$.

16. Divide $2\frac{1}{2}$ by $1\frac{3}{4}$.

SOLUTION. Reduce both numbers to improper fractions: $2\frac{1}{2}\div1\frac{3}{4}=\frac{5}{2}\div\frac{7}{4}=\frac{5}{2}\times\frac{4}{7}=\frac{10}{7}=1\frac{3}{7}$.

17. $\frac{5}{6}\div1\frac{1}{2}$.
18. $4\frac{1}{4}\div5\frac{2}{3}$.
19. $5\frac{1}{2}\div\frac{11}{12}$.
20. $5\frac{2}{5}\div2\frac{7}{10}$.
21. $3\frac{1}{3}\div1\frac{1}{2}$.
22. $1\frac{1}{6}\div\frac{3}{5}$.
23. $4\frac{3}{8}\div2\frac{2}{7}$.
24. $\frac{3}{8}\div2\frac{1}{4}$.
25. $2\frac{1}{5}\div1\frac{1}{10}$.
26. $3\frac{2}{3}\div\frac{3}{5}$.
27. $3\frac{1}{5}\div1\frac{1}{3}$.
28. $4\frac{1}{3}\div2\frac{1}{2}$.

WRITTEN.

29. Divide $\frac{11}{12}$ by $\frac{12}{17}$.
30. Divide $\frac{25}{27}$ by $\frac{35}{81}$.
31. Divide $\frac{19}{36}$ by $\frac{38}{63}$.
32. Divide $\frac{31}{51}$ by $\frac{155}{306}$.
33. Divide $\frac{17}{142}$ by $\frac{68}{71}$.
34. Divide $8\frac{3}{10}$ by $7\frac{4}{9}$.
35. Divide $12\frac{2}{3}$ by $5\frac{8}{11}$.
36. Divide $16\frac{5}{6}$ by $7\frac{3}{7}$.
37. Divide $24\frac{6}{11}$ by $12\frac{3}{11}$.
38. Divide $15\frac{3}{4}$ by $8\frac{7}{8}$.

Complex Fractions.

282. 1. Simplify $\frac{\frac{3}{8}}{\frac{5}{7}}$ ·

Solution. This is equivalent to $\frac{3}{8} \div \frac{5}{7} = \frac{3}{8} \times \frac{7}{5} = \frac{21}{40}$. Observe that the numerator of the product $\frac{21}{40}$ is the product of the extremes 3 and 7, that is, of the numerator of the numerator, and the denominator of the denominator, of the complex fraction $\frac{\frac{3}{8}}{\frac{5}{7}}$. Also that the denominator of the product $\frac{21}{40}$ is the product of the means 8 and 5. Hence,

To simplify a complex fraction,

Divide the product of the extremes by the product of the means.

2. Simplify $\frac{\frac{3}{4}}{\frac{2}{3}}$. 4. Simplify $\frac{\frac{2}{5}}{\frac{7}{9}}$. 6. Simplify $\frac{2\frac{4}{5}}{1\frac{3}{4}}$.

3. Simplify $\frac{\frac{5}{8}}{\frac{3}{7}}$. 5. Simplify $\frac{12\frac{1}{2}}{37\frac{1}{2}}$. 7. Simplify $\frac{1\frac{1}{5}}{2\frac{1}{3}}$.

Reciprocals.

283. Two numbers are reciprocals of each other when their product is 1. Thus, $\frac{1}{2}$ is the reciprocal of 2; 3 is the reciprocal of $\frac{1}{3}$; $\frac{3}{5}$ is the reciprocal of $\frac{5}{3}$.

The reciprocal of any number is 1, divided by that number. Thus, the reciprocal of 6 is $\frac{1}{6}$, that is, 1 divided by 6; the reciprocal of $\frac{1}{6}$ is $1 \div \frac{1}{6} = 6$. So, the reciprocal of $\frac{3}{8}$ is $1 \div \frac{3}{8} = \frac{8}{3}$. The reciprocal of any fraction is found by inverting its terms; the reciprocal of $\frac{7}{9}$ is $\frac{9}{7}$; the reciprocal of $\frac{15}{8}$ is $\frac{8}{15}$, etc.

1. What is the reciprocal of 5? Of 9? Of 12?
2. What is the reciprocal of $\frac{1}{8}$? Of $\frac{1}{10}$? Of $\frac{1}{25}$?
3. What is the reciprocal of $\frac{4}{5}$? Of $\frac{6}{7}$? Of $\frac{7}{12}$? Of $\frac{9}{5}$?

284. To divide by any number is to multiply by its reciprocal. Thus, to divide by $\frac{1}{2}$ is to multiply by 2; as $8 \div \frac{1}{2} = 16$, and $8 \times 2 = 16$; $10 \div \frac{3}{2} = \frac{20}{3}$, and $10 \times \frac{2}{3} = \frac{20}{3}$; $12 \div 2$

$=6$, and $12\times\frac{1}{2}=6$. Therefore, a general rule for division, whether the divisor is an integer or a fraction, is

Multiply the dividend by the reciprocal of the divisor.

The three cases of Division of Fractions are included in this principle; and all examples therein may be solved by the rule just given.

Solve each of the following examples by multiplying the dividend by the reciprocal of the divisor :

1. $18\div2$.
2. $25\div5$.
3. $48\div6$.
4. $15\div\frac{1}{4}$.
5. $20\div\frac{1}{5}$.
6. $30\div\frac{1}{6}$.
7. $6\div\frac{2}{3}$.
8. $9\div\frac{3}{5}$.
9. $12\div\frac{4}{7}$.

285. To reduce a decimal fraction to a common fraction.

1. Reduce .25 to a common fraction.

SOLUTION. $.25=\frac{25}{100}=\frac{1}{4}$.

2. Reduce .025 to a common fraction.

SOLUTION. $.025=\frac{25}{1000}=\frac{1}{40}$. Therefore,

To reduce a decimal fraction to a common fraction :

Erase the decimal point. Write the denominator under the given numerator, and reduce to lowest terms.

NOTE 1. The denominator is always 1, followed by as many ciphers as there are figures in the decimal fraction.

NOTE 2. When the decimal point is erased, the ciphers on the *left* of the numerator become valueless, and are omitted.

Reduce to common fractions :

ORAL.

3. .5.
4. .05.
5. .005.
6. .50.
7. .125.
8. $.12\frac{1}{2}$.
9. .8.
10. .80
11. .15.
12. .35.
13. .45.
14. .6.
15. .65.
16. .06.
17. .75.
18. .075.
19. $.33\frac{1}{3}$.
20. $.16\frac{2}{3}$.
21. $.37\frac{1}{2}$.
22. $.62\frac{1}{2}$.
23. $.83\frac{1}{3}$.
24. $.66\frac{2}{3}$.
25. $.87\frac{1}{2}$.
26. .875.

WRITTEN.

27. .0064.
28. .0896.
29. .575.
30. .0125.
31. .5675.
32. .0225.
33. .448.
34. .0248.
35. .0675.
36. .0256.
37. .9375.
38. .15625.

286. To reduce a common fraction to a decimal fraction.

1. Reduce $\frac{3}{8}$ to a decimal fraction.

PROCESS.

$$\begin{array}{r} 8)3.000 \\ \hline .375 \end{array}$$

ANALYSIS. Since a numerator is a dividend of which the denominator is the divisor, the fraction $\frac{3}{8}$ is equivalent to $\frac{1}{8}$ of 3 units, or $\frac{1}{8}$ of 30 tenths, or $\frac{1}{8}$ of 300 hundredths, etc. $\frac{1}{8}$ of 3 units is .375.

2. Reduce $\frac{1}{16}$ to a decimal fraction.

SOLUTION. $\begin{array}{r} 16)1.0000 \\ \hline .0925. \end{array}$ Hence,

To reduce a common fraction to a decimal fraction :
Divide the numerator by the denominator.

Reduce to decimal fractions :

3. $\frac{1}{4}$.
4. $\frac{5}{8}$.
5. $\frac{2}{25}$.
6. $\frac{7}{8}$.
7. $\frac{11}{25}$.
8. $\frac{9}{20}$.
9. $\frac{3}{40}$.
10. $\frac{4}{5}$.

287. If the denominator of a fraction, when the fraction is reduced to its lowest terms, contains any factor except 2 or 5, the division of the numerator by the denominator will not be exact.

Thus: $\frac{1}{3}$=1.0÷3= $3\frac{1}{3}$, or .3333$\frac{1}{3}$.

Reduce to decimals of four figures each.

1. $\frac{2}{3}$.
2. $\frac{1}{6}$.
3. $\frac{1}{9}$.
4. $\frac{4}{9}$.
5. $\frac{1}{7}$.
6. $\frac{5}{9}$.
7. $\frac{1}{12}$.
8. $\frac{5}{6}$.

288. Reduce to decimal fractions or mixed numbers, obtaining five fractional places in each result, unless the division terminates sooner.

1. $\frac{19}{32}$.
2. $\frac{43}{200}$.
3. $\frac{73}{80}$.
4. $\frac{12}{26}$.
5. $\frac{63}{24}$.
6. $\frac{35}{8}$.
7. $\frac{49}{60}$.
8. $\frac{16}{19}$.
9. $\frac{24}{21}$.
10. $\frac{81}{35}$.
11. $\frac{25}{13}$.
12. $\frac{3}{65}$.

EXAMPLES ILLUSTRATING PRECEDING PRINCIPLES.

ORAL.

289. 1. Change $9\frac{3}{8}$ to 8ths; $5\frac{11}{12}$ to 12ths.
2. Reduce $7\frac{4}{9}$ to 9ths; $10\frac{10}{11}$ to 11ths.
3. Change $\frac{60}{12}$ to integers; $\frac{57}{9}$ to a mixed number.
4. Reduce $\frac{75}{9}$ to an integer or mixed number.
5. How many 36ths in 1? In $\frac{1}{2}$? In $\frac{1}{9}$? In $\frac{1}{12}$?
6. Change $\frac{5}{8}$ to 16ths; to 48ths; to 96ths.
7. Change $\frac{8}{24}$ to 3rds; to 12ths; to 48ths; to 6ths.
8. Reduce to lowest terms $\frac{81}{108}$; $\frac{60}{75}$; $\frac{96}{144}$.
9. Reduce to a common denominator $\frac{3}{9}$ and $\frac{8}{12}$.
10. Reduce to least common denominator $\frac{3}{7}$ and $\frac{7}{12}$.
11. Find sum of $\frac{5}{9}$ and $2\frac{1}{4}$.
12. What is the difference of $3\frac{2}{3}$ and $2\frac{1}{4}$?
13. From the sum of $\frac{3}{5}$ and $\frac{11}{12}$ subtract $\frac{5}{6}$.
14. The sum of two numbers is $8\frac{7}{9}$; one of the numbers is $4\frac{3}{5}$; what is the other number?
15. The difference of two numbers is $2\frac{4}{5}$; the greater number is $5\frac{11}{12}$; what is the less?
16. Find the sum of two numbers, the less of which is $3\frac{4}{5}$, the difference of the two being $\frac{3}{4}$.
17. A man walked $12\frac{1}{2}$ miles on Monday, $14\frac{3}{4}$ miles on Tuesday, and $13\frac{5}{7}$ miles on Wednesday. What distance did he walk in the three days?
18. A man collected \$$35\frac{4}{5}$. He spent \$$14\frac{3}{4}$, and lost \$$3\frac{1}{2}$. How much of the money collected did he still have?
19. Two bins contained respectively $16\frac{3}{8}$ bushels, and $12\frac{3}{4}$ bushels. If $9\frac{1}{2}$ bushels are removed from either bin, how many bushels remain in both?
20. A has \$$3\frac{1}{2}$, B has \$$1\frac{1}{4}$ more than A, and C has as much as both A and B. How much money had they all?

EXAMPLES ILLUSTRATING PRECEDING PRINCIPLES.

WRITTEN.

290. 1. Change $98\frac{5}{32}$ to 32nds.

2. Change $129\frac{25}{49}$ to 49ths.

3. Reduce $\frac{84960}{24}$ to integers.

4. Reduce $\frac{95767}{256}$ to a mixed number.

5. How many 50ths in $259\frac{14}{25}$?

6. How many 9ths in $187\frac{27}{81}$?

7. Reduce to lowest terms $\frac{459}{486}$.

8. Reduce to lowest terms $\frac{1530}{3960}$.

9. Reduce to least common denominator $\frac{5}{16}$, $\frac{3}{4}$, $\frac{7}{12}$.

10. Reduce to least common denominator $\frac{35}{49}$, $\frac{56}{63}$, $\frac{48}{96}$.

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11. Find sum of $\frac{15}{16}$, $\frac{3}{20}$, and $\frac{49}{60}$.

12. Find sum of $48\frac{9}{10}$, $17\frac{25}{28}$, and $3\frac{5}{7}$.

13. Find the difference of $89\frac{37}{54}$ and $60\frac{87}{90}$.

14. The sum of three numbers is $125\frac{9}{16}$. Two of the numbers are respectively $31\frac{3}{4}$ and $47\frac{5}{8}$; what is the third number?

15. A ship sailed $54\frac{11}{12}$ miles on one day, $63\frac{15}{22}$ miles on another, and $96\frac{7}{11}$ miles on the third day. How many miles were sailed in the three days?

16. The difference of two numbers is $18\frac{4}{9}$; the less number is $58\frac{2}{7}$; what is the sum of the two numbers?

17. Half the sum of two numbers is $25\frac{7}{9}$; half their difference is $5\frac{11}{12}$; what are the two numbers?

18. A lady having $\$125\frac{7}{8}$, paid $\$1\frac{3}{4}$ for a pair of gloves, $\$87\frac{5}{8}$ for a dress, and $\$5\frac{1}{2}$ for an umbrella. How much money did she retain?

19. A house was bought for $\$5962\frac{3}{8}$, and sold at a loss of $\$375\frac{11}{12}$. What was it sold for?

21. 8 is $\frac{1}{5}$ of what number? $\frac{2}{5}$ of what number.
22. 12 is $\frac{1}{11}$ of what number? $\frac{6}{11}$ of what number?
23. $9\frac{4}{5}$ is $\frac{1}{6}$ of what number? $\frac{1}{7}$ of what number.
24. 16 is $\frac{2}{3}$ of some number; what is $\frac{5}{6}$ of that number?
25. 54 is $\frac{9}{11}$ of some number; what is $\frac{3}{10}$ of that number?
26. \$$8\frac{4}{5}$ is $\frac{1}{9}$ of the price of a shawl; what is the price of the shawl?
27. James, who is 12 years old, is $\frac{6}{7}$ the age of Richard; how old is Richard?
28. If $\frac{3}{4}$ of a ton of coal cost \$6, what is the cost of 1 ton?
29. \$28 is $\frac{7}{12}$ of John's money; how much has he?
30. Henry spent $\frac{3}{7}$ of his money, and then had \$28. How much money had he at first?

31. A sold $\frac{5}{8}$ of his farm, and then had 350 acres; how many acres did he sell?
32. $\frac{3}{4}$ is $\frac{5}{9}$ of what number?
33. If $\frac{3}{8}$ of a yard of cloth is worth \$$1\frac{1}{2}$, what is one yard worth?
34. The multiplicand is $\frac{5}{9}$, the multiplier is $2\frac{1}{5}$; what is the product?
35. The divisor is $\frac{8}{11}$, the quotient is $\frac{6}{7}$; what is the dividend?
36. $18\frac{17}{25}$ is the sum of what two equal numbers?
37. $19\frac{6}{7}$ is the sum of what three equal numbers?
38. $4\frac{2}{3}$ is $\frac{7}{9}$ of what number?
39. If $\frac{5}{8}$ of a vessel is worth \$25,000, what is the vessel worth?
40. After $\frac{2}{9}$ of a farm was sold, the remainder was valued at \$6,300. What was the value of the part sold?
41. The dividend is $8\frac{2}{5}$, the divisor is $\frac{7}{5}$; what is the quotient?

20. A man having $630\frac{3}{4}$ acres of land, sold $96\frac{11}{16}$ acres to one man, and $156\frac{7}{12}$ acres to another. How many acres did he yet own?

21. $120\frac{5}{9}$ is $\frac{5}{27}$ of what number?
22. $242\frac{8}{11}$ is $\frac{30}{77}$ of what number?
23. $84\frac{9}{10}$ is $\frac{3}{20}$ of some number. What is $\frac{3}{7}$ of that number?
24. Mr. A. spent $\$62\frac{2}{5}$, which was $\frac{8}{9}$ of his money. How much money did he still have?
25. A man bought a farm with $\frac{3}{5}$ of his money, and had $\$840\frac{3}{10}$ remaining. How much money had he before purchasing the farm?
26. Mrs. C. lost $\$832\frac{4}{5}$, and had $\frac{5}{9}$ of her money left. How much money had she at first?
27. The salary of the President of the United States, $50,000, is $4\frac{16}{21}$ times the salary of the Chief Justice of the United States. What is the salary of the Chief Justice?
28. The area of Illinois, 56650 square miles, is $45\frac{8}{25}$ times the area of Rhode Island. What is the area of Rhode Island?
29. If $\frac{17}{24}$ of a ship is worth $68,000, what is the ship worth?
30. What is $\frac{7}{125}$ of a number, $\frac{4}{3}$ of which is 5000?
31. $849\frac{11}{12}$ is $\frac{5}{17}$ of what number?
32. If $\frac{15}{16}$ of a farm is worth $4575, what is $\frac{7}{9}$ of it worth?
33. $845\frac{15}{22}$ is the sum of what 3 equal numbers?
34. The sum of three numbers is $230\frac{10}{11}$; one of the numbers is $31\frac{5}{9}$; the other two numbers are equal; what is one of them?
35. $\frac{1}{3}$ of my property is in real estate, $\frac{2}{5}$ in bank stock; the remainder, $8,460, is cash. What is the value of my entire property?

42. The product is $5\frac{1}{4}$, the multiplicand is $2\frac{1}{3}$; what is the multiplier?

43. If $6\frac{2}{3}$ yards of ribbon are worth 80 cents, what is 1 yard worth?

44. If $4\frac{3}{4}$ pounds of coffee cost 95 cents, what does 1 pound cost?

45. How many barrels of flour at \$$5\frac{1}{4}$ a barrel, can be bought for \$84?

46. Mr. D. owned $\frac{4}{7}$ of a mill, and sold $\frac{2}{5}$ of his share. What part of the mill did he sell? What part did he still own?

47. How many bottles, each containing $1\frac{3}{4}$ pints, may be filled from a vessel containing $18\frac{2}{3}$ pints?

48. If $\frac{7}{12}$ of an acre of land is worth \$$15\frac{3}{4}$, what is 1 acre worth? What are $2\frac{1}{2}$ acres worth?

49. If $\frac{3}{4}$ of a sack of flour is worth $1\frac{1}{2}$, what is 1 sack worth? What are 3 sacks worth?

50. If $\frac{4}{9}$ of a ton of hay cost \$8, what will $\frac{3}{4}$ of a ton cost?

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51. The difference between $\frac{1}{5}$ and $\frac{1}{6}$ of a number is 5; what is the number?

SUGGESTION. $\frac{1}{5}-\frac{1}{6}=\frac{1}{30}$. $5=\frac{1}{30}$ of the number.

52. \$$3\frac{3}{5}$ is the difference between $\frac{3}{4}$ and $\frac{3}{5}$ of John's money. How much money has he?

53. $5\frac{2}{3}$ years is the difference between $\frac{2}{3}$ and $\frac{3}{4}$ of Mr. B.'s age; how old is he?

54. A number increased by $\frac{3}{5}$ of itself is 32; what is the number?

SUGGESTION. The number itself is $\frac{5}{5}$; $\frac{5}{5}+\frac{3}{5}=\frac{8}{5}$: $32=\frac{8}{5}$ of the number.

55. A number diminished by $\frac{2}{7}$ of itself is 45; what is the number?

36. Find the quotient of $\frac{11}{16}$ divided by one half of its reciprocal.

37. The divisor is $8\frac{5}{9}$; the quotient is $3\frac{2}{3}$; the remainder is $2\frac{1}{2}$; what is the dividend ?

38. The multiplier is $5\frac{3}{4}$; the multiplicand is $17\frac{4}{15}$; what is the product?

39. Simplify $\frac{11}{12}\times 10\frac{2}{3}\times 2\frac{1}{16}\times\frac{24}{121}\times\frac{1}{4}$.

40. Simplify $(1\frac{2}{3}$ of $4\frac{1}{2}$ of $2\frac{2}{3})\div(\frac{4}{5}$ of $8\frac{1}{4}$ of $\frac{5}{11})$.

41. If $20\frac{1}{2}$ tons of coal are worth \$$110\frac{3}{16}$, what are $3\frac{3}{4}$ tons worth?

42. How many bushels of oats at $30\frac{3}{4}$ cents a bushel, can be bought for \$61.50?

43. How many baskets of pears, worth \$$1.12\frac{1}{2}$ a basket, equal in value 73 pounds of butter, worth $37\frac{1}{2}$ cents a pound?

44. If $3\frac{3}{4}$ tons of coal cost \$$22\frac{1}{2}$, what is the value of $117\frac{5}{8}$ tons of coal of the same quality?

45. If a man can mow $1\frac{7}{8}$ acres in 9 hours, in how many hours can he mow $1\frac{2}{3}$ acres?

46. What is the average speed per hour made by a train running $81\frac{7}{8}$ miles in $2\frac{1}{2}$ hours?

47. The product is $215\frac{25}{63}$; the multiplier is $25\frac{5}{9}$; what is the multiplicand ?

48. $249\frac{3}{11}$ is $\frac{3}{25}$ of what number?

49. If $\frac{27}{28}$ of an estate is worth \$13,554, what is the value of one half of the estate?

50. The quotient is $49\frac{2}{5}$; the divisor is $81\frac{3}{7}$; the remainder is $\frac{3}{7}$. What is the dividend ?

51. The difference of $\frac{1}{12}$ and $\frac{1}{20}$ of a number is 29 ; what is the number?

52. The difference between $\frac{5}{8}$ and $\frac{4}{9}$ of a number is 260 ; what is the number?

56. $\frac{1}{4}$ of 48 is what part of 96?
57. $3\frac{5}{6}$ is the product of $3\frac{1}{3}$, of $\frac{3}{10}$, and of what other number.
58. What is the reciprocal of $\frac{5}{7}$? Of $\frac{3}{8}$? Of $8\frac{5}{9}$? Of $\frac{6}{11}$?
59. Simplify $\dfrac{\frac{3}{4}}{\frac{5}{6}}$; $\dfrac{\frac{2}{9}}{\frac{3}{7}}$; $\dfrac{2\frac{1}{2}}{\frac{5}{8}}$; $\dfrac{\frac{3}{5}}{1\frac{1}{10}}$.
60. How many dollars in $\frac{3}{8}$ of $\frac{2}{7}$ of $5600?
61. What is $\frac{1}{8}$ of $3\frac{1}{3}$?
62. $\frac{4}{5}$ of 45 is what part of 144 ?
63. A number increased by $\frac{5}{9}$ of itself equals 56; what is the number?
64. A number diminished by $\frac{2}{9}$ of itself equals 49 ; what is the number?
65. Henry sold a knife for $1.75, which was $\frac{7}{6}$ of what he had paid for it; how much had he paid for the knife?
66. The price of a book, $1.25, is $\frac{5}{4}$ of its cost What is the profit? What part of the cost is the profit?
67. If $\frac{3}{4}$ of an acre of ground is worth $75, what is $\frac{5}{9}$ of it worth?
68. From $\frac{3}{7}$ of 28 subtract $\frac{4}{5}$ of 5.
69. If $4\frac{4}{5}$ bushels of corn are worth $1.92, how much is 1 bushel worth?
70. Divide the reciprocal of $\frac{7}{2}$ by the reciprocal of 5.
71. What fraction of 63 is 92?
72. What fraction of $3\frac{4}{5}$ is $5\frac{2}{5}$?
73. What fraction of $4\frac{1}{2}$ is $6\frac{2}{3}$?
74. Of what number is 15 two thirds?
75. I had ten horses after having sold $\frac{3}{5}$ of my drove ; how many horses did I sell ?
76. At $$1\frac{3}{4}$ a yard, how many yards of cloth can be bought for $28?

53. The area of North Dakota, 75,000 square miles, is $2\frac{1}{7}$ times the area of Maine. How many square miles does Maine contain?
54. $\frac{17}{26}$ of 2340 is what part of 12240?
55. A number diminished by $\frac{15}{39}$ of itself is 7296; what is the number?
56. What is the reciprocal of $329\frac{11}{19}$?
57. A horse and carriage are worth $1463; the horse is worth $\frac{2}{5}$ as much as the carriage. What is the value of each?
58. The product of three numbers is $34\frac{8}{21}$; one of the numbers is $6\frac{2}{7}$, another is $\frac{3}{14}$; what is the third?
59. The sum of three numbers is $461\frac{9}{11}$; one of the numbers is $103\frac{3}{7}$; the other two are equal. What is one of the equal numbers?
60. Simplify $1\frac{1}{2}$ of $\frac{5}{6}\times\dfrac{\frac{2}{3}\times\frac{3}{5}}{\frac{5}{8}\times\frac{3}{10}}$.
61. Simplify $3\frac{1}{3}\times5\frac{2}{5}-\dfrac{\frac{2}{5}+\frac{4}{9}}{\frac{4}{51}+\frac{8}{27}}$.
62. Multiply the reciprocal of $\frac{5}{121}$ by the reciprocal of $24\frac{1}{5}$.
63. A house and lot cost $9000. The cost of the lot was $\frac{3}{15}$ of the cost of the house. What was the cost of each?
64. A number increased by $\frac{4}{25}$ of itself equals 1073; what is the number?
65. Simplify $(\frac{4}{5}+\frac{5}{9}+\frac{7}{15})-(\frac{2}{3}-\frac{3}{5})$.
66. John received $\frac{1}{4}$ of his father's estate, and James received $\frac{1}{5}$ of it. John's portion was $750 more than James' portion. How many dollars did each receive? What was the value of the estate?

67. If 1 barrel of flour cost $\$7\frac{2}{9}$, what is $\frac{3}{10}$ of it worth?

68. Divide $\frac{17}{18}$ into two parts, one of which is 2 times the other.

69. Divide $\frac{17}{18}$ into two parts, one of which is $2\frac{1}{2}$ times the other.

70. If $5\frac{3}{8}$ tons of hay cost $\$28\frac{2}{3}$ how many tons will $\$85\frac{1}{3}$ buy?

71. The sum of two numbers is $\frac{8}{9}$, their difference is $\frac{1}{7}$; what are the numbers?

72. If 42 is $\frac{3}{11}$ of some number, what is $\frac{6}{7}$ of the same number?

73. Divide $\frac{3}{4}$ of $\frac{8}{11}$ of $7\frac{3}{7}$ by $\frac{5}{9}$ of $\frac{18}{19}$ of $5\frac{4}{7}$.

74. Multiply $\frac{3}{4}+\frac{5}{6}-\frac{7}{8}$ by $\frac{4}{5}+\frac{3}{4}-\frac{9}{10}$, and divide the product by $3\frac{2}{5}$.

75. I bought 10 cords of wood for $\$5\frac{1}{4}$ a cord, and paid $\$1\frac{1}{3}$ a cord for sawing, splitting and piling it. What did the wood cost a cord ?

76. How many pounds of cheese, at $16\frac{2}{3}$ cents a pound, may be exchanged for 24 yds. of muslin worth $8\frac{1}{3}$ cents a yard ?

77. A farmer having $126\frac{3}{4}$ bushels of wheat, sold $\frac{5}{8}$ of it at $\$1\frac{3}{5}$ a bushel, and the remainder at $\$1\frac{2}{3}$ a bushel. How much did he receive for his wheat ?

78. A and B engaged in business, A furnishing $\frac{5}{12}$ of the capital. If he had added $\$596\frac{3}{4}$, he would have furnished $\frac{1}{2}$ of the entire capital. What was the entire capital ?

79. Divide the product of $28\frac{5}{7}$ and $13\frac{4}{9}$ by the difference of $8\frac{5}{12}$ and $4\frac{4}{5}$.

80. If a locomotive move $\frac{5}{8}$ of a mile in $\frac{11}{12}$ of an hour, what is its speed per hour ?

81. Change to decimals and find the sum of $\frac{4}{5}$, $\frac{5}{8}$, $\frac{11}{16}$, $\frac{9}{20}$, $\frac{41}{50}$.

THIRTY-THREE DIFFICULT PROBLEMS.

To be used at the discretion of the teacher.

291. 1. A and B can build a fence in 6 days. B would require 10 days to build it, working alone. In how many days could A alone build it?

SUGGESTIONS. A and B build $\frac{1}{6}$ of the fence in one day. B would build $\frac{1}{10}$ of it in 1 day. What part of it could A build in one day? If A could build $\frac{1}{15}$ of the fence in 1 day, in how many days could he build 15 fifteenths?

2. A and B can do a certain work in 10 days. B would do it in 18 days. In how many days could A do it?

3. C can do a certain piece of work in 9 days; D can do it in 12 days. In how many days can they do it working together?

4. A, B and C can build a barn in 12 days. A and B can build it in 20 days. In how many days can C alone build it?

5. D can build a wall in 30 days; E works twice as fast as D; F works one third as fast as E. In how many days can they all build the wall, working together?

6. A can do $\frac{3}{8}$ of a piece of work in one day; B can do $\frac{2}{5}$ of it in one day; C $\frac{7}{8}$ of it; D $\frac{4}{5}$ of it. In what part of a day can they do the entire work, all working together?

7. A can do a piece of work in $\frac{3}{8}$ of a day; B can do it in $\frac{2}{5}$ of a day; C can do it in $\frac{7}{8}$ of a day; D can do it in $\frac{4}{5}$ of a day. In what part of a day can they do the work, all working together?

8. E can build a boat in $1\frac{3}{4}$ months; F in $2\frac{1}{4}$ months; G in $2\frac{1}{2}$ months. E charges $2.50 a day; F charges $2 a day; G charges $1.75 a day. Assuming 24 working days in each month, will it be cheaper to employ E and F, or F and G?

9. The hour and minute hands of a clock are together at 12 o'clock M. When will they be together the fourth time after noon?

Analysis. The minute hand passes the hour hand 11 times in 12 hours. Since the motion is uniform, the minute hand will pass the hour hand the first time in $\frac{1}{11}$ of 12 hours; the second time in $\frac{2}{11}$ of twelve hours; the fourth time in $\frac{4}{11}$ of 12 hours. 12 hours$\times\frac{4}{11}=\frac{48}{11}$h.$=$4h. $21\frac{9}{11}$ m. P. M.

10. When will the two hands of a clock be together between 5 and 6 o'clock? Between 9 and 10 o'clock?

11. A man bought some oranges at the rate of 3 for 2 cents, and an equal number at the rate of 4 for 3 cents. He sold them all at the rate of 2 for 5 cents, gaining \$4.30. How many oranges did he buy?

Suggestions Find the average cost of the oranges. Divide the total gain by the gain on one orange.

12. A peddler bought some pears at the rate of 5 for 4 cents, and an equal number at the rate of 4 for 2 cents. He sold them all at the rate of 3 for 10 cents, thereby gaining \$3.22. How many pears did he buy?

13. If 3 oranges and 8 pears cost 47 cents, and 6 oranges and 7 pears cost 58 cents, what is the cost of 1 orange? Of 1 pear?

Operation.

$$\begin{array}{lr} (a)\ \text{Twice 1st lot,} & 6\text{ oranges}+16\text{ pears}=94\text{ cents.} \\ (b)\qquad\quad \text{2nd lot,} & \underline{6\text{ oranges}+\ 7\text{ pears}=58\text{ cents.}} \\ \text{Subtract }(b)\text{ from }(a) & 0\text{ oranges}+\ 9\text{ pears}=36\text{ cents.} \\ & 1\text{ pear}\ =\ 4\text{ cents.} \end{array}$$

Analysis. Take twice the first lot, that the number of oranges may be the same in each lot. From lot (a) subtract lot (b), and thus cause the oranges and their cost to disappear, leaving 9 pears$=$36 cents.

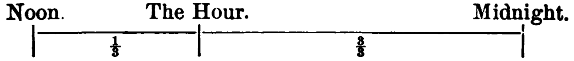
Find the cost of 1 orange by substituting in (b) 28 cents, the cost of 7 pears. Then (b) 6 oranges$+$28 cents$=$58 cents. Subtract 28 cents from each member of this equation, and 6 oranges$=$30 cents. 1 orange$=$5 cents.

14. A farmer bought 5 calves and 4 sheep for \$37, and 4 calves and 2 sheep for \$26. Find cost of each animal.

15. Mr. C. bought 10 bushels of wheat and 7 bushels of corn for \$18.10, and 6 bushels of wheat and 8 bushels of corn for \$13.90. Find cost of 1 bushel of each kind of grain.

16. The time past noon is $\frac{1}{3}$ of the time to midnight. What is the hour?

SUGGESTION. The time from noon to midnight, 12 hours, is to be divided into two parts, one of which is $\frac{1}{3}$ of the other.



17. The time past noon is $\frac{2}{3}$ of the time to midnight. What is the hour?

18. The time past noon is $\frac{1}{3}$ of the time past midnight. What is the hour?

19. One fourth of the time from now to midnight equals the time past noon. What time is it?

20. A flag staff broke into two pieces, one piece being $\frac{5}{8}$ as long as the other. The difference in their lengths was 15 feet. How long was the staff ?

21. If $\frac{5}{16}$ of A's money is equal to $\frac{5}{8}$ of B's money, and B has \$40, how many dollars has A?

22. A gave $\frac{1}{3}$ of his money to B, $\frac{1}{6}$ to C, and the remainder, which is \$18 more than $\frac{1}{4}$ of the whole, to D. How much money had A at first ?

23. A drover bought sheep for \$3$\frac{2}{5}$ each. If he had paid \$4$\frac{1}{4}$ each, the sheep would have cost him \$120.70 more. How many sheep did he buy?

24. Divide \$9840 among E, F, G, and H, in such a manner that E shall have half as much as F; that F shall have twice as much as G, and that H shall have \$1000 less than the other three together.

25. A cistern can be filled by three pipes in one hour. By the first and second it may be filled in 2 hours ; by the first and third, in $1\frac{1}{2}$ hours. In what time could it be filled by each alone?

26. A cistern has three receiving and three discharging pipes. The first three will fill it in 24, 36, and 48 minutes respectively; the last three will empty it in 30, 40, and 60 minutes respectively. If the cistern is empty, and all the pipes be opened at the same moment, in how many minutes will the cistern be filled?

27. Divide 72 into two such parts that one part shall be $37\frac{1}{2}$ greater than the other.

28. If 5 be subtracted from a certain number, $\frac{2}{3}$ of the remainder is 40. What is the number?

29. Divide 128 into three such parts that $\frac{1}{3}$ of the first part shall equal the second, and the third shall be $\frac{4}{7}$ of the second.

30. A man divided $500 among his three boys, so that the second had $\frac{5}{7}$ as much as the first, and the third $\frac{4}{5}$ as much as the second. How many dollars had each?

31. Five men can do a certain work in 20 days; three boys can do the same work in 50 days. In how many days could two men and two boys do it?

32. The head of a fish is 6 inches long; his tail is as long as his head and $\frac{1}{2}$ his body; his body is as long as his head and tail together. How long is the fish?

33. Mr. E sold $\frac{1}{5}$ of his farm at one time, $\frac{2}{7}$ of the remainder at another, and $\frac{1}{3}$ of what then remained at another, and had 240 acres left. How many acres did he have at first?

292. Examples in Fractions, to be used at the discretion of the teacher.

REDUCTION. Reduce Fractions in third column to lowest terms.

ADDITION: As $\frac{2}{3}+\frac{4}{5}+\frac{3}{8}$.

MULTIPLICATION: As $\frac{2}{3}\times\frac{4}{5}\times\frac{3}{8}$.

DIVISION: Use third fraction as divisor, as $\frac{2}{3}\times\frac{4}{5}\div\frac{3}{8}$.

1	$\frac{2}{3}$	$\frac{4}{5}$	$\frac{3}{8}$	26	$\frac{3}{5}$	$\frac{11}{12}$	$\frac{7}{15}$	51	$\frac{64}{70}$	$\frac{40}{44}$	$\frac{39}{39}$
2	$\frac{5}{9}$	$\frac{3}{7}$	$\frac{21}{25}$	27	$\frac{7}{8}$	$\frac{5}{4}$	$\frac{5}{16}$	52	$\frac{45}{50}$	$\frac{56}{63}$	$\frac{44}{27}$
3	$\frac{5}{12}$	$\frac{3}{4}$	$\frac{3}{10}$	28	$\frac{3}{7}$	$\frac{21}{28}$	$\frac{3}{2}$	53	$\frac{64}{72}$	$\frac{77}{88}$	$\frac{48}{40}$
4	$\frac{2}{3}$	$\frac{7}{8}$	$\frac{5}{14}$	29	$\frac{5}{12}$	$\frac{2}{3}$	$\frac{20}{36}$	54	$\frac{20}{25}$	$\frac{64}{56}$	$\frac{60}{66}$
5	$\frac{11}{12}$	$\frac{3}{4}$	$\frac{6}{12}$	30	$\frac{6}{7}$	$\frac{24}{60}$	$\frac{35}{56}$	55	$\frac{28}{35}$	$\frac{64}{72}$	$\frac{64}{49}$
6	$\frac{3}{5}$	$\frac{4}{15}$	$\frac{10}{12}$	31	$\frac{11}{12}$	$\frac{3}{4}$	$\frac{20}{48}$	56	$\frac{36}{45}$	$\frac{44}{48}$	$\frac{84}{77}$
7	$\frac{2}{9}$	$\frac{16}{20}$	$\frac{27}{24}$	32	$\frac{56}{64}$	$\frac{63}{81}$	$\frac{25}{30}$	57	$\frac{48}{54}$	$\frac{36}{32}$	$\frac{18}{21}$
8	$\frac{8}{15}$	$\frac{6}{30}$	$\frac{20}{24}$	33	$\frac{11}{14}$	$\frac{36}{42}$	$\frac{28}{77}$	58	$\frac{49}{54}$	$\frac{49}{63}$	$\frac{64}{24}$
9	$\frac{9}{15}$	$\frac{21}{28}$	$\frac{10}{18}$	34	$\frac{23}{46}$	$\frac{34}{96}$	$\frac{24}{20}$	59	$\frac{42}{57}$	$\frac{76}{95}$	$\frac{52}{39}$
10	$\frac{6}{11}$	$\frac{2}{9}$	$\frac{33}{36}$	35	$\frac{32}{40}$	$\frac{56}{24}$	$\frac{81}{90}$	60	$\frac{58}{87}$	$\frac{69}{46}$	$\frac{48}{54}$
11	$\frac{5}{6}$	$\frac{18}{30}$	$\frac{9}{4}$	36	$\frac{17}{51}$	$\frac{27}{36}$	$\frac{25}{30}$	61	$\frac{68}{85}$	$\frac{45}{28}$	$\frac{48}{60}$
12	$\frac{8}{9}$	$\frac{7}{10}$	$\frac{3}{27}$	37	$\frac{18}{27}$	$\frac{14}{42}$	$\frac{16}{40}$	62	$\frac{15}{60}$	$\frac{560}{490}$	$\frac{121}{144}$
13	$\frac{5}{3}$	$\frac{4}{2}$	$\frac{21}{5}$	38	$\frac{35}{49}$	$\frac{63}{54}$	$\frac{35}{70}$	63	$\frac{42}{72}$	$\frac{81}{91}$	$\frac{69}{108}$
14	$\frac{3}{8}$	$\frac{7}{12}$	$\frac{6}{49}$	39	$\frac{24}{36}$	$\frac{51}{21}$	$\frac{20}{25}$	64	$\frac{325}{450}$	$\frac{84}{96}$	$\frac{126}{262}$
15	$\frac{5}{21}$	$\frac{9}{30}$	$\frac{8}{14}$	40	$\frac{27}{54}$	$\frac{39}{40}$	$\frac{18}{63}$	65	$\frac{135}{205}$	$\frac{82}{27}$	$\frac{100}{125}$
16	$\frac{27}{29}$	$\frac{4}{9}$	$\frac{2}{3}$	41	$\frac{33}{77}$	$\frac{81}{27}$	$\frac{37}{74}$	66	$\frac{32}{56}$	$\frac{216}{279}$	$\frac{16}{20}$
17	$\frac{15}{40}$	$\frac{1}{2}$	$\frac{8}{6}$	42	$\frac{15}{45}$	$\frac{18}{27}$	$\frac{46}{42}$	67	$\frac{217}{497}$	$\frac{71}{31}$	$\frac{108}{144}$
18	$\frac{5}{7}$	$\frac{3}{12}$	$\frac{20}{25}$	43	$\frac{13}{39}$	$\frac{49}{14}$	$\frac{16}{48}$	68	$\frac{296}{512}$	$\frac{264}{390}$	$\frac{60}{72}$
19	$\frac{3}{7}$	$\frac{35}{24}$	$\frac{4}{5}$	44	$\frac{45}{65}$	$\frac{34}{48}$	$\frac{60}{75}$	69	$\frac{121}{132}$	$\frac{196}{392}$	$\frac{36}{40}$
20	$\frac{13}{16}$	$\frac{4}{5}$	$\frac{10}{8}$	45	$\frac{49}{63}$	$\frac{19}{38}$	$\frac{41}{82}$	70	$\frac{500}{623}$	$\frac{140}{280}$	$\frac{35}{49}$
21	$\frac{12}{15}$	$\frac{3}{4}$	$\frac{5}{3}$	46	$\frac{84}{96}$	$\frac{34}{51}$	$\frac{38}{19}$	71	$\frac{185}{296}$	$\frac{432}{648}$	$\frac{8}{10}$
22	$\frac{16}{18}$	$\frac{20}{24}$	$\frac{15}{60}$	47	$\frac{48}{60}$	$\frac{50}{75}$	$\frac{35}{25}$	72	$\frac{368}{828}$	$\frac{296}{189}$	$\frac{12}{20}$
23	$\frac{17}{34}$	$\frac{6}{7}$	$\frac{8}{3}$	48	$\frac{42}{49}$	$\frac{60}{63}$	$\frac{44}{52}$	73	$\frac{130}{208}$	$\frac{216}{270}$	$\frac{81}{96}$
24	$\frac{3}{10}$	$\frac{2}{3}$	$\frac{25}{40}$	49	$\frac{18}{33}$	$\frac{22}{40}$	$\frac{60}{15}$	74	$\frac{162}{216}$	$\frac{168}{189}$	$\frac{81}{96}$
25	$\frac{2}{9}$	$\frac{4}{8}$	$\frac{30}{45}$	50	$\frac{45}{48}$	$\frac{26}{13}$	$\frac{39}{3}$	75	$\frac{252}{288}$	$\frac{198}{215}$	$\frac{48}{16}$

293. Examples in Fractions, to be used at the discretion of the teacher.

ADDITION: As $\frac{3}{16}+\frac{4}{5}$. SUBTRACTION: As $\frac{4}{5}-\frac{3}{16}$.

MULTIPLICATION: As $\frac{3}{16}\times\frac{4}{5}$. DIVISION: As $\frac{3}{16}\div\frac{4}{5}$.

1	$\frac{3}{16}$	$\frac{4}{5}$	28	$\frac{5}{4}$	$\frac{3}{11}$	55	$\frac{25}{36}$	$\frac{12}{45}$	82	$33\frac{4}{25}$	$7\frac{3}{4}$
2	$\frac{5}{9}$	$\frac{7}{20}$	29	$\frac{6}{7}$	$\frac{1}{2}$	56	$\frac{84}{16}$	$\frac{75}{14}$	83	$51\frac{2}{9}$	$6\frac{4}{13}$
3	$\frac{13}{40}$	$\frac{17}{18}$	30	$\frac{3}{9}$	$\frac{4}{3}$	57	$\frac{31}{90}$	$\frac{41}{80}$	84	$89\frac{3}{5}$	$4\frac{2}{7}$
4	$\frac{20}{21}$	$\frac{3}{16}$	31	$\frac{8}{5}$	$\frac{6}{11}$	58	$\frac{15}{60}$	$\frac{97}{81}$	85	$37\frac{3}{44}$	$5\frac{11}{12}$
5	$\frac{4}{15}$	$\frac{30}{39}$	32	$\frac{5}{12}$	$\frac{3}{8}$	59	$\frac{27}{61}$	$\frac{43}{17}$	86	$6\frac{4}{6}$	$15\frac{14}{25}$
6	$\frac{8}{21}$	$\frac{11}{12}$	33	$\frac{11}{15}$	$\frac{3}{4}$	60	$\frac{82}{96}$	$\frac{13}{65}$	87	$7\frac{8}{9}$	$26\frac{12}{50}$
7	$\frac{15}{8}$	$\frac{12}{25}$	34	$\frac{2}{3}$	$\frac{20}{21}$	61	$\frac{4}{20}$	$\frac{5}{14}$	88	$5\frac{3}{18}$	$14\frac{7}{20}$
8	$\frac{4}{19}$	$\frac{20}{3}$	35	$\frac{5}{9}$	$\frac{3}{30}$	62	$\frac{12}{18}$	$\frac{24}{60}$	89	$7\frac{3}{5}$	$54\frac{11}{16}$
9	$\frac{5}{18}$	$\frac{13}{12}$	36	$\frac{81}{17}$	$\frac{17}{25}$	63	$\frac{22}{33}$	$\frac{18}{40}$	90	$81\frac{2}{25}$	$9\frac{17}{24}$
10	$\frac{14}{17}$	$\frac{10}{21}$	37	$\frac{14}{51}$	$\frac{18}{56}$	64	$\frac{5}{25}$	$\frac{6}{18}$	91	$39\frac{6}{7}$	$4\frac{8}{11}$
11	$\frac{8}{15}$	$\frac{24}{28}$	38	$\frac{35}{60}$	$\frac{5}{18}$	65	$\frac{3}{4}$	$\frac{40}{60}$	92	$63\frac{16}{21}$	$3\frac{9}{25}$
12	$\frac{9}{20}$	$\frac{12}{7}$	39	$\frac{41}{42}$	$\frac{13}{14}$	66	$\frac{26}{52}$	$\frac{4}{9}$	93	$80\frac{5}{12}$	$4\frac{3}{11}$
13	$\frac{20}{25}$	$\frac{4}{5}$	40	$\frac{29}{70}$	$\frac{16}{11}$	67	$\frac{5}{7}$	$\frac{42}{49}$	94	$17\frac{15}{16}$	$5\frac{2}{19}$
14	$\frac{5}{11}$	$\frac{7}{12}$	41	$\frac{62}{84}$	$\frac{25}{85}$	68	$\frac{8}{11}$	$\frac{56}{64}$	95	$49\frac{4}{7}$	$8\frac{6}{29}$
15	$\frac{8}{34}$	$\frac{15}{17}$	42	$\frac{24}{60}$	$\frac{29}{90}$	69	$\frac{2}{3}$	$\frac{72}{96}$	96	$5\frac{3}{7}$	$21\frac{2}{3}$
16	$\frac{19}{27}$	$\frac{5}{6}$	43	$\frac{45}{72}$	$\frac{3}{19}$	70	$\frac{15}{35}$	$\frac{8}{11}$	97	$18\frac{2}{5}$	$7\frac{8}{9}$
17	$\frac{3}{7}$	$\frac{40}{49}$	44	$\frac{73}{91}$	$\frac{13}{20}$	71	$12\frac{5}{6}$	$8\frac{3}{7}$	98	$6\frac{4}{21}$	$15\frac{3}{8}$
18	$\frac{12}{81}$	$\frac{24}{56}$	45	$\frac{16}{25}$	$\frac{30}{64}$	72	$14\frac{2}{3}$	$5\frac{5}{9}$	99	$29\frac{1}{4}$	$3\frac{16}{17}$
19	$\frac{27}{50}$	$\frac{60}{72}$	46	$\frac{5}{80}$	$\frac{24}{90}$	73	$21\frac{6}{7}$	$3\frac{6}{11}$	100	$4\frac{5}{18}$	$21\frac{4}{5}$
20	$\frac{13}{39}$	$\frac{14}{15}$	47	$\frac{32}{51}$	$\frac{39}{34}$	74	$91\frac{4}{5}$	$8\frac{5}{8}$	101	$10\frac{10}{11}$	$3\frac{2}{7}$
21	$\frac{29}{30}$	$\frac{33}{58}$	48	$\frac{26}{40}$	$\frac{81}{60}$	75	$29\frac{2}{7}$	$9\frac{5}{12}$	102	$36\frac{8}{9}$	$5\frac{11}{20}$
22	$\frac{11}{80}$	$\frac{16}{20}$	49	$\frac{95}{99}$	$\frac{45}{25}$	76	$8\frac{3}{4}$	$15\frac{4}{9}$	103	$4\frac{5}{6}$	$18\frac{3}{4}$
23	$\frac{65}{70}$	$\frac{4}{19}$	50	$\frac{22}{30}$	$\frac{63}{72}$	77	$15\frac{7}{15}$	$3\frac{1}{11}$	104	$29\frac{3}{5}$	$3\frac{7}{16}$
24	$\frac{3}{61}$	$\frac{21}{40}$	51	$\frac{82}{94}$	$\frac{29}{60}$	78	$29\frac{4}{17}$	$2\frac{1}{11}$	105	$5\frac{4}{5}$	$19\frac{20}{26}$
25	$\frac{15}{19}$	$\frac{14}{11}$	52	$\frac{16}{38}$	$\frac{44}{36}$	79	$36\frac{5}{29}$	$4\frac{3}{7}$	106	$33\frac{2}{9}$	$5\frac{2}{5}$
26	$\frac{5}{6}$	$\frac{24}{25}$	53	$\frac{28}{16}$	$\frac{5}{92}$	80	$3\frac{5}{40}$	$11\frac{16}{21}$	107	$21\frac{4}{7}$	$3\frac{5}{8}$
27	$\frac{15}{20}$	$\frac{4}{5}$	54	$\frac{70}{88}$	$\frac{21}{49}$	81	$5\frac{9}{16}$	$31\frac{4}{17}$	108	$36\frac{3}{5}$	$2\frac{4}{9}$

CHAPTER XI.

DENOMINATE NUMBERS.

MEASURES AND WEIGHTS.

Article 294. Length, or distance, is ascertained by applying to the object to be measured a measure of length, as an inch, a foot, a yard, etc. The length of this page is about 8 inches.

The principal unit of length is the **yard**, which is divided into feet and into inches.

The standard yard of the United States is a brass bar, which is deposited in the Office of Weights and Measures at Washington. Copies of this standard yard have been distributed to the United States Custom houses; other copies have been given to the State Governments.

The U. S. standard yard was obtained from the British standard yard.

MEASURES OF LENGTH.

295.

LINEAR (OR LONG) MEASURE.

TABLE.

12 inches (*in.*) = 1 foot, (*ft.*)

3 feet = 1 yard, (*yd.*)

5½ yards
16½ feet, } = 1 rod, (*rd.*)

320 rods = 1 mile, (*mi.*)

UNIT EQUIVALENTS.

mi.		*rd.*		*yd.*		*ft.*		*in.*
1	=	320	=	1760	=	5280	=	63360.
		1	=	5½	=	16½	=	198.
				1	=	3	=	36.
						1	=	12.

Note. The English mile (given in the above table) is much shorter than the mile of other European nations. Thus, a German mile is 8106 yards, that is, $4\frac{3}{5}$ English miles. A Geographical, or Nautical mile, or knot), is 1.15 English miles, nearly.

SURVEYORS' LINEAR MEASURE.

296. A Gunter's Chain, used by land surveyors, is 4 rods or 66 feet long, and consists of 100 links.

The unit is the **chain**; and the table consists of divisors and multiples of this unit.

TABLE.

7.92 inches (*in.*) = 1 link, (*l.*)
25 links, = 1 rod, (*rd.*)
4 rods, or 66 feet, = 1 chain, (*ch.*)
80 chains, = 1 mile, (*mi.*)

UNIT EQUIVALENTS.

mi.		*ch.*		*rd.*		*l.*		*in.*
1	=	80	=	320	=	8000	=	63360
		1	=	4	=	100	=	792
				1	=	25	=	198
						1	=	7.92

MEASURES OF SURFACE.—SQUARE MEASURE.

297. Surface, or area, is measured by square units, as square inches, square feet, square miles, etc. These square units are the squares of linear units. The principal unit of surface is the **square yard**. Area is a product of which length and breadth are the factors. Thus, the number of square inches in a square foot is the product of the length and breadth of a square foot, expressed in inches, that is, 12 × 12 = 144 square inches.

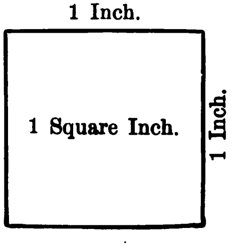


TABLE.

144 square inches (*sq. in.*) = 1 square foot, (*sq. ft.*)
9 " feet = 1 " yard, (*sq. yd.*)
30¼ " yards = 1 " rod, (*sq. r.*)
160 " rods = 1 acre, (*A.*)
640 acres = 1 square mile, (*sq. mi.*)

UNIT EQUIVALENTS.

sq. mi.	*A.*	*sq. rd.*	*sq. yd.*	*sq. ft.*	*sq. in.*
1=	640=	102400=	3097600 =	27878400 =	4014489600
	1=	160=	4840 =	43560 =	6272640
		40=	1210 =	10890 =	1568160
		1=	30¼=	272¼=	39204
			1 =	9 =	1296
				1 =	144

SURVEYORS' SQUARE MEASURE.

298. This measure is used by surveyors.

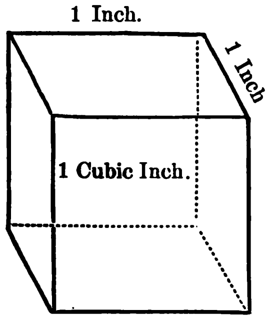
TABLE.

625 square links (*sq. l.*) = 1 pole, (*P.*)
16 poles = 1 square chain, (*sq. ch.*)
10 square chains = 1 acre, (*A.*)
640 acres = 1 square mile, (*sq. mi.*)
36 square miles (6 miles sq.) = 1 township, (*Tp.*)

NOTES. A square mile of land, as surveyed by U. S. Government surveyors, is called a *Section*. The "section lines," that is, the lines bounding the sections, extend North and South, and East and West. 36 sections constitute a township. A tier, or row, of townships extending North and South, is called a *Range*. Ranges are numbered East and West of a *Principal Meridian*. The townships in each range are numbered North and South of a *Base-line*, which is the parallel of latitude forming the base of the survey.

MEASURES OF VOLUME.

299. Volume, or solidity, is the product of three dimensions, length, breadth, and thickness.



The number of cubic inches in a cubic foot is the product of the length breadth, and thickness of a cubic foot expressed in inches: 12×12×12 =1728 cubic inches.

Volume is measured by units of volume, or cubical units; as cubic inches, cubic feet, etc. These cubical units are cubes of linear units.

The standard of volume or solidity is the **Cubic Yard.**

CUBIC MEASURE.

300. Cubic measure is used in finding the volume of solids, such as logs, stones, etc; the capacity of rooms, bins, etc.

TABLE.

1728 cubic inches (*cu. in.*)	=	1 cubic foot,	(*cu. ft.*)
27 cubic feet	=	1 cubic yard,	(*cu. yd.*)
128 cubic feet	=	1 cord of wood	(*Cd.*)

NOTES. Observe that as 1728 is the cube of 12, as stated above, so 27, the number of cubic feet in one cubic yard, is the cube of 3, the number of feet in one linear yard. A pile of wood or stone 8 feet long, 4 feet wide and 4 feet high, contains one cord. A cord foot is ⅛ of the length of the pile, and contains 16 cubic feet.

A perch of stone, or of masonry, contains 24¾ cubic feet.

For examples in linear, square, and cubic measures, see Art. 315, examples 1 to 40.

LIQUID MEASURE.

301. Liquid Measure is used in measuring liquids, such as milk, oil, etc.

TABLE.

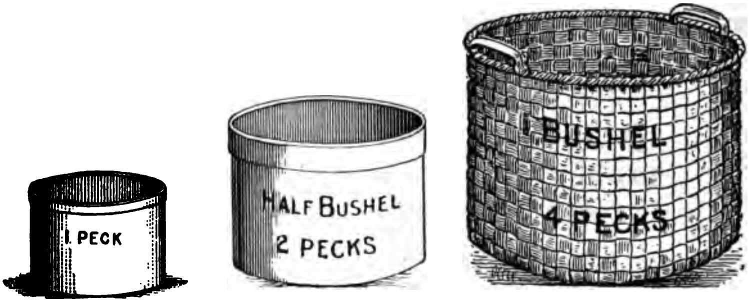
4 gills (*gi.*) = 1 pint, (*pt.*)
2 pints = 1 quart, (*qt.*)
4 quarts = 1 gallon, (*gal.*)

UNIT EQUIVALENTS.

gal.		*qt.*		*pt.*		*gi.*
1	=	4	=	8	=	32.
		1	=	2	=	8.
				1	=	4.

NOTES. A barrel formerly was regarded as a measure containing 31½ gallons; and a hogshead equalled 2 barrels. Barrels now frequently contain 50 gallons or more.

1 gallon=231 cubic inches.



DRY MEASURE.

302. Dry Measure is used in weighing such articles as grain, fruits, etc.

TABLE.

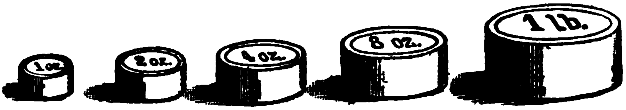
2 pints (*pts.*) = 1 quart, (*qt.*)
8 quarts = 1 peck, (*pk.*)
4 pecks = 1 bushel, (*bu.*)

UNIT EQUIVALENTS.

bu.		*pk.*		*qt.*		*pt.*
1	=	4	=	32	=	64.
		1	=	8	=	16.
				1	=	2.

NOTES. 1 bushel=2150.4 cubic inches. Wheat and other cereals, though nominally measured by the bushel, are actually, especially in large quantities, bought and sold by weight. (See page 334.)

Examples 41 to 64, Art. 315, illustrate Liquid and Dry measures.


1 oz.
2 oz.
4 oz.
8 oz.
1 lb.

AVOIRDUPOIS WEIGHT.

303. Avoirdupois Weight is used for all ordinary purposes of weighing.

TABLE.

16 ounces (*oz.*) = 1 pound, (*lb.*)
100 lb. = 1 hundred weight, (*cwt.*)
20 cwt., or 2000 lbs. = 1 ton, (*T.*)

UNIT EQUIVALENTS.

T.		*cwt.*		*lbs.*		*oz.*
1	=	20	=	2000	=	32000
		1	=	100	=	1600
				1	=	16

NOTES. The "long ton" of 2240 lbs. is used in Great Britain, in U. S. Custom Houses, and at the Pennsylvania coal mines.

TROY WEIGHT.

304. Troy Weight is used in weighing gold, silver and jewels.

TABLE.

24 grains, (*gr.*)	= 1 pennyweight,	(*pwt.*)
20 pennyweights	= 1 ounce,	(*oz.*)
12 ounces	= 1 pound,	(*lb.*)

UNIT EQUIVALENTS.

lb.		*oz.*		*pwt.*		*gr.*
1	=	12	=	240	=	5760
		1	=	20	=	480
				1	=	24

A *carat* is a weight of about $3\frac{1}{5}$ grains, used in weighing diamonds. The word *carat* also signifies one twenty-fourth part, when used to denote the purity of manufactured gold. Thus, a ring of "18 carats fine" is $\frac{18}{24}$ pure gold and $\frac{6}{24}$ alloy.

APOTHECARIES' WEIGHT.

305. Apothecaries' Weight is used by apothecaries and physicians in compounding dry medicines; but drugs are bought and sold by avoirdupois weight.

TABLE.

20 grains (*gr.*)	= 1 scruple,	(*sc.* or ℈.)
3 scruples	= 1 dram,	(*dr.* or ʒ.)
8 drams	= 1 ounce,	(*oz.* or ℥.)
12 ounces	= 1 pound,	(*lb.* or ℔.)

UNIT EQUIVALENTS.

lb.		*oz.*		*dr.*		*sc.*		*gr.*
1	=	12	=	96	=	288	=	5760
		1	=	8	=	24	=	480
				1	=	3	=	60
						1	=	20

APOTHECARIES' FLUID MEASURE.

306. The measures for fluids, as adopted by apothecaries and physicians in the United States, to be used in compounding medicines and preparing for market, are given in the following

TABLE.

60 minims, (*m*) = 1 flüidrachm, (fʒ.)
8 fluidrachms = 1 fluidounce, (f℥.)
16 fluidounces = 1 pint, (O.)
8 pints = 1 gallon, (Cong.)

NOTES. O stands for *Octavius*, the Latin for *one eighth*. Cong. is for *Congius*, the Latin for *gallon*.

COMPARISON OF WEIGHTS.

307. The pound, ounce and grain, troy, are equal, respectively, to the pound, ounce and grain apothecaries' weight. The pound avoirdupois is $\frac{175}{144}$ of the pound troy.

TABLE.

1 lb. troy = 5760 grains troy.
1 lb. apoth. = 5760 " "
1 lb. avoir. = 7000 " "
1 oz. troy = 480 " "
1 oz. apoth. = 480 " "
1 oz. avoir. = 437½ " "

Examples 41 to 51, Art. 315, illustrate the tables of weights.

MISCELLANEOUS TABLE.

308.

12 units = 1 dozen.
12 dozen = 1 gross.
12 gross = 1 gt. gross.
20 units = 1 score.

24 sheets paper = 1 quire.
20 quires " = 1 ream.
10 reams " = 1 bale.

ANGULAR OR CIRCULAR MEASURE.

309. **Angular, or circular, measure is used in measuring arcs and angles. This is done by a variety of instruments.**

310. **An arc measures the angle formed by two radii drawn to the extremities of the arc.**

The angle D C E is formed by the two radii D C and E C meeting at the point C. which is the center of the circle. The angle D C E is measured by the arc D E.

The angle D C B is a right angle; it is measured by the arc D B, which contains 90 degrees. The angle D C E, one half of the angle D C B, is measured by the arc D E. which is one half of the arc D B.

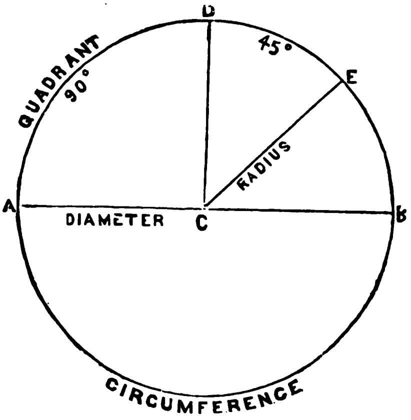


TABLE.

60 seconds (″) = 1 minute, (′)
60 minutes = 1 degree, (°)
360 degrees = 1 circumference, (*cir.*)

Cir.		°		′		″
1	=	360	=	21600	=	1296000
		1	=	60	=	3600
				1	=	60

NOTES. The unit of angular measure is the degree. A degree is measured by one of the 360 equal parts of the circumference of a circle. The length of a degree varies with the diameter of the circle, but the angle which is measured by an arc of one degree does not vary. Thus, if the circumference measures 360 inches, an arc of one degree is 1 inch; if the circumference measures 720 inches, the arc of one degree measures 2 inches; but the angle at the centre measured by the arc, is the same in both circles.

A quadrant is one fourth of the circumference, or 90°.

A sextant is one sixth of the circumference, or 60°.

A sign is one twelfth of the circumference, or 30°.

A minute of the circumference of the earth is a geographical, or nautical mile, and is 1.15 common miles, nearly.

A degree of longitude on the equator, is 69.16 common, or statute miles; at 30° of latitude it is 59.81 miles; at 60° of latitude it is 34.53 miles.

A degree of latitude varies from 68.72 miles near the equator, to 69.34 in the polar regions. It is sufficiently accurate to remember it as 69.16 miles.

TABLE OF TIME.

311.

60 seconds (*sec.*) = 1 minute, (*min.*)
60 minutes = 1 hour, (*h.*)
24 hours = 1 day, (*d.*)
7 days = 1 week, (*wk.*)
365 days, or =
12 months, or = } 1 year, (*yr.*)
52 weeks, 1 day =
366 days = 1 leap year.
100 years = 1 century.

NOTES. A Solar year is the time occupied by the earth in completing one revolution in its orbit. Its exact length is 365 days, 5 hours, 48 minutes, 46.05 seconds; nearly 365¼ days.

The common year is 365 days. Leap year which is every fourth year, contains 366 days, February having, in that year, 29 days, instead of 28.

Every year whose number is divisible by 4 is leap year, unless its number ends with two ciphers, when, to be a leap year, the number must be divisible by 400. Thus, 1896 and 2,000 will be leap years: 1900 will not be a leap year.

Examples 51 to 70, Art. 315, illustrate the three preceding tables.

REDUCTION OF DENOMINATE NUMBERS.

312. The **Reduction** of a denominate number is a change of its denomination without change of its value.

Thus, 4 quarts may be changed, or reduced, to 8 pints, or to 1 gallon. 1 yd. 2 ft. may be expressed as 5 feet, or as 60 inches.

313. There are two cases of reduction of denominate numbers:

I. The reduction of a denominate number from a higher to a lower denomination.

II. The reduction of a denominate number from a lower to a higher denomination.

The reduction of feet to inches, of gallons to pints, etc., illustrates the first case.

The reduction of inches to feet, of pints to gallons, etc., illustrates the second case.

314. Since a unit of any denomination always contains two or more units of a lower denomination, the reduction of a number from a higher to a lower denomination is always a process of *Multiplication.*

Thus, feet are always reduced to inches by multiplying the number of inches in one foot by the number of feet to be changed to inches; as, 8 ft. = 8 times 12 inches = 96 inches. $\frac{1}{3}$ ft. = $\frac{1}{3} \times 12$ inches = 4 inches.: $\frac{1}{4}$ lb. = $\frac{1}{4} \times 16$ oz. = 4 oz.: $2\frac{1}{2}$ bu. = $2\frac{1}{2} \times 32$ qt. = 80 qt.

Observe, that the above statement is true whether the number to be reduced is integral or fractional.

315. For the reason stated in the preceding article, the reduction of a number from a lower to a higher denomination is always a process of *Division.*

Thus, since there are 3 feet in 1 yard, in any number of feet there are *one third* as many yards: as, 18 ft. = (18 ÷ 3) yd. = 6 yd.: 9 cu. ft. = (9 ÷ 27) cu. yd. = $\frac{1}{3}$ cu. yd.: $\frac{1}{3}$ gal. = ($\frac{1}{3} \div 4$) qt. = $\frac{1}{12}$ qt.

Observe, as in preceding article, *the number to be reduced may be either integral or fractional.*

ORAL.

Examples 1 to 40 illustrate linear, square and cubic measures.

1. Reduce 6 feet to inches.

SOLUTION. Since there are 12 inches in 1 foot, in 6 feet there are 6 times 12 inches, which are 72 inches.

2. How many inches in 5 ft.? In 8 ft.? In 12 ft.?
3. How many feet in 7 yd.? In 9 yd.? In 15 yd.?
4. How many feet in 60 inches?

SOLUTION. Since 1 inch is one-twelfth of 1 foot, in 60 inches there are one-twelfth as many feet, which are 5 feet.

5. How many yards in 15 feet? In 24 ft.? In 30 ft.?
6. How many sq. yd. in 18 sq. ft.? In 27 sq. ft.
7. How many inches in 2 yd.? In 3 yd.?

SUGGESTIONS. Reduce the yards to feet; then reduce the feet to inches. Or, multiply the number of inches in 1 yd. by the number of yds.

8. How many inches in 4 yd.? In 5 yd.?
9. Reduce 2 yd. 1 ft. 6 in. to inches.

SOLUTION. In 2 yd. there are 6 ft.: 6 ft.+1 ft.=7 ft.: 7 ft.=84 in. 84 in.+6 in.=90 inches.

10. Reduce 1 yd. 2 ft. 8 in. to inches.
11. How many cu. ft. in 10 cu. yd. 5 cu. ft.?
12. How many sq. ft. in 5 sq. yd. 7 sq. ft.?
13. How many sq. in. in 2 sq. ft. 12 sq. in.?
14. How many sq. rd. in 1 sq. mi. 1 A. 100 sq. rd.?
15. How many sq. rd. in 1 acre? In $\frac{1}{2}$ of an acre? In $\frac{3}{4}$ A.?
16. How many sq. in. in $\frac{1}{2}$ sq. ft.? In $\frac{1}{4}$ sq. ft.?
17. How many sq. ft. in $\frac{1}{3}$ sq. yd.? In $3\frac{1}{3}$ sq. yd.?
18. How many cu. ft. in $\frac{2}{3}$ cu. yd.? In $2\frac{2}{3}$ cu. yd.?
19. How many cu. in. in $\frac{1}{2}$ cu. ft.? In $\frac{1}{4}$ cu. ft.?
20. How many in. in .25 ft.? In .25 yd.?
21. How many sq. in. in 1.25 sq. ft.?
22. How many sq. rd. in $.12\frac{1}{2}$ A.? In $.37\frac{1}{2}$ A.?

SUGGESTION. $.12\frac{1}{2}=\frac{1}{8}$.

23. How many acres in $.87\frac{1}{2}$ sq. mi.?
24. How many sq. yd. in 4 sq. rd.?
25. How many sq. ft. in 1 sq. rd.?
26. How many sq. rd. in $\frac{1}{4}$ A. 20 sq. rd.?
27. How many cu. ft. in $\frac{3}{4}$ of a cord?
28. How many sq. rd. in $\frac{7}{10}$ A.? In 1.75 A.?
29. How many cu. ft. in $10\frac{1}{3}$ cu. yd.?
30. How many sq. ft. in 288 sq. in.?
31. How many cu. yd. in 54 cu. ft.?
32. How many cords in 256 cu. ft.?

33. How many feet in 30 in.? In 40 in.? In 50 in.?

34. How many yards in 8 ft.? In 16 ft.? In 25 ft.?

35. What part of a foot is 1 inch? 3 inches? 8 inches?

36. What part of a foot is $\frac{1}{2}$ inch? $\frac{1}{3}$ inch? $\frac{1}{4}$ inch?

37. What part of a yard is $\frac{1}{2}$ of a foot?

SUGGESTION. What part of a yard is 1 foot?

38. What part of a yard is $\frac{1}{4}$ of a foot?

NOTE. The solution of this question is merely an application of the general principle stated in Art. 315, that a lower denomination is reduced to a higher denomination by *division*. Since feet are reduced to yards by dividing the given number of feet by 3, so a fraction of a foot is reduced to the fraction of a yard by division by 3. $\frac{1}{4} \div 3 = \frac{1}{12}$: therefore, $\frac{1}{4}$ of a foot is $\frac{1}{12}$ of a yard.

39. What part of a foot is $\frac{1}{2}$ of an inch?

40. What part of a square yard is $\frac{1}{3}$ of a sq. ft.?

Examples 41 to 64 illustrate Liquid and Dry measures.

41. How many pints in 8 qt.? In 12 qt.?

42. How many quarts in 8 pt.? In 12 pt.?

43. How many pints in 5 gal.? In 7 gal.?

44. How many gal. in 16 qt.? In 20 qt.?

45. How many quarts in 3 pk.? In 3 bu.?

46. How many quarts in 2 bu., 1 pk., 3 qt.?

47. How many gi. in 2 gal., 2 qt., 1 pt., 3 gi.?

48. How many pints in 3 gal., 2 qt., 1 pt.?

49. How many pints in $\frac{3}{4}$ of a peck?

50 How many quarts in $\frac{1}{4}$ peck, 5 qt.?

51. How many pecks in $\frac{3}{4}$ bu. 7 pk.?

52. How many quarts in $\frac{1}{2}$ bu. $\frac{1}{2}$ pk.?

53. How many gills in $\frac{1}{4}$ gal. $\frac{1}{2}$ pt.?

54. How many cu. in. in 2 gal.? In $\frac{1}{3}$ gal.?

55. How many gal. in 4 bbl., each containing $31\frac{1}{2}$ gal.?

56. How many cu. in. in 10 bu.?

57. Reduce 32 gills to pints; to quarts.

58. Reduce 64 pints to quarts; to pecks.

59. Reduce 64 quarts to pecks; to bushels.
60. Reduce 128 gills to gallons.
61. What part of a bushel is 1 peck? $\frac{1}{4}$ peck?
62. What fraction of a quart is $\frac{1}{2}$ pint? $\frac{1}{2}$ gill?
63. What fraction of a gallon is $\frac{2}{3}$ qt.? $\frac{3}{5}$ qt.?
64. What decimal of a qt. is 1 pt.? $\frac{1}{2}$ pt.?

Examples 65 to 86 illustrate the tables of weights.

65. How many oz. in 10 lb. avoir.? In $10\frac{3}{4}$ lb.?
66. How many lb. in 10 cwt. 80 lb.?
67. How many oz. in 5 lb. 8 oz. troy? In 5 lb. 8 oz. avoir.?
68. How many oz. in 3 cwt.? In 2 cwt. 10 lb.?
69. How many pwt. in 5 $\frac{1}{2}$ oz. troy? In 1 lb. 1 oz. troy?
70. How many gr. in 2 oz. 10 pwt. troy?
71. How many lb. in 64 oz. avoir.? In 128 oz. avoir.?
72. How many cwt. in 1800 lb.? In 5900?
73. How many tons in 8000 lb.? In 24000?
74. How many lb. troy in 96 oz.? In 144 oz.?
75. How many oz. in 120 pwt.? In 180 pwt.?
76. Reduce 72 gr. to pwt. 960 gr. to oz.
77. What part of a ton is 1 cwt.? $\frac{1}{4}$ cwt.?
78. What part of a lb. are 4 oz.? 8 oz.? 12 oz.? (Avoir.)
79. What part of a lb. is $\frac{1}{2}$ oz.? $\frac{3}{4}$ oz.? $\frac{2}{3}$ oz.? (Avoir.)
80. What part of a lb. is $\frac{1}{5}$ pwt.? $\frac{4}{5}$ pwt.?
81. What fraction of an ℥ is ℈ 1?
82. How many gr. in 5℈? In 8℈?
83. How many ℈ in ℥5 ʒ1?
84. How many ʒ in lb. 3 ℥ 4?
85. How many grains in $\frac{1}{10}$ lb. troy?
86. How many grains troy in $\frac{1}{7}$ lb. avoir.?

Examples 87 to 99 illustrate the Miscellaneous Table, the Table of Angular Measure, and the Table of Time.

87. How many units in a gross? In 3 doz.?
88. How many dozen in $5\frac{3}{4}$ gross?

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89. How many sheets in $\frac{1}{2}$ ream? In 5 quires 12 sheets?
90. How many quires in 120 sheets? In 240 sheets?
91. How many seconds in $5\frac{3}{4}$ minutes?
92. Change $8\frac{4}{7}$ weeks to months of 30 days each.
93. Change 231 days to weeks.
94. How many hours from 9 o'clock A. M. to 3 o'clock P. M.?
95. How many hours from 10:30 A. M. to 5:15 P. M.?
96. How many degrees in $\frac{1}{2}$ of a circumference? In $\frac{3}{4}$?
97. How many degrees in $\frac{5}{6}$ of a cir.? In $\frac{4}{9}$? In $\frac{5}{12}$?
98. How many degrees in 3600″?
99. How many seconds in 1° 10′ 20″?

WRITTEN.

100. Reduce 18 rods to inches.

SOLUTION. In 18 rods there are $18 \times 16\frac{1}{2}$ ft. $= 297$ ft. In 297 ft there are 297 times 12 inches $= 3564$ in.

101. Reduce 124 sq. rods to sq. ft.
102. Reduce 16 acres to sq. yd.
103. Reduce 5 A. 40 sq. rd. 28 sq. ft. to sq. ft.

SOLUTION. Change the 5 acres to sq. rd.: add the 40 sq. rd.: change this sum to sq. ft.: add the 28 sq. ft. Hence,

To reduce a denominate number from higher to lower denominations:

1. *Multiply the units of the highest denomination of the given number by the value of one of that denomination in units of the next lower.*

To this product add the given number, if any, of this lower denomination.

2. *Multiply the sum thus obtained by the number necessary to reduce it to units of the next lower denomination, and add the given number, if any, of this denomination.*

So proceed, until the number is reduced to the denomination required.

104. How many cu. ft. in 5 cords, 32 cord feet, 8 cu. ft.?
105. How many yd. in 3 mi. 80 rd. 3 yd.?
106. How many sq. in. in 27 sq. yd. 5 sq. ft. 49 sq. in.?
107. How many cu. in. in 12 cu. yd. 7 cu. ft. 84 cu. in.?
108. Reduce 2 mi. 16 rd. 7 ft. to ft.
109. Reduce 3 sq. mi. 160 A. to acres.
110. Reduce $\frac{1}{8}$ mi. to feet.
111. How many feet in .75 of a mile?
112. How many sq. feet in $\frac{1}{4}$ of an acre?
113. How many cu. in. in $\frac{4}{9}$ of a cu. yard?

Compare closely the statement and the solution of example 114 with the statement and solution of example 115.

114. Reduce .65 of a sq. rod to sq. inches.

PROCESS.

$.65 \times 30\frac{1}{4} = 19.66\frac{1}{4}$ sq. yd.
$19.66\frac{1}{4} \times 9 = 176.96\frac{1}{4}$ sq. ft.
$176.96\frac{1}{4} \times 144 = 25482.6$ sq. in.

ANALYSIS. The .65 of 1 sq. rd. are changed to sq. yd.

The $19.66\frac{1}{4}$ sq. yd. are changed to sq. ft.

The $176.96\frac{1}{4}$ sq. ft. are changed to sq. in.

Observe, that the *entire product* of each multiplication is used as a factor in the next multiplication.

Result sought, 25482.6 sq. in.

115. Reduce .65 of a sq. rd. to integers of lower denominations.

PROCESS.

$.65 \times 30\frac{1}{4} = \underline{19}.66\frac{1}{4}$ sq. yd.
$.66\frac{1}{4} \times 9 = \underline{5}.96\frac{1}{4}$ sq. ft.
$.96\frac{1}{4} \times 144 = \underline{138.6}$ sq. in.

ANALYSIS. The .65 of 1 sq. rd. are changed to $19.66\frac{1}{4}$ sq. yd. as before.

The 19 sq. yd., being integers, are reserved and the $.66\frac{1}{4}$ sq. yd. are changed to $5.96\frac{1}{4}$ sq. ft.

The 5 sq. ft. being integers, are reserved, and the $.96\frac{1}{4}$ sq. ft. are changed to 138.6 sq. in.

Result sought, 19 sq. yd. 5 sq. ft, 138.6 sq. in.

Of course, the *value* of the two results is the same.

Reduce to integers of lower denominations :

116. .84 of an acre.
117. .57 of a mile.
118. .38 of a square mile.
119. .73 of a cubic yard.
120. $\frac{3}{7}$ of rod.
121. $\frac{4}{9}$ of a square rod.
122. $\frac{3}{5}$ of a cord.
123. $\frac{3}{4}$ of a cubic yard.

124. Reduce 10000 gr. troy to higher denominations.

PROCESS.

10000 gr.÷24=416 pwt. 16 gr.
416 pwt.÷20=20 oz. 16 pwt.
20 oz.÷12=1 lb. 8 oz.
10000 gr.=1 lb. 8 oz. 16 pwt. 16 gr.

ANALYSIS. 1. There are 24 gr. in 1 pwt.; in 10000 gr. there are $\frac{10000}{24}$ pwt.=416 pwt. 16 gr.

2. There are 20 pwt. in 1 oz.; in 416 pwt. there are $\frac{416}{20}$ oz.=20 oz. 16 gr.

3. There are 12 oz. in 1 lb.; in 20 oz. there are $\frac{20}{12}$ lb =1 lb 8 oz.

125. Reduce 596 sq. ft. to higher denominations.

PROCESS.

596 sq. ft.÷9=66 sq. yd. 2 sq. ft.

66 sq. yd.÷30$\frac{1}{4}$=264÷121=2 sq. rd. 5$\frac{1}{2}$ sq. yd.

$\frac{1}{2}$ sq. yd.=4$\frac{1}{2}$ sq. ft. ; 4$\frac{1}{2}$ sq. ft.+2 sq. ft.(remainder from first division)=6$\frac{1}{2}$ sq. ft.

596 sq. ft.=2 sq. rd. 5 sq. yd. 6$\frac{1}{2}$ sq. ft. Hence,

To reduce a denominate number from a lower to higher denominations:

1. *Divide the number given by the number of its denomination which equals a unit of the next higher denomination. The quotient will be the number of that denomination.*
2. *Divide this quotient by the number of its denomination which equals a unit of the next higher denomination. So proceed. The last quotient, with the several remainders, will be the result sought.*

NOTE. Reduction Ascending is the converse of Reduction Descending. One is proof of the other.

126. How many miles, etc., in 18000 rods?
127. How many A. in 598464 sq. ft.?
128. How many T. in 794863 lbs.?
129. How many gal. in 156970 gills?
130. How many lbs. troy in 16759 grains?
131. How many quarter sections in 151200 sq. rods?
132. How many lbs in 3975 ℈?
133. How many ℥ in 7957 gr.
134. How many bush. in 800 pints?
135. How many cords in 9684 cu. ft.?
136. How many cords in 1898000 cu. in.?
137. How many cu. ft. in 75894 cu. in.?
138. How many perches of stone in 27546 cu. ft.?
139. How many A. in 625000 sq. links?
140. How many miles in 84694 chains?
141. How many ch. in 484675 inches?
142. How many rods in 92600 inches?
143. How many rods in 49862 feet?
144. How many sq. miles in 8000000 sq. rods?
145. How many cords in 849670 cu. ft.?
146. 1 ft. is what fraction of a mile?

SOLUTION. In one mile there are 5280 ft.; therefore 1 foot is $\frac{1}{5280}$ of a mile.

147. 5 lb. 8 oz. is what part of a ton?

SOLUTION. In 1 T. there are 32000 oz; in 5 lb. 8 oz. there are 88 oz.: therefore 5 lb. 8 oz are $\frac{88}{32000}$ T. or $\frac{11}{4000}$ T.

148. What fraction of a cord is $\frac{5}{9}$ cu. ft.?

SOLUTION. In 1 cd. there are 128 cu. ft. or $\frac{1152}{9}$ cu. ft. $\frac{5}{9}$ cu. ft. $\div \frac{1152}{9} = \frac{5}{9}$ cu. ft. $\times \frac{9}{1152} = \frac{5}{1152}$ cd.

149. 140 lb. 8 oz. is what part of a ton?
150. Reduce 3 yd. 1 ft. 5 in. to a fraction of a rod.
151. Reduce 3 A. 20 sq. rd. to a fraction of a sq. mile.
152. What fraction of a cord are 864 cu. in.?
153. What fraction of a ream is a sheet of paper?

154. Reduce 3 inches to a decimal of a foot.

SOLUTION. 3 inches=$\frac{3}{12}$ ft. or $\frac{1}{4}$ foot; $\frac{1}{4}$ ft.=.25 ft.

155. Reduce $1\frac{1}{2}$ feet to the decimal of a yard.

156. Reduce $2\frac{3}{4}$ yd. to the decimal of a rod.

157. Reduce $4\frac{1}{8}$ ft. to decimal of a rod.

158. Reduce 18 h. 15 min. to a decimal of a year.

159. Reduce 13 A. 26 sq. rd. 8 sq. ft. 16 sq. in. to decimal of mile sq.

160. What fraction of a lb. is $\frac{1}{2}$ oz. av.?

SOLUTION 1 oz.=$\frac{1}{16}$ lb.; $\frac{1}{2}$ oz.=($\frac{1}{2}\times\frac{1}{16}$) lb.=$\frac{1}{32}$ lb.

161. $\frac{4}{5}$ of a lb. is what part of a cwt.?

SOLUTION. 1 lb. is $\frac{1}{100}$ cwt.; $\frac{1}{5}$ lb. is $\frac{1}{500}$ cwt.; $\frac{4}{5}$ lb. is $\frac{4}{500}$ cwt. or $\frac{1}{125}$ cwt.

162. What decimal of a yard is $\frac{1}{2}$ ft.?

SOLUTION. 1 ft.=$\frac{1}{3}$ yd.; $\frac{1}{2}$ ft.=($\frac{1}{2}\times\frac{1}{3}$) yd.=$\frac{1}{6}$ yd.=.16+ yd.

163. What fraction of a foot is $\frac{7}{8}$ in.?

164. What fraction of a bu. is $\frac{3}{4}$ pk.?

165. What fraction of a mile is $\frac{1}{2}$ rod?

166. What decimal of a sq. m. is $\frac{4}{5}$ A.?

167. What decimal of a degree is $\frac{1}{2}$ minute?

168. What decimal of a cd. is $\frac{1}{4}$ cu. in.?

169. What decimal of a T. is $\frac{3}{4}$ oz.?

170. Change 50 lbs. troy to lbs. av.

SOLUTION. I. 1 lb. troy=5760 grains; 50 lb troy=50×5760 gr.=288000 gr.; 288000 gr.÷7000 (1 lb. av.)=$41\frac{1}{7}$ lb. av.

SOLUTION. II. 50 lbs. troy×$\frac{144}{175}$=$41\frac{1}{7}$ lb. av.

171. Change 50 lbs. av. to lbs. troy.

SOLUTION. I. 50 lbs. av.=50×$\frac{7000}{5760}$=$60\frac{55}{72}$ lbs. troy.

SOLUTION. II. 50 lbs. av.=50×$\frac{175}{144}$=$60\frac{55}{72}$ lbs. troy.

172. Change 144 lbs. troy to lbs. av.

173. Change 375 lbs. av. to lbs. troy.

174. Change 15 lbs. apoth. to lbs. av.

175. Change 81 lbs. av. to lbs. apoth.

176. Change 168 lbs. troy to lbs. av.

177. Change 262 lbs. av. to lbs. troy.

Examples to be used at the discretion of the teacher:

316. REDUCTION. Regard the following numbers as

Inches,	Pints (*Dry*),	Grains (*Troy*),
Square Inches,	Drams (*Av.*),	Gills (*Liq.*),
Cubic Inches	Seconds of Time.	

1	75574	6	92437	11	84375	16	49873
2	89320	7	54692	12	91248	17	29675
3	45673	8	37849	13	65927	18	65720
4	98437	9	83726	14	82543	19	80964
5	83743	10	95433	15	75135	20	59720

317. DENOMINATE NUMBERS AND U. S. MONEY.

	Find Cost of	at	per		Find Cost of	at	per
1	718 bu.	$ .08	pk.	21	4 T.	$ .18	lb.
2	574 gal.	.15	qt.	22	5 hhd.	.12½	qt.
3	175 A.	8.	sq. rd.	23	3 sq. mi.	1.05	sq. rd.
4	351 lbs.	40.	T.	24	7 cd.	.87½	cu. ft.
5	480 gal.	63.	hhd.	25	8 A.	1.16	sq. sd.
6	224 sq. yd	18.	sq. ft.	26	4 bu.	1.60	bu.
7	118 T.	75.	cwt.	27	5 lb.	15.	lb.
8	960 lbs.	5.25	pwt.	28	3 bbl.	20.	hhd.
9	189 cu. ft.	.005	cu. in.	29	12 A.	5.00	sq. rd.
10	487 lbs.	.125	oz.	30	8 cwt.	100.	T.
11	384 cd.	.20	cu. ft.	31	3 pk.	1.60	bu.
12	640 cu. ft.	8.75	cord.	32	8 oz.	80c	oz.
13	864 sq. rd.	16.	A.	33	25 gal.	40.	gal.
14	329 qts.	.64	bu.	34	8 sq. ft.	9.00	sq. yd.
15	720 qts.	.48	gal.	35	9 sq. rd.	16.	A.
16	196 sq. mi	.80	sq. rd.	36	4 T.	4.	cwt.
17	325 bu.	.12	qt.	37	3 hhd.	30.	bbl.
18	320 sq. rd	50.	A.	38	2 hhd.	10c	qt.
19	840 T.	9.	cwt.	39	3 R.	40.	A.
20	102 cwt.	.30	lb.	40	.6 bbl.	.05	pt.

Money and Currencies.

English Money.

318. English or Sterling Money is the currency of Great Britain. The unit is the **pound sterling.**

Table.

4 farthings (*far.* or *qr.*) = 1 penny, (*d.*)
12 pence = 1 shilling, (*s.*)
20 shillings = 1 pound or sovereign, (£ or *sov.*)

Notes. The sovereign is equal in value to $4.86. Its value may be roughly estimated as five dollars. £10 8/6 is read ten pounds, eight shillings and sixpence.

Canada Money.

319. The currency of the Canadian provinces is decimal; the table and denominations are the same as those of United States money.

Note. The decimal currency was adopted by the Canadian Parliament in 1858, and the Act took effect in 1859. Previous to the latter year the money of Canada was reckoned in pounds, shillings, and pence, the same as in England.

French Money.

320. The currency of France is a decimal currency, of which the unit is the franc.

Table.

10 millimes (*m.*) = 1 centime, (*ct.*)
10 centimes = 1 decime, (*dc.*)
10 decimes = 1 franc, (*fr.*)

Notes. A franc is equal in value to 19.3 cents, U. S. money. It may be roughly estimated as 20 cents, a fifth of a dollar. Centimes are written as hundredths of a franc. Thus, 3.65 francs = 3 francs and 65 centimes.

The *Lira* of Italy, the *Peseta* of Spain, and the *Drachma* of Greece, have the same value as the franc.

GERMAN MONEY.

321. The unit of the money of the German Empire is the "mark" (Reichsmark), which is divided into 100 pennies (pfennige).

NOTE. The mark is equal in value to 23.85 cents, U. S. money. It may be roughly estimated as worth 25 cents.

EXAMPLES.

1. How many farthings in one shilling and six pence?
2. How many pence in 1 pound 10 shillings?
3. How many centimes in 1 franc?
4. How many pennies in 4 marks?
5. What is the value in U. S. money of 1 shilling English money?
6. What is the value in U. S. money of a five franc piece?
7. What is the value in U. S. money of a five mark piece?
8. Estimate roughly the value in U. S. money of £150. Of £600.
9. Estimate roughly the value of 5000 francs. Of 750 francs.
10. Estimate roughly the value of 400 marks. Of 1200 marks.

ADDITION AND SUBTRACTION.

322. The principles which are used in the addition, subtraction, multiplication and division of simple numbers apply to the corresponding processes in denominate numbers. In the latter, however, these operations are more difficult, since the tables of denominate numbers are seldom constructed on the principle that ten units make one unit of the next higher denomination.

Compare the table of U. S. money with the table of English money, Art. 318.

323. 1. Find the sum of 17 cwt. 15 lb. 11 oz.; 2 cwt. 9 lb 8. oz; 90 lb. 12 oz.

PROCESS.

T.	*cwt.*	*lb.*	*oz.*
	17	15	11
	2	9	8
		90	12
1	0	15	15

ANALYSIS. 1. Write the numbers to be added so that units of the same denomination shall stand in the same column.

2. Find the sum of the lowest column (oz.) which is 31 oz.=1 lb. 15 oz. Write the 15 oz. under the column of oz.; add the 1 lb. to the column of lbs.

3. Find the sum of the column of lbs., which is 115 lbs.=1 cwt. 15 lbs. Write the 15 lbs. under the column of lbs.; add the 1 cwt. to the column of cwt.

4. Find the sum of the column of cwt., which is 20 cwt.=1 T. Write 1 T. 0 cwt.

2. Add 20 T. 17 cwt. 40 lb. 5 oz. ; 5 T. 15 cwt. 7 lb. 10 oz.; 8 cwt. 8 oz.; 1 T. 50 lb.; 11 T. 89 lb. 15 oz.

3. Add 7 m. 2 fur. 6 rd. 1 yd. 3.5 in.; 3 m. 5 fur. 38 r. 5 yd. 7.75 in.; 18 m. 1 fur. 25 rd. 3 yd. 11.4 in.; 19 r. 8 in.; 32 rd. 3 yd.

4. Add 18 bu. 3 pk. 5 qt.; 6 bu. 2 qt. 1 pt.; 11 bu. 7 qt. 2 pt.; 6 bu. 2 pk. 4 qt. 1 pt.; 21 bu. 3 pk. 5 qt.

5. Add 11° 49′ 59″; 16° 17′ 8″; 40′ 21″; 3° 9′ 29″; 48′ 6″; 11′ 38″.

6. From 18 yr. 5 mo. 17 d. subtract 12 yr. 9 m. 14 d.

PROCESS.

yr.	*mo*	*d.*
18	5	17
12	9	14
5	8	3

ANALYSIS. 1. Write the subtrahend under the minuend, units of the same denomination in same column.

2. Subtract, beginning with the lowest denomination: 17d.−14 d.=3 d.

3. It is impossible to take 9 mo. from 5 mo.; therefore from the 18 yr. take 1 yr.=12 mo.; 5 mo.+12 mo.=17 mo. 17 mo.−9 mo.=8 mo.

4. The difference between 17 yr. and 12 yr. is 5 yr.

7. From 20 cu. yd. 4 cu. ft. 1600 cu. in. take 5 cu. yd. 7 cu. ft. 1590 cu. in.

8. From 89 T. take 70 T. 17 cwt. 49 lb. 1.8 oz.
9. From 13 lb. 6 oz. 5 pwt. take 9 oz. 8 pwt. 15 gr.
10. From 20 gal. 1$\frac{1}{4}$ pt. take 3 qt. 1$\frac{5}{8}$ pt.
11. From 49 bu. 1 pk. 1 qt. 1 pt. take 30 bu. .75 pt.
12. Difference of time between June 19, 1870, and Dec. 5, 1873? (30 days to a month).
13. Subtract a quadrant from a semicircle. How many degrees remain?
14. Subtract 9° 8′ 49.05″ from a quadrant.
15. From 1 mi. subtract 7 fur. 39 rd. 5 yd. 1 ft. 5 in.
16. Find the sum of £$\frac{3}{8}$ and $\frac{4}{5}$ shilling.

SOLUTION. £$\frac{3}{8}$=$\frac{3}{8}$ of 20 s.=$\frac{60}{8}$ s.=7$\frac{1}{2}$ s.=7s. 6d.
$\frac{4}{5}$ s.=$\frac{4}{5}$ of 12 d.=$\frac{48}{5}$=9$\frac{3}{5}$ d.
7s. 6d.+9$\frac{3}{5}$ d.=8 s. 3$\frac{3}{5}$ d.=8 s. 3 d. 2$\frac{2}{5}$ qr.

17. Add £$\frac{5}{6}$ $\frac{3}{4}$ s. and $\frac{3}{8}$ d.
18. Add $\frac{2}{5}$ mi.; 4 fur. 8$\frac{1}{4}$ rd.; 38 rd. 13$\frac{1}{3}$ ft.; 1$\frac{3}{8}$ mi. 3$\frac{1}{7}$ fur.
19. Add 10 A. 17.35 sq. rd. 16.5 sq. yd. 7.25 sq. ft. 90.19 sq. in.
20. From 9 sq. ft. 18.37 sq. in. take .46$\frac{2}{3}$ sq. yd.

SOLUTION. .46$\frac{2}{3}$ sq. yd.=(.46$\frac{2}{3}$×9) sq. ft.=4.2 sq. ft.=4 sq. ft. 28.8 sq. in. 9 sq. ft. 18.37 sq. in.—4 sq. ft. 28 8 sq. in.=4 sq. ft. 133.57 sq. in.

21. From $\frac{11}{15}$ A. take $\frac{9}{10}$ sq. rd.
22. From 1 hhd. take 24.15 gal.
23. From 1$\frac{5}{9}$ mi. take 3.47 yd.

MULTIPLICATION.

324. 1. Multiply £8 5 s. 10 d. 3 qr. by 8.

PROCESS.

£.	*s.*	*d.*	*qr.*
8	5	10	3
			8
66	7	2	0

1. Begin with the lowest denomination; 8 times 3 qrs. are 24 qrs.=6 d 0 qr.
2. 8 times 10 d. are 80 d.; 80 d.+6 d. are 86 d.=7 s. 2 d.
3. 8 times 5 s. are 40 s.; 40 s.+7 s. are 47 s.=£2 7s.
4. 8 times £8 are £64; 64 £+£2=£66.

2. Multiply 31 T. 4 cwt. 49 lb. 8 oz. by 4.

3. Multiply 19 m. 2 fur. 18 rd. 10 ft. 11 in. by 12.
4. Multiply 13 oz. 13 pwt. 13 gr. by 10.
5. Multiply 10½ bu. 3¾ pk. 1⅖ qt. by 11.
6. Multiply 80° 16′ 25.52″ by 30.

DIVISION.

325. 1. Divide 5 hhd. 40 gal. 3 qt. by 3.

PROCESS.

$$\begin{array}{r}3)\underline{5\text{ hhd.}\quad 40\text{ gal.}\quad 3\text{ qt.}\quad\quad}\\ 1\text{ hhd.}\quad 55\text{ gal.}\quad 2\text{ qt.}\quad \tfrac{2}{3}\text{ pt.}\end{array}$$

ANALYSIS. 1. Begin with the highest denomination; one third of 5 hhds. is 1 hhd., with remainder of 2 hhds.

2. 2 hhds.=126 gal.; 126 gal.+40 gal.=166 gal.; one third of 166 gal. is 55 gal. with remainder of 1 gal.
3. 1 gal.=4 qt.; 4 qt +3 qt.=7 qt.; one third of 7 qt. is 1 qt., with remainder of 1 qt.
4. 1 qt. =2 pt.; one third of 2 pints is ⅔ pint.

2. Divide 20 lb. 7 oz. 13 pwt. 11 gr. by 6.
3. Divide 14 T. 7 cwt. 12 oz. by 8.
4.. Divide 354 cu. yd. 8 cu. ft. 1009 cu. in. by 24.
5. Divide 28 sq. mi. 19 A 10 sq. rd. 7 sq. ft. by 15.

PROBLEMS.

326. 1. A had 50 bu. 3 pk. 1 qt. of corn; B 16 bu. 1 pk. 3 qt.; C 27 bu. 2 pk. 7 qt. 1 pt.; D 30 bu. 5 qt. They give ⅛ of the whole amount to a widow and divide the remainder equally among 5 children. What does each child receive?

2. 5 men had each 29 gal. 3 qt. 1 pt. of wine; they fill all the bbls. that it will fill, and divide the remainder equally among two dozen neighbors. How much wine did each neighbor receive?

3. From a bbl. of wine 18 qt. 1.4 pt. are drawn every day for 6 days. What remains?

4. A grocer's sales are as follows: Jan., 245 £17 s. 10d; Feb., £201 18 s. 9 d.; Mar., £285 14 s. 7 d.; Ap.,

£305 2 s. 2 d.; May, £346 7 s. 8d.; June, £400 15s. 3 d. What did his sales average per month?

5. A package weighing 5 lb. 7 oz. 7 dr. 2 sc. 3 gr. is composed of smaller packages, each weighing 3 oz. 7 dr. 2 sc. 19 gr. How many small packages are there?

SUGGESTION. Reduce to grains.

6. The driving wheels of a locomotive are each 5 yd. 2 ft. 9 in. in circumference, and average 3 revolutions per second. Find the rate of the train per hour.
7. If a cannon ball travel 1250 ft. per second, how long would it be in passing from the earth to the moon, 240000 miles?
8. Sound travels 1142 feet per second; what is the distance of a cannon whose report follows the flash at an interval of 9 seconds?

Measurement of Rectangles and Rectangular Solids.

For definitions and principles consult Chapter XV.

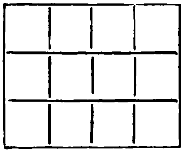
Review Articles 297, 299, and 300.

327. A plane surface bounded by four straight lines, and having four square corners, is a **rectangle**.

A visiting card, a sheet of writing paper, the surfaces of a brick, are familiar examples of rectangles.

328. When the four sides of a rectangle are equal, the rectangle is a *square*.

329. Take 12 pieces of stiff paper, each piece an inch square. Arrange them in 3 rows of 4 pieces each, as represented, making a rectangle.



The number of square inches in this rectangle is the product of its two dimensions.

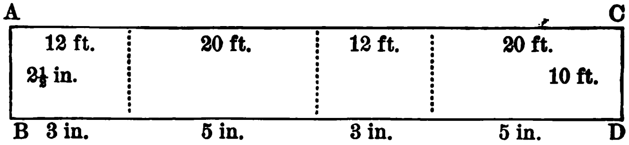
This statement is true, if the squares be arranged in other ways; as in 2 rows of 6 squares each; or 6 rows of 2 squares each. It is a general truth that

330. *The area of a rectangle equals the product of its two dimensions.*

ORAL.

1. How many square feet in the surface of a blackboard 12 feet long, 4 feet high?
2. How many square inches in the page of a book 5 in. wide, 9 in. long?
3. The ceiling of a room is 12 feet square; what is its area in square feet? In square yards?
4. How many square inches on one side of a board 12 inches by 30 inches? How many square inches on both sides?
5. How many square yards in a lot 15 by 40 yards? How many square feet?
6. How many square feet in a floor 20 feet long and 18 feet wide? How many square yards?
7. How many square feet in one side of a wall 120 feet long and 9 feet high? How many square yards?
8. How many square feet in the four walls of a room 20 feet long, 12 feet wide, and 10 feet high?

SUGGESTIONS. To represent the walls of a room, take a piece of stiff paper, or thin pasteboard, 16 inches long and 2½ inches wide. Each foot of the wall will be represented by ¼ of an inch of the paper or board. Fold the paper at right angles at the dotted lines. This will bring the edges A B and C D together, and the four walls of the room will be represented by the folded paper.



The line A C (or B D) is the *perimeter* (measure around) of the room and the area of the four walls is the product of the two dimensions A C and A B.

9. How many square feet in the four walls of a room 12 feet long, 10 feet wide, and 8 feet high?

10. How many square feet in the four walls of a room 15 feet long, 12 feet wide, 10 feet high? How many square yards? How many square feet in the floor of the room?

11. How many feet long is a surface which contains 600 square feet, and is 3 feet wide?

12. How many sq. in. in a surface 9 in. long by 8 inches wide? What part of a sq. ft. is this surface? What part of a linear foot is its length? Its width?

13. What part of a square foot is a surface $\frac{5}{8}$ of a foot long by $\frac{1}{3}$ of a foot wide? $\frac{3}{4}$ ft. by $\frac{5}{6}$ ft.? $\frac{2}{5}$ ft. by $\frac{3}{10}$ ft.?

331. A solid bounded by six rectangles is a **rectangular solid.**

A common brick is a rectangular solid.

332. The rectangles which bound a rectangular solid are called its **faces.** The opposite faces of a rectangular solid are equal. Any two opposite faces may be called the upper and lower **bases** of the solid. The lines in which two faces meet is called an **edge.**

333. A rectangular solid whose six faces are equal squares is a **cube.**

How many edges has a cube? Are they equal? Why?

334. The entire surface of a rectangular solid is the sum of the areas of its faces.

The lateral surface of a rectangular solid is the sum of four of its faces, its two bases being excluded.

1. An edge of a cube is 2 inches; how many square inches in one of its faces? How many square inches on its

six faces? What, then, is the entire surface of a cube whose edge is 2 inches?

2. What is the entire surface of a cube whose edge is 3 inches?

3. What is the entire surface of a cube whose edge is 10 inches?

4. What is the entire surface of a cube whose edge is 20 inches?

5. What is the lateral surface of a brick, 8 in. by 4 in. by 2 in., when its largest faces are considered its bases?

SUGGESTIONS. Find the *perimeter* of the brick, (2×8 in.)+(2×4 in.)= 24 in.; multiply this perimeter by the height, or thickness, of the brick, which is 2 inches. Compare this example with example 8, Art. 330, and observe that the process is the same in both. In example 8, the room is really considered as a rectangular solid, whose *interior* dimensions are measured. In this example, the *exterior* dimensions of the rectangular solid are measured.

6. Common brick are made in a wooden mold, or box, without top or bottom. What are the interior dimensions of the mold for a brick 8 in. by 4 in. by 2 in.? How many square inches in the interior lateral surface of such a mold?

7. What is the lateral surface of a block of marble 6 ft. long, 4 ft. wide, 3 ft. high, when its smallest faces are considered its bases? What is its entire surface?

8. What is the difference in the processes of finding the entire surface of a rectangular solid, and the area of the walls, ceiling and floor of a rectangular room?

9. How many square inches on the sides and ends of a box 10 inches long, 8 inches wide, and 4 inches high? How many square inches on the top and bottom of the box? How many square inches on the entire surface?

10. A glass paper weight is 4 in. by 2 in. by 1½ in. What is its entire surface?
11. A polished block of granite, 4 ft. by 3 ft. by 2 ft. was shipped in a heavy plank box. What was the interior surface of the box in sq. ft.?
12. The granite pedestal of a certain monument is 10 ft. long, 5 ft. wide, 6 ft. high, the four sides being polished. What is the area of the polished surfaces?
13. A block of writing paper is 8 in. by 4 in. by ½ in. What is its entire surface?
14. A bureau drawer measures, inside, 3 ft. by 2 ft. by 8 in. (⅔ ft.) What is its interior surface?

WRITTEN.

15. How many square feet in the floor of a hall 126 ft. long, 87 feet wide? How many square yards?
16. How many square rods in a field 320 rods long, 80 rods wide? How many acres?
17. The base of the great pyramid of Egypt is a square, 746 feet on each side. How many acres does the pyramid cover?
18. The capitol at Washington is 751 feet long and 348 feet wide. How many acres does it cover?
19. How many square feet in 60 boards, each 16 feet long and 1 foot and 3 inches wide?

SUGGESTION. Consider 3 inches as ¼ of a foot.

20. How many square feet in 40 boards, each 20 feet long and 1 foot 9 inches wide?
21. How many acres in a street 4 rods wide and 1 mile long ?
22. A surveyor divided a quarter section of land into fields each 20 chains long and 10 chains wide. How many fields did he make?

23. What is the value of a township of government land worth $1.25 an acre?

24. If $\frac{3}{4}$ of a section of land is sold for $2520, what is the price per acre?

25. How many tiles, each 8 inches square, will cover a hall 24 feet long and 9 feet wide?

SUGGESTION. Use cancellation whenever possible. Thus $24\times9\times144$ represents the area of the hall in square inches. Divide this by 8×8, the surface of one tile; the quotient will be the number of tiles required: $\frac{24\times9\times144}{8\times8}$.

26. How many tiles, each $2\frac{1}{4}$ by 4 inches, are needed to make a hearth 6 ft. long and 2 ft. 4 in. wide?

27. How many bricks, each 8 in. by 4 in. are needed to lay a walk 60 ft. long and 6 feet wide?

28. How many sheets of tin, each 8 in. by 10 in. will cover a roof 40 feet long and 24 ft. wide?

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29. How many square yards of plastering on the walls and the ceiling of a room 15 feet long, 12 feet wide, and 9 feet high?

30. What is the cost, at 30 cents a sq. yd. of plastering the walls and ceiling of a room 28 ft. 8 in. long, 18 ft. wide, 12 ft. high, deducting 20 sq. yds. for openings?

31. Find the cost, at 35 cents a sq. yd., of plastering the walls and ceiling of a room 36 ft. long, 24 ft. wide, 14 ft. high, deducting for 3 openings, each 10 ft. high by 7 ft. wide, and for 8 openings each 7 ft. high by $3\frac{1}{2}$ feet wide.

32. How many rolls of paper, each 24 ft. long and 18 in. wide, will be needed to paper the ceiling of a room 12 yd. by 15 yd.?

SUGGESTION. Consider the surface of the ceiling as a product, and the area covered by one roll as one of the factors of the product. The number of rolls is the other factor. Divide $12\times 15\times 9$, the number of sq. feet in the ceiling, by $1\frac{1}{2}\times 24$, the number of sq ft. in one roll.

NOTE. Wall paper is usually 18 in. wide, and a roll is 24 ft. long.

33. Find the cost of papering the walls of a room which is 51 ft. in perimeter and 9 ft. high, with paper at $1.50 a roll.

34. Paper costing $1.75 a roll, is to be used in papering the walls of a room 17 ft. long, 13 ft. wide, 9 ft. high. If 16 sq. ft. are deducted for openings, what will be the cost?

35. The walls of a room 20 ft. long, 15 ft. wide, 10 ft. high, are to be papered with paper at 50 cents a roll. The ceiling is to be papered with paper costing 35 cents a roll. Deductions are to be made for a base board 12 in. wide, and a frieze 16 in, wide, each extending around the room. Add 25 cents a linear foot for frieze. What will be the total cost?

36. Find cost, at $1.75 per square of 100 sq. ft., of painting the walls of a house 60 ft. long, 24 ft. wide, 25 ft. high?

37. Find cost, at $9.60 a square of 100 sq. ft. of slating a roof, each half of it being 40 ft. long and 16 ft. wide.

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38. How many yards in length, of carpet $\frac{3}{4}$ yd. wide, are needed to cover a floor 6 yds. long by 4 yds. wide, no allowance being made for matching the pattern?

SOLUTION. If the carpet is to "run" *across* the room, there will be as many strips, or "breadths," needed as the width of the carpet is contained times in the length of the room. 6 yds. =24 qrs.; 24 qrs. ÷3 qrs. =8, which is the number of breadths needed, each breadth being as long as the room is wide. $8\times 4=32$, the number of yds. in length needed.

If the carpet is to "run" lengthwise of the room, the number of breadths needed is found by dividing the width of the room, 4 yds., by the width of the carpet, ¾ yd.; that is, 5⅓ breadths. To avoid cutting the carpet lengthwise, 6 breadths, each 6 yards long, are necessary; that is, 36 yds. in length.

39. How many yds. in length of a carpet ¾ yd. wide are necessary to cover a floor 6¾ yds. long, 4½ yds. wide, if the carpet is laid lengthwise of the room? How many yds. if laid across the room?

40. How many yds., in length, of carpet ⅞ yd. wide will cover a floor 31 ft. long, 16 ft. wide, if the carpet is laid lengthwise of the room, and 4 inches on each breadth are needed to match the pattern? Find cost of the carpet at $1.25 a yd.

41. A room is 5 yd. 1 ft. 8 in. long by 3 yd. 2 ft. 3 in. wide. The carpet to cover it is ¾ yd. wide. What will be the cost of the carpet, at 90 cents a yard, if laid most economically? How much money is lost by laying the carpet the other way?

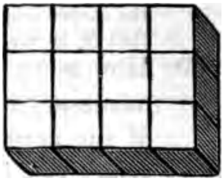
42. Which is the more economical way of laying carpet 2 ft. 3 in. wide on a floor 26 ft. long by 19 ft. 6 in. wide?

43. A rug, 6 yds. long by 5 yds. wide, covers the floor of a parlor, except a strip 18 in. wide around the room, between the rug and the walls. How many square feet of the floor are exposed?

44. At 20 cents a square foot, what will be the cost of laying a walk 6 ft. wide on the four sides of a city block 600 ft. long and 300 ft. wide, the walk to be placed next to the curb?

45. What will be the cost, at 22 cents a sq. ft., of laying a stone walk 7 ft. wide on the four sides of a block 400 ft. by 300 ft. the outer edge of the walk to be 6 feet within the curb?

335. Take 48 blocks, each a cubic inch. Place 12 of them on a table in 3 rows of 4 blocks each, as represented, making a rectangular solid 4 in. long, 3 in. wide, 1 in. high. The number of cubic in. in the rectangular solid thus constructed is equal to the product of its three dimensions, 4×3×1=12.



Place 12 other blocks, arranged in the same manner as the first 12, directly upon them, making a rectangular solid 4 inches long, 3 inches wide, 2 inches high. The number of cubic inches in this second solid is also equal to the product of its three dimensions, 4×3×2=24.

Place 12 other blocks in like manner, making a third rectangular solid, whose cubical contents are likewise the product of its three dimensions, 4×3×3=36.

Place the other 12 blocks in like manner, making a fourth solid, whose cubical or solid contents equal the product of its three dimensions, 4×3×4=48 cu. in.

This is true in whatever manner the blocks are arranged. Make 2 rows of 8 blocks each, 3 layers high ; or 6 rows of 4 blocks each, 2 layers high ; or 2 rows of 2 blocks each, 12 layers high. In each case, the product of the three dimensions equals 48 cubic inches. It is a general truth that

336. *The volume of a rectangular solid equals the product of its three dimensions.*

ORAL.

1. Blocks, each a cubic inch, are piled as a rectangular solid, 5 in. by 4 in. by 6 in. How many blocks are in the pile?

2. How many cubic inches in a block 8 in. by 10 in. by 9 in.?
3. How many cubic feet in a box car 30 ft. long, 7 ft. wide, 10 ft. high?
4. 72 blocks, each a cubic inch, are arranged as a rectangular solid, with a base 3 in. by 4 in. How many inches high is the solid?
5. The base of a rectangular marble block is 4 ft. by 5 ft.; its volume is 200 cu. ft. What is its height?
6. What is the volume of an envelope box 15 inches long, 6 inches wide, 4 inches high? What is its entire surface?
7. My waste basket is 8 inches square, 10 inches high; what is its volume? What is its lateral surface?
8. How many cubic inches in a cube 10 inches on each edge? How many square inches on its entire surface?

WRITTEN.

9. How many cubic yards of earth are excavated from a cellar 90 ft. long, 27 ft. wide, 10 ft. deep?
10. Find the value, at $5.50 a cord, of a pile of wood 64 ft. long, 4 ft. wide, 6 ft. high.
11. A stone wall 260 ft. long, 6 ft. high, $2\frac{1}{2}$ feet thick was built at an expense of $16 a perch. What did it cost?
12. How many cubic feet in a wall 60 ft. long, 10 ft. high, 18 in. thick? How many bricks, each 8 in. by 4 in. by 2 in., could be used in this wall, if no mortar were used?
13. How many bricks in the above wall, if built in the usual way, with mortar, 20 bricks to the cubic foot?
14. Find the cost, at $8 a thousand, of the bricks necessary to build a wall 88 ft. by 12 ft. by 2 ft., 20 bricks to the cu. ft.

15. How many brick are required for the walls of a house 36 ft. long, 24 ft. wide, 21 ft. high, 12 in. thick?

NOTES. Consider the perimeter of the house, outside, as the length of the wall. Make no allowance for openings. This is the masons' usual method of estimating, though evidently in excess of the number of brick actually required. Unless otherwise stated, calculate 22½ bricks to the cubic foot.

16. Find cost of erecting the brick walls of a foundry 120 ft. long, 40 ft. wide, 16 ft. high, at $8.25 per thousand brick, "laid in the wall." Walls 18 in. thick.

17. Find cost of erecting the brick walls of a church, on a stone foundation. The building is 90 ft. long, 48 ft. wide, 36 ft. high, above the stone work. The brick walls are 16 in. thick; the stone wall is 3 ft. thick for 2 feet, and 30 in. thick for 8 ft. high. The bricks cost $9 a thousand, laid; the stone wall cost $18 a perch.

18. How many cords of stone per acre in a quarry, if the stone is 18 ft. deep?

19. How many tons of coal in a vein three fourths of a mile long, 9 yds. wide, 4 ft. thick, if a cubic yard of coal weighs 1.316 tons?

20. How many cubic feet in a bin holding 9 tons of anthracite coal, if each cubic foot weighs 90 lbs.?

21. How many tons of ice can be stored in an ice house whose interior dimensions are 160 ft. by 48 ft. by 38 ft., if a cu. ft. of ice weighs 57 lbs.?

22. How many tons of hay in a mow 27 ft. long, 15 ft. high, averaging 12 ft. wide, estimating 450 cubic ft. to the ton?

23. How many gallons of water may be contained by a rectangular cistern 6 ft. square and 7 ft. deep?

NOTE. A gallon is 231 cubic inches.

24. How many barrels, each holding 31½ gallons, must be

taken from a cistern 7 ft. long, 5 ft. wide, in order to lower the water $3\frac{1}{2}$ feet?

25. How many cubic feet are necessary in order to store 50 barrels of water, each containing $31\frac{1}{2}$ gallons? If stored in a tank 7 ft. deep, how many sq. ft. in the floor of the tank?
26. How many bushels of oats may be placed in a bin 8 ft. by 6 ft. by 4 ft.?

NOTE. A bushel is 2150.4 cu. in.

27. How many cu. ft. are necessary in which to store 1000 bushels of grain?
28. How many bushels of grain may be put in a wagon box 10 ft. long, 3 ft. wide, 20 in. deep?
29. A certain lake vessel carries 72000 bushels of wheat. How many cu. ft. does the cargo occupy?

LONGITUDE AND TIME.

337. When the sun is over the meridian of any place it is noon, 12 o'clock M., at that place. Thus, it is noon at Chicago when the sun is over the meridian of Chicago. It is *later* in the day, P. M., at places *East* of Chicago—at New York, Boston, etc.,—the sun having already passed the meridians of those places; since the movement of the earth on its axis from West to East causes the sun to *appear* to move from East to West.

On the contrary, the time of places *west* of Chicago is *earlier*, since the sun has not yet reached their meridians.

The earth revolves upon its axis once in 24 hours. This causes the sun to *appear* to complete in that time the circumference (360°) of a circle of which the earth is the centre. That is, the daily motion of the sun is at the rate of 15° (360°÷24) per hour. So that the time of a place 15° *east* of Chicago is one hour *later* than Chicago time. Thus, when it is noon at Chicago, it is 1 o'clock P. M. at Montpelier, Vermont, which is 15° east of Chicago.

The time of a place 15° *west* of Chicago is one hour *earlier* than Chicago time, etc. When it is noon at Chicago, the time at San Diego, Cal., which is 30° west of Chicago, is 10 o'clock A. M.

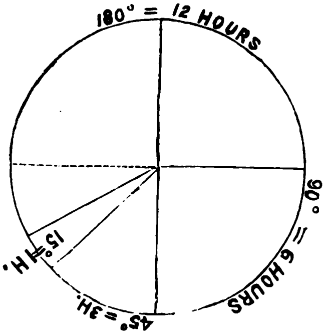


TABLE.

Time.		*Longitude.*
1 hour	is equivalent to	15°
1 minute	“ “	15′
1 second	“ “	15″

338. Therefore, the difference in longitude being known, to find the difference in time :

Divide the difference in longitude by 15 ; the quotient is the difference in time.

The difference of longitude expressed in *degrees*, divided by 15, equals the difference of time in *hours* or fractions of an hour. The difference of longitude expressed in *minutes* of arc, divided by 15, equals the difference of time in *minutes* or fractions of a minute, etc.

339. The difference in time being known, to find the difference in longitude :

Multiply the difference in time by 15 ; the product is the difference in longitude.

The difference of time expressed in *hours*, multiplied by 15, equals the difference of longitude in *degrees*. The difference of time expressed in *minutes*, multiplied by 15, equals the difference in longitude expressed in *minutes* of arc.

What is the difference in time between places whose difference in longitude is

1. 15°? 45°? 90°? 75°? 105°? 180°?
2. 15′? 30′? 45′? 1°? 2°? 4°? 8°?
3. 15° 30′? 30° 15′? 31°? 29°?
4. 7° 30′? 25°? 40°? 60°? 70°? 80°?
5. When it is 6 o'clock A. M. at Greenwich, what is the hour 90° E.? 90° W.? 180° E.? 180° W.?

What is the difference in longitude between places whose difference of time is

6. 1 hour? 2 h.? 6 h.? 8 h.? 10 h.? 12 h.?
7. 30 m.? 1 h. 30 m.? 5 h. 30 m.?
8. 1 m.? 4 m.? 8 m.? 12 m.? 16 m.? 24 m.? 40 m.?
9. When I travel E. do I find my watch too fast or too slow?
10. Upon landing at New York I found that my watch, which had been set at London, did not agree with New York time. Was it fast or slow?
11. If a ship's chronometer at noon reads 9 A. M., in what direction, and how many degrees has she sailed?

340. TABLE OF LONGITUDES.

	From Greenwich.		*From Greenwich.*
Boston - -	71° 3′ 30″ W.	Washington	77° 0′ 15″ W.
Chicago - -	87° 37′ 47″ W.	Canton -	113° 14′ E.
Cincinnati -	84° 27′ W.	London -	5′ 48″ W.
New Orleans	90° W.	Paris - -	2° 20′ 22″ E.

1. What is the difference of time between London and Chicago?

SOLUTION. Chicago is 87° 37′ 47″ W.
London is 5′ 48″ W.
The difference in longitude is 87° 31′ 59″.
Divide the difference in longitude by 15. 15) 87° 31′ 59″
Quotient is difference in time, 5 h. 50 m. $7\frac{14}{15}$ sec.

2. When it is noon in Chicago, what is the hour in London?

SOLUTION. Since London is E. of Chicago, London time is later than Chicago time. By the preceding example the *difference* in time is 5 h. 50 m. $7\frac{14}{15}$ sec London time is therefore 5 h. 50 m. $7\frac{14}{15}$ sec. *later* than Chicago time, or 5 o'clock 50 m. $7\frac{14}{15}$ sec. P. M.; or 9 m. $52\frac{1}{15}$ sec. *of* 6 o'clock P. M.

3. When it is noon in London, what is the hour in Chicago?

SOLUTION. Since Chicago is W. of London, Chicago time is *earlier* than London time. By example 1, the *difference* is 5 h. 50 m. $7\frac{14}{15}$ sec. Chicago time is therefore 5 h. 50 m $7\frac{14}{15}$ sec. *before* 12 M.; or 9 m. $52\frac{1}{15}$ sec. past 6 o'clock A. M.

4. What is the difference in longitude between Chicago and St. Petersburg, if it is 51 m. $47\frac{2}{15}$ sec. past 4 o'clock P.M. in St. Petersburg when it is 9 A. M. in Chicago?

SOLUTION. The *difference* in time is 7 h. 51 m. $47\frac{2}{15}$ sec. (that is, the time *after* noon+the time *before* noon.)

This difference multiplied by 15=117° 56′ 47″, which is the difference in longitude.

5. What is the longitude of St. Petersburg ?

SOLUTION. Since (Ex. 4) the time in St. Petersburg is *later* than Chicago time, St. Petersburg must be E. of Chicago. And (also by Ex.4) St. Petersburg is 117° 56′ 47″ E. of Chicago.

Since Chicago is 87° 37′ 47″ W., St. Petersburg is 117° 56′ 47″—87° 37′ 47″, or 30° 19′ E.

6. What is the difference in time between Boston and London?

7. What is the difference in time between Boston and Canton?

8. When it is 7 o'clock A. M. in Cincinnati, what is the hour in Washington?

9 The difference in time between two places is 3h. 43m.; what is their difference in longitude?

10. When it is 11 o'clock A. M. in St. Petersburg, it is 8 h. 46 m. A. M. in Edinburg; what is the longitude of Edinburg ?

11. When it is 6 o'clock P. M. at Washington it is 13 m. 33 sec. past midnight at Vienna; what is the longitude of Vienna?

12. When it is 10 A. M. at New Orleans, it is 27 m. 48 sec. past 10 A. M. in Columbus; what is the longitude of Columbus?

13. What is the time at London when it is 9 P. M. at Boston?

NOTE. The above examples illustrate the use of local clock time, which has been largely superseded in the United States by "Standard Time," which recognizes the clock time of the meridians of 75°, 90°, 105°, and 120° West of Greenwich, only.

MISCELLANEOUS PROBLEMS.

341. 1. In 1 yr. 30 da. 6 h. 30 mi. how many seconds?

2. In 2 mi. 31 ch. 62 l. how many links?

3. In 3 A. 24 sq. rd. how many sq. miles?

4. In 126720 inches how many miles?

5. In 5760 gr. how many pounds troy?

6. What part of 4 gal. 3 qt. is 2 qt. 1 pt.?

7. Change $\frac{4}{27}$ cu. yd. to the decimal of a cord.

8. How many common miles in $\frac{5}{12}$ of the equator?

9. How many sq.rds.of land does a man retain who buys $\frac{3}{8}$ A. and sells $\frac{4}{11}$ sq. rod ?

10. Reduce 5 cwt. 50 lbs. to the decimal of a ton.

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11. Find the cost of 25 lbs. if 50 cwt. cost $125.

12. A section of land was sold at the uniform price of $3.50 an acre. What did Mr. J. pay for the half of a quarter section?

13. A merchant buys seed for $2 a bushel, and sells it at 25 cents a pint. How much does he make on 3 pecks?
14. A druggist pays 60 cents an ounce (av.) for quinine, and charges 30 cents for 10 powders of 3 grains each. What is his gain on 4 drams of quinine?
15. What is the cost of sawing 3.75 cords of wood at 80 cents a cord ?
16. What is the cost of building a fence inclosing a rectangular field 80 rods by 120 rods, at 12 cents a linear foot ?
17. What is the value, at $1.20 a bushel, of 3 loads of wheat averaging 38 bu. 2 pks. each?
18. What is the value of a dozen silver spoons, each weighing 2 oz. 15 pwt., if the silver is worth $1.40 an ounce?
19. A man exchanged 40 chests of tea, averaging 23 lb. 6 oz., and valued at 32 cents a pound, for 59.84 tons of hay. What was the estimated value of the hay?
20. If an ounce of pure gold is worth $20.672, what is .625 of an ounce worth?

Questions 21 to 25 inclusive, have reference to a room 24 ft. long, 18 ft. wide, 10 ft. high.

21. Find the cost, at $36 per m. sq. ft. of flooring the room with 6 in. matched boards.

NOTE. When 6 in. matched flooring is to be used, add ⅙ of the area to be floored.

22. Find the cost of lathing and plastering the room at 36 cents a square yard.
23. Find the cost of wainscoting the room to the height of 30 in., with 3 in. matched lumber, at 6 cents a sq. ft.

NOTE. When 3 in. matched lumber is used, add ⅓ of the area to be covered.

24. Find the cost, at 50 cents a roll (24 ft. long, 18 in. wide) of papering the room, above the 30 inch wainscoting.
25. Find the cost of carpeting the room with carpet $\frac{3}{4}$ yd. wide, worth $1.50 a (linear) yard. The breadths of carpet are to be laid lengthwise of the room.

Questions 26 to 30 inclusive refer to a house 48 ft. by 36 ft., having stone foundation wall 9 ft. high, 2 ft. thick, and brick wall above, 22 ft. high, 12 in. thick.

26. Find cost, at 35 cents a cu. yd., of excavating cellar to depth of 6 ft.
27. Find cost of building the stone wall, at $6 a cord, for the stone, laid in the wall.
28. Find the cost of the brick work, at $8.50 per M.
29. Find cost of painting the exterior of the brick walls, at $1.75 a square of 100 sq. ft.
30. The roof of this house consists of two parts, each 50 ft. long by 24 ft. wide. Find cost of shingling it, at at $5.50 per m. shingles, each thousand shingles covering 100 sq. ft.

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31. A certain tank is 22.5 ft. long, 16.8 ft. wide, 4.16 ft. deep. How many tons of water will it hold, a cu. ft. of water weighing 62.5 lbs.?
32. The tank described in example 31 will contain how many gallons?
33. If a cubic foot of anthracite coal weighs 95 lbs., how many lbs. may be carried on a car having a box 32 ft. long, 6 ft. wide, 2 ft. deep?
34. If a cubic foot of bituminous coal weighs 77 lbs.,

how many tons can be put into a wagon box 10 ft. long, 3 ft. 6 in. wide, 2 ft. 9 in. high?

35. How many bushels will be contained in a bin 12 ft. 7 in. long, 6 ft. 3 in. wide, 3 ft. 9 in. deep?
36. How many bushels may be put in a cistern that will hold 537.6 gallons?
37. A tank 4 ft. wide, 6 ft. deep, is made to hold 9000 lbs. of water. A cu. ft. of water weighs 1000 oz. What is the length of the tank?
38. The ice on a lake is 24 inches thick, and weighs .92 as much as the same volume of water. How many tons in one acre of this ice?

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39. Find the sum of 25 lb. 7 oz. 15 pwt. 20 gr. ; 78 lb. 11 oz. 19 pwt. 23 gr.; 9 oz. 17 gr.; 16 lb. 14 pwt.
40. From 72 mi. 3 yd. subtract 56 mi. 126 rd. 2 ft. 7 in.
41. How many cords in 16 piles of wood, each containing 9 cd. 106 cubic feet ?
42. The moon moves around the earth in 29 da. 12 h. 44 mi. In what time does it move 12 degrees?
43. John Milton was born Dec. 9, 1608, and died Nov. 8, 1675. At what age did he die?
44. Daniel Webster was born Jan. 18, 1782, and delivered his oration on the completion of the Bunker Hill Monument June 17, 1843. How old was he at the latter date?
45. A note was given Feb. 16, 1890, and was paid Sept. 24, 1890. How many days did it run?
46. A note was given June 20, and was paid in 93 days. On what date was it paid ?

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47. What is the difference in time between Philadelphia (75° 9′ 5″ W.) and New Orleans?

48. What is the difference in time between Philadelphia and Paris?

49. When it is 7:30 P. M. in Chicago, it is $3{:}43\frac{1}{10}$ A. M in Jerusalem. What is the longitude of Jerusalem?

50. A sea captain finds it is 2 h. 25 min. 30 sec. P. M. by solar time, when it is 11 h. 32 min. 42 sec. A. M. by his chronometer, which has Greenwich time. What is his longitude?

342. 1. Find the number of square feet in a floor 9 ft. 6 in. by 8 ft. 3 in.

SUGGESTION. Reduce the inches to a common fraction, or a decimal fraction, of a foot, and multiply thus: $9\frac{1}{2}\times8\frac{1}{4}$, or 9.5×8.25.

2. Find the number of square feet in a floor 12 ft. 9 in. by 7 ft. 8 in.

3. How many square feet in a surface 3 yd. 2 ft. 8 in. by 10 yd. 1 ft. 9 in.?

SUGGESTION. Change to feet and fractions of a foot.

4. What is the area in square feet of a surface 16 yd. 2 ft. 3 in. by 20 yd. 1 ft. 4 in.?

5. What is the area in square feet of a surface 10 ft. $8\frac{1}{2}$ in. by 7 ft. $5\frac{3}{4}$ in.?

SUGGESTIONS. Since the area is desired in sq. ft., proceed as before. 10 ft. $8\frac{1}{2}$ in.$=10\frac{17}{24}$ ft.; 7 ft. $5\frac{3}{4}$ in.$=7\frac{23}{48}$ ft. If the area were desired in sq. in., reduce each factor to inches and fractions of an inch. 10 ft. $8\frac{1}{2}$ in$=128.5$ in.; 7 ft. $5\frac{3}{4}$ in.$=89.75$ in. Then multiply.

6. What is the area in sq. ft. of a surface 5 ft. $6\frac{5}{8}$ in. by 6 ft. $8\frac{3}{4}$ in.?

7. What is the area in sq. in. of a surface 7 ft. $4\frac{7}{8}$ in. by 4 ft. $2\frac{1}{2}$ in.

8 What is the area in sq. yd. of a lot 9 yds 2 ft. 4 in. by 40 yd. 1 ft. 6 in.?

9. What is the volume, in cu. ft., of a rectangular solid having a base of 8sq. ft. 36 sq. in., and a height of 3 ft. 8 in.?

10. What is the volume, in cu. yds., of a rectangular piece of masonry 4 yd. 1 ft. 6 in. long, 3 yd. 2 ft. 3 in. wide, 2 yd. 1 ft. high?

11. How many boards, each having a surface of 7 sq. ft. 60 sq. in., are equal in area to a surface measuring 37 sq. yd. 108 sq. in.

SUGGESTION. Reduce each quantity to sq. in.

12. How many tiles, each having a surface $5\frac{1}{5}$ sq. in., will cover the floor of a hall 173 sq. ft. 48 sq. in. in area?

13. Into how many pieces, each containing 1 sq. ft. 36 sq. in., may a roll of paper measuring 13 sq. yd. 8 sq. ft. be cut?

SOME PRACTICAL RULES.

343. The following practical rules are used by builders and others for rapid computation. The results are only approximately correct, but sufficiently accurate for estimates.

344. Lathing. A lath is 4 ft. long, and usually $1\frac{1}{2}$ in. or $1\frac{3}{8}$ in. wide. There are 50 laths in a bundle. In making estimates, contractors usually deduct one half of the area of openings for doors and windows, and adopt this rule:

One bundle of lath will cover 3 square yards.

1. How many bundles of lath are needed to cover a ceiling 15 ft. by 21 ft.?

2. How many bundles of lath are needed for the walls of a room 18 ft. long, 15 ft. wide, 10 ft. high, making the usual deduction for 3 doors, each $2\frac{3}{4}$ ft. by 7 ft., and 4 windows, each $2\frac{1}{2}$ ft. by 6 ft.?

3. How many bundles of lath are needed for the walls and ceiling of a room 16 ft. wide by 28 ft. long, 9 ft.

high, with 2 doors, each 7 ft. by 6 ft., and 8 windows, each 3 ft. by 6 ft.?

SUGGESTION. When a room is 9 ft. high, the number of sq. yards in its walls is equal to the number of feet in its perimeter.

345. **Shingling.** Shingles are considered as averaging 4 in. in width. A bunch contains 250 shingles. With due allowance for waste, the usual rule is :

Laid 4 inches to the weather, 1000 shingles will cover 100 square feet.

1. How many bunches of shingles, laid 4 in. to the weather, will cover the roof, each half of which is 40 ft. by 20 ft.?

NOTE. Dealers do not sell less than a bunch. The result should contain no fractions.

2. A double roof is 54 ft. long ; the rafters are 24 ft. long. Find cost of necessary shingles, at $3.75 per M.

346. **Flooring** and **Siding.** To find the number of sq. ft. of matched lumber to cover a given area:

I. For 6 inch matched lumber, find six fifths of the number of sq. ft. to be covered.

II. For 3 inch matched lumber, find four thirds of the number of sq. ft. to be covered.

1. How many sq. ft. of 6 in. matched lumber are needed to cover a floor 56 ft. by 30 ft.?
2. Find cost of siding, with 6 in. matched lumber, the walls of a house 60 ft. long by 36 ft. wide by 24 ft. high, the lumber costing $22 per M.
3. Find cost of 3 in. matched flooring, at $38 per M., sufficient to floor 6 rooms, each 15 ft. by 21 ft.

347. **Board Measure.** Sawed lumber is generally sold by Board Measure. A **board foot** is the measure of a board 1 foot long, 1 foot wide, 1 inch thick.

A board 12 ft. long, 1 ft. wide, 1 in. thick, contains 12 ft. board measure. If 2 inches thick, it contains 2× 12 ft. =24 ft. If $1\frac{1}{2}$ inches thick, it contains $1\frac{1}{2}$×12 ft. =18 ft. board measure.

A board less than 1 inch thick is considered as measuring the same as if it were 1 inch thick. Thus, a board 12 ft. long, 1 ft. wide, $\frac{1}{2}$ in. thick, contains 12 board feet.

To find the number of board feet in sawed lumber:

Multiply the length of the piece by its width, both expressed in feet; then multiply this product by its thickness expressed in inches, if the thickness is 1 inch or more.

1. How many feet board measure in 60 boards, each 16 ft. long, 9 inches wide, $\frac{3}{4}$ in. thick?
2. How many feet board measure in 40 planks, each 12 ft. long, 14 in. wide, $1\frac{3}{4}$ in. thick?
3. How many feet board measure in 36 joists, each 18 ft. long, 10 in. wide, $2\frac{1}{2}$ in. thick?
4. Find cost, at $40 per M., of 12 scantling, each 24 ft. long, by 8 in. by 8 in.

348. **Stone Masonry.** To find the number of perches of stone masonry:

Divide the number of cubic feet by 25.

1. How many perches of masonry in an abutment 20 ft. wide, 15 ft. thick, 35 ft. high?

349. **Brickwork.** To find the number of bricks laid in a wall:

Multiply the number of cubic feet in the wall by $22\frac{1}{2}$.

1. How many bricks in a wall 80 ft. by 10 ft. by $2\frac{1}{2}$ ft.?
2. How many bricks necessary for the 12 inch walls of a house 48 ft. long, 24 ft. wide, 22 ft. high, deducting 56 cu. ft. for openings?

350. To Find the Number of Gallons in a Cistern.

1728 (the no. of cu. in. in a cu. ft.)÷231 (the no. of cu. in. in a gal.) = 7.48, which is nearly $7\frac{1}{2}$. Hence,

Multiply the number of cu. feet in the bin by $7\frac{1}{2}$.

1. How many gallons in a cistern 7 ft. deep, the base of which has an area of 38 sq. ft.?
2. How many gallons in a tank 6 ft. by 8 ft. by 4 ft.?
3. How many gallons in a tub having a base of 4 sq. ft., and a depth of 15 inches?
4. How many gallons in a bucket having a base of 72 sq. in., and a depth of 12 inches?

351. To Find the Number of Bushels in a Bin.

1728 ÷ 2150.4 (the number of cu. in. in a bush.) = $\frac{4}{5}$ nearly. Hence,

Take four fifths of the number of cubic feet in the bin.

1. How many bushels in a bin whose cubical contents are 9250 cu. ft.?
2. How many bushels in a bin which is 20 ft by 10 ft. by 6 ft.?
3. How many cu. ft. will be occupied by 3416 bushels of grain?
4. How many cu. ft. are necessary to store 2596 bushels of grain?

352. **To Measure Coal.** A ton (2000 lbs.) of anthracite coal (Lehigh, range) measures about 34 cu. ft.; a ton (2000 lbs.) of bituminous coal measures about 41 cu. ft.

1. How many tons of anthracite coal in a bin 8 ft. by $8\frac{1}{2}$ ft. by 4 ft?
2. How many tons of bituminous coal in a bin $10\frac{1}{4}$ ft. by 5 ft. by 3 ft?
3. How many cu. ft. in a bin holding $5\frac{3}{17}$ tons of anthracite coal? In one holding $7\frac{25}{41}$ tons of bituminous coal?
4. How many cu. ft. in the bunkers of a steam vessel that uses $12\frac{28}{41}$ tons of bituminous coal a day, and carries enough coal to last 8 days?

CHAPTER XII.

PERCENTAGE.

Article 353. 1. A farmer gave to the poor 6 bushels out of every 100 bushels of grain which he raised. How many bushels did he give when his crop was 500 bushels? When it was 800 bushels? How many *hundredths* of his grain did he give to the poor?

2. A student spends for books $12 of every $100 of his income. How many *hundredths* of his income does he spend for books? How many dollars does he spend for books when his income is $600?

3. A fruit grower who had 400 trees, lost 8 out of every hundred of them. How many trees did he lose? How many hundredths of his trees did he lose?

4. A farmer who had 50 trees lost 10 hundredths of them. How many trees did he lose? At the same *rate*, how many would he have lost if he had had 100 trees? 200? 300? Would he have lost in each case, the same number of *hundredths?*

Instead of saying, as in example 3, "8 out of every hundred," or, "8 hundredths," we say "8 *per cent.*" Therefore,

354. Per cent means *hundredths.*

1 per cent is $\frac{1}{100}$; 6 per cent is 6 hundredths, etc.

Instead of the words "*per cent,*" the sign % is frequently used.

355. Rate per cent means a certain number of hundredths.

In the expression *6 per cent*, 6 is the *rate.* The rate is the numerator of a fraction of which the denominator is always 100.

PERCENTAGE.

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1. How many hundredths of a number are 2 per cent of it? 3 per cent of it? 4 % of it? 15% of it?
2. How many sheep are 5% of a flock of 200 sheep?
3. How many dollars are 10% of \$800? Of \$900?
4. How many per cent of a number are $\frac{5}{100}$ of it? $\frac{8}{100}$ of it? .06 of it? .09 of it? .25 of it?

356. Rate per cent may be written

As a common fraction, as $\frac{1}{100}$.
As a decimal fraction, as .01.
With the sign %, as 1%.

So, $\frac{6}{100}=.06=6\%$. $\frac{12\frac{1}{2}}{100}=.12\frac{1}{2}$, or $.125,=12\frac{1}{2}\%$.

$\frac{125}{100}=1.25=125\%$. $\frac{\frac{1}{2}}{100}=.005=\frac{1}{2}\%$.

$\frac{1}{2}\%$ should always be read *one half of one per cent*

357. Express as per cent:

1. $\frac{7}{100}$.
2. $\frac{9}{100}$.
3. $\frac{25}{100}$.
4. $\frac{40}{100}$.
5. $\frac{75}{100}$.
6. $\frac{150}{100}$.
7. $\frac{175}{100}$.
8. $\frac{112\frac{1}{2}}{100}$.
9. $\frac{8\frac{1}{3}}{100}$.
10. $\frac{16\frac{2}{3}}{100}$.
11. .08.
12. .10.
13. $.16\frac{2}{3}$.
14. $.37\frac{1}{2}$.
15. $.62\frac{1}{2}$.
16. 1.06.
17. 2.50.
18. .005.
19. .0075.
20. .008.

358. Express decimally:

1. 10%.
2. 15%.
3. 3%.
4. 6%.
5. 9%.
6. $6\frac{1}{4}\%$.
7. $5\frac{3}{8}\%$.
8. $12\frac{1}{2}\%$.
9. $37\frac{1}{2}\%$.
10. $33\frac{1}{3}\%$.
11. 100%.
12. 125%.
13. 400%.
14. 525%.
15. 708%.
16. 250%.
17. $162\frac{1}{2}\%$.
18. $237\frac{1}{2}\%$.
19. $113\frac{1}{3}\%$.
20. $266\frac{2}{3}\%$.

359. Express as hundredths, and as per cent:

1. $\frac{1}{2}$.
2. $\frac{1}{4}$.
3. $\frac{1}{5}$.
4. $\frac{1}{10}$.
5. $\frac{1}{6}$.
6. $\frac{2}{5}$.
7. $\frac{3}{5}$.
8. $\frac{3}{4}$.
9. $\frac{4}{5}$.
10. $\frac{3}{2}$.
11. $\frac{1}{3}$.
12. $\frac{2}{3}$.
13. $\frac{1}{6}$.
14. $\frac{5}{6}$.
15. $\frac{3}{4}$.
16. $\frac{1}{8}$.
17. $\frac{1}{7}$.
18. $\frac{1}{9}$.
19. $\frac{5}{8}$.
20. $\frac{9}{8}$.

360. The following table should be made familiar to every pupil.

TABLE.

$1\% = \frac{1}{100}$.
$2\% = \frac{1}{50}$.
$4\% = \frac{1}{25}$.
$5\% = \frac{1}{20}$.
$10\% = \frac{1}{10}$.
$20\% = \frac{1}{5}$.

$25\% = \frac{1}{4}$.
$50\% = \frac{1}{2}$.
$75\% = \frac{3}{4}$.
$6\frac{1}{4}\% = \frac{1}{16}$.
$8\frac{1}{3}\% = \frac{1}{12}$.
$12\frac{1}{2}\% = \frac{1}{8}$.

$37\frac{1}{2}\% = \frac{3}{8}$.
$62\frac{1}{2}\% = \frac{5}{8}$.
$87\frac{1}{2}\% = \frac{7}{8}$.
$16\frac{2}{3}\% = \frac{1}{6}$.
$33\frac{1}{3}\% = \frac{1}{3}$.
$66\frac{2}{3}\% = \frac{2}{3}$.

361. 1. Find 6% of 300 books.

SOLUTION. 6% of a number is .06 of it. 300 books × .06 = 18 books.

This is an example of the multiplication by a decimal fraction, 300 books being the multiplicand, 6% (or .06) being the multiplier, and 18 books being the product. In examples in *Percentage*,

The *Multiplicand* is called the **Base**;

The *Multiplier* is called the **Rate per cent**;

The *Product* is called the **Percentage.**

Observe that the word *Percentage* has two meanings, viz.: 1, a department in arithmetic, and, 2, the product of the base multiplied by the rate per cent.

2. The base is 800 dollars; the rate is 5%; what is the percentage?

SOLUTION. Since the percentage is *always* the product of the two factors base and rate per cent, \$800 × .05 = \$40.

NOTE. In performing operations such as the above, it is best to write the rate per cent as a *decimal.* Unless it is a fractional rate, it is written as *hundredths.* See Art. 356.

3. What is the percentage when the base is 400 cows, and the rate is 9 per cent?

4. What is 7 per cent of 300 men?

Such examples are included under

CASE I.

362. Base and Rate per cent given, to find the Percentage: *Multiply the Base by the Rate expressed as a decimal.*

ORAL.

1. Find 8% of 900 ounces; of 400 ounces.
2. Find 3% of 500 inches; of 600 feet.
3. What is 10% of 750 lbs. of sugar.
4. What is 20% of 40 oranges?
5. The base is 800 bricks; the rate is 25%; what is the percentage?
6. A man lost 50% of his fortune, $16000. How many dollars did he lose?
7. Find $12\frac{1}{2}$% of 24.
   $12\frac{1}{2}\% = .12\frac{1}{2} = \frac{1}{8}$.
8. Find $37\frac{1}{2}$% of 48; of 64; of 56.
9. Find $62\frac{1}{2}$% of 32; of 72; of 80.
10. Find $33\frac{1}{3}$% of 15; of 21; of 60.
11. Find $66\frac{2}{3}$% of 27; of 48; of 96.
12. The base is 96; the rate is $87\frac{1}{2}$%; what is the percentage?

WRITTEN.

13. Find 56% of $4984.
14. Find 72% of 3757 acres.
15. Find 98% of 345 bushels of oats.
16. Find 35% of 6400 oxen.
17. A merchant invested 24% of his capital of $120,000 in dry goods; how many dollars did he invest in dry goods?
18. A collector receives 5% of a bill of $2980; how many dollars does he receive?

19. A man bought a farm for $18000, and sold it at a loss of 16%; how many dollars did he lose?
20. Mr. E. having $60,000, invested 28% of it in a mill, 13% of it in a schooner, and the remainder in a mine. How much did he invest in the mine?
21. Mr. A. receives 8% of an investment of $3750, and Mr. B. receives 7% of an investment of $4235. Which receives the larger amount from his investment ?
22. 11$\frac{1}{9}$%, by weight, of water is hydrogen ; how many pounds of hydrogen in a ton of water?

363. Since the percentage is the product of two factors (the base and the rate) when the base and the percentage are given, the rate may be found by dividing the percentage by the base.

1. The percentage is 15, the base is 60; what is the rate per cent ?

Solution. The percentage÷the base=15÷60=$\frac{15}{60}$=$\frac{1}{4}$=.25=25%.

2. The base is 592, the percentage is 222 ; what is the rate per cent ?

Solution. 222÷592=.375=37$\frac{1}{2}$%.

3. A farmer raised 8432 bushels of grain, of which 1517.76 bushels were wheat. What per cent of his grain was wheat ?

Solution. 1517.76÷8432=.18=18%.

Such examples are included under

Case II.

364. Base and Percentage given, to find the Rate per cent: *Divide the Percentage by the Base.*

Oral.

1. The base is 16, the percentage is 8; what part of the

base is the percentage? How **many** hundredths of it? What per cent of it?

2. The base is 25, the percentage is 5; what is the rate?
3. Divide 40 by 80; what is the quotient? What part, then, is 40 of 80? How many per cent is 40 of 80?
4. If I have 40 dollars and lose 10 dollars, what per cent of my money do I lose?
5. If I lose $\frac{1}{6}$ of my trees, what per cent of them do I lose?
6. What per cent is 12 of 12? Is 12 of 24? Is 12 of 48? Is 12 of 6?
7. What per cent of 50 is 25? Is 10?
8. What per cent of 1 is 2? Is 3? Is 4?
9. A man lost all his sheep, what per cent did he lose?
10. What per cent of 120 miles are 60 miles? Are 30 miles?
11. What per cent of 36 yards are 12 yards? 9 yards? 4 yards?
12. A coal merchant had sold $\frac{2}{5}$ of his coal by January 1. What per cent of his coal remained unsold?
13. A debtor owing $360, pays $120. What per cent of the debt remains unpaid?
14. A's money is twice B's; what per cent of A's money is B's? What per cent of B's money is A's?
15. John's age is 5 years; James' age is 15 years. What per cent of John's age is James' age? What per cent of James' age is John's age?

WRITTEN.

16. Mr. A. has an annual income of $1800, and pays $378 rent. What per cent of his income does he pay for rent?

17. An agent charges $73.68 for collecting a debt of $1842. What per cent does he charge ?
18. A man failed in business owing $8496, and paid $6372. What per cent did he pay?
19. Mr. C. paid $21 for the use of $350 ; what per cent did he pay?
20. Mr. F. paid $225 for insuring his house for $15000 ; what per cent did he pay?
21. If the diameter of the moon is 2160 miles and the diameter of the earth is 7925 miles, what per cent of the earth's diameter is the moon's diameter?
22. What per cent of a pound avoirdupois is a pound troy?
23. What per cent of a pound troy is a pound avoirdupois?
24. In a campaign an army of 84000 men lost 12000 men; what was the per cent of the loss?

365. Since the base and the rate are the factors, of the percentage, when the percentage and the rate are given, the base may be found by dividing the percentage by the rate.

1. The percentage is 15, the rate is 5%; what is the base?

SOLUTION. The percentage÷the base=15÷.05=300.

2. The percentage is 250; the rate is 125%; what is the base?

SOLUTION. 250÷1.25=200.

3. A man sold 50 sheep, which were 25% of his flock. How many sheep had he?

SOLUTION. I. 50÷25=200.

SOLUTION. II. Since division by any number produces the same result as multiplication by the reciprocal of that number (Art. 284.)dividing by .25, or ¼, is equivalent to multiplying by 4, the reciprocal of ¼. Therefore, 50÷.25=50×4=200.

Such examples are classed under

CASE III.

366. Percentage and Rate per cent given, to find the Base:
Divide the Percentage by the Rate per cent expressed as a decimal.

ORAL.

1. The percentage is 40, the rate is 50% ; what is the base?

See Solution II of Example 3, Art. 365.

2. The percentage is 80; the rate is 25% ; what is the base?
3. The rate is 20% ; the percentage is 60; what is the base?
4. 49 is 10% of what number?
5. 30 is $12\frac{1}{2}$% of what number?
6. 50 is $14\frac{2}{7}$% of what number?
7. My house is worth $500, and its value is $33\frac{1}{3}$% of the value of the lot. What is the value of the lot ?
8. Mary's age, 12 years, is $37\frac{1}{2}$% of her mother's age. What is the age of the mother?
9. Oats were worth 45 cents a bushel, which was $62\frac{1}{2}$% of the price of wheat. What was the price of wheat?
10. 60 is $66\frac{2}{3}$% of what number?
11. I drew out of bank $300, which was 75% of the amount I had on deposit. What was that amount ?
12. The width of a lot, 25 ft., is $16\frac{2}{3}$% of its depth. What is its depth?
13. A house is rented for $1800 a year, which is 6% of its value. What is the value of the house?

SUGGESTION. $1800÷.06=(1800÷6)×100.

14. A travels 450 miles, a journey which is 90% of the distance which B travels. How many miles does B travel?

15. The population of a city, 5000, is 125% of its population 3 years ago. What was the population then?
16. A bin holds 90 bushels of corn, which is 2% of the owner's crop. How many bushels did he have?
17. A gentleman bequeathed to a friend $2000, which was 4% of the estate. What was the value of the estate?
18. A man failing in business paid $900, which was 3% of his indebtedness. What was his indebtedness?

WRITTEN.

19. 792 is 5% of what number?
20. 960 is $12\frac{1}{2}$ per cent of what number?
21. Mr. A. gained last year $4896, which was 16% of his capital. What was his capital?
22. Mr. B. sold a house for $9000, which was 60% of what he had paid for it. What had he paid for it?
23. In the year 1760, a certain college had a library of 1281 volumes, which was $1\frac{83}{100}\%$ of its library in 1890. How many volumes did the library contain in 1890?
24. If I draw 15% of my bank deposit to pay a note of $1350, how much money did I have in bank?
25. A vessel lost 760 bushels of wheat, which was 19% of her cargo. How many bushels in her cargo?
26. The melting point of zinc is 425 degrees Centigrade, which is $26\frac{9}{16}\%$ of the melting point of wrought iron. What is the melting point of wrought iron?
27. A piano string producing the musical pitch called "middle C," vibrates 256 times in a second, which is $66\frac{2}{3}\%$ of the number of vibrations necessary to produce "G." How many vibrations per second will produce "G."?
28. The pay of a major-general of the U. S. Army is

$7500 a year, which is 57$\frac{9}{13}$% of the pay of the Admiral of the U. S. Navy. What is the pay of the Admiral?

367. 1. 40 is 125% of what number?

Solution. 40 is the percentage; 125% is the rate, but is here the sum of the base (100%) and the rate, 25%. 40÷1.25=32.

2. 40 is 25% more than what number?

Solution. I. This is another method of asking the preceding question, and assumes the number sought as the base, or 100%; therefore 40 is 125% of the number required, that is, of the base. 40÷1.25=32.

Solution. II. Since 125%=$\frac{5}{4}$, 40 is $\frac{5}{4}$ of some number; 40 is $\frac{5}{4}$ of 40×$\frac{4}{5}$=32.

In such examples, the number given, as 40, is called the **Amount**, since it is the sum of the base and the percentage. The number sought is the base, which is always 100%. Hence,

Case IV.

368. Amount and Rate per cent given, to find the Base: *Divide the Amount by 100% plus the Rate per cent expressed as a decimal.*

Oral.

1. 60 is 150% of what number?
2. 75 is 50% more than what number?
3. 24 is 20% more than what number?
4. 80 is 33$\frac{1}{3}$% more than what number?
5. John's age, plus 25% of itself, is 15 years; what is his age?
6. The height of a tree, plus 12$\frac{1}{2}$% of itself, is 90 feet; how high is the tree?
7. Henry has $120, which is 33$\frac{1}{3}$% more money than his sister has; how many dollars has his sister.
8. What number is that, which, being increased by 40% of itself, equals 140?

9. A clerk's salary is $4500, which is 80% more than a high school principal's salary. What is the salary of the latter?

10. The price charged for a horse, $160, is 60% more than his cost; what was his cost?

11. My library this year numbers 1500 volumes, which is an increase of 25% over last year. How many volumes had I last year?

12. After adding 37½% to his farm, a farmer had 1100 acres. How many acres had he before?

13. A man's horse and wagon cost $250, the horse costing 50% more than the wagon. What was the cost of each?

SUGGESTIONS. The wagon cost 100%; the horse cost 150%; the two cost 100%+150%=250%.

14. $4800 were divided between two sons in such a manner that the elder received 40% more than the younger. How much did each receive?

15. A log, 100 ft. long, was cut into two pieces, so that the length of one piece was 50% greater than the length of the other. What was the length of each?

16. The sum of the ages of two sisters is 30 years. One of them is 14$\frac{2}{7}$% older than the other. How old is each sister?

17. A book and a slate cost 90 cents, the book costing 87½% more than the slate. What was the cost of each?

WRITTEN.

18. 8960 is 28% more than what number?

19. 72760 is 36% more than what number?

20. A merchant's capital this year is $64750, which is 116⅔% of what it was last year. What was his capital last year?

21. What number, increased by 62½% of itself, equals 3942¼?

22. What number, increased by 11⅑% of itself, equals 875?

23. 1215 is some number plus 35 per cent of itself. What is the number?

24. The Mississippi River is 4200 miles long, and is 5% longer than the Nile, which is 6⅔% longer than the Amazon. How long is the Nile? The Amazon?

25. The standing army of Russia is 975,000 men, which is 95% greater than the standing army of France. How many men in the latter?

26. The area of Delaware is 2050 square miles, which is 64% greater than the area of Rhode Island. What is the area of Rhode Island?

27. The diameter of the planet Jupiter, 88000 miles, is 14$\frac{2}{7}$% greater than the diameter of the planet Saturn. What is Saturn's diameter?

28. A tract of 8360 acres of land was divided between two purchasers, so that one had 9% more than the other. How many acres had each?

29. The territory of Dakota contained 152,000 square miles, and was divided into the two States of North Dakota and South Dakota, so that South Dakota has 2⅔% greater area than North Dakota. What is the area of each State?

369. 1. 40 is 80% of what number?

2. 40 is 20% less than what number?

SOLUTION. I. This is another form of the preceding example. The number sought is the base, or 100%, of which number 40 is 80%, that is, 100%−20%. 40÷.80=50.

SOLUTION. II. Since 80%=⅘ 40 is ⅘ of some number; 40 is ⅘ of 40×$\frac{5}{4}$ =50.

In such examples, the number given is called **the Difference,** since it is the difference of the base and the rate. The number sought is the base. Such examples give rise to

CASE V.

370. Difference and Rate per cent given, to find the Base.

Divide the Difference by 100% minus the Rate per cent expressed as a decimal.

371. Cases III, IV, and V are essentially one. In each of them a product and a multiplier are given, and the number to be found is a multiplicand.

In Case III, the given product is the percentage.

In Case IV, the given product is a percentage, which includes 100%, representing the base itself.

In Case V, the given product is also a percentage, the remainder of 100% after a given per cent has been subtracted.

In all of the cases named, the Base is sought by dividing a product (Percentage, Amount or Difference) by the number expressing the rate per cent which the product is of the number sought.

ORAL.

1. 12 is 75% of what number?
2. 12 is 25% less than what number?
3 36 is 10% less than what number?
4. 63 is $12\frac{1}{2}$% less than what number?
5. A woman's age is 30 years, which is $14\frac{2}{7}$% less than her husband's age. How old is her husband ?
6. A man sold a buggy for $80, which was 60% less than it cost him. What did it cost him?
7. What number decreased by 20% of itself equals 28 ?

8. What number decreased by 90% of itself equals 32 ?
9. A boy gave to his playmates 75% of his apples, and had 6 left. How many had he at first ?
10. When 80% of a journey had been finished, 50 miles remained. How long was the journey?
11. After 44% of a cargo of potatoes had been sold, 560 bushels remained. How many bushels were in the cargo?
12. The sum of the ages of two boys is 14 years. The age of one of them is 25% less than the age of the other. How old is each?

SUGGESTION. The sum of the ages is 175%, or $\frac{7}{4}$ of the age of the older boy.

13. Two horses are valued at $1500, one being worth $12\frac{1}{2}$% less than the other. What is the value of each?
14. Two casks contain together 48 gallons, one of them holding 40% less than the other. How many gallons does each hold ?

WRITTEN.

15. 7360 is 92% of what number?
16. 1320 is 12% less than what number?
17. 1445 is 15% less than what number?
18. What number, decreased by 23% of itself equals 1155?
19. What number, decreased by 46% of itself equals 1152?
20. A drover sold 34% of his cattle, and had 990 left; how many did he sell ?
21. A man worth $1,220,000 is unhappy because his fortune is 39% less than his neighbor's. What is his neighbor's fortune ?
22. Mt. St. Elias is 17900 feet high, and is 38.4% less than Mt. Everest. What is the height of Mt. Everest?
23. The Rio Grande and Yukon rivers are, together, 3100

miles long ? The Rio Grande is 6.25% less in length than the Yukon. How long is each river?

24. A cubic foot of anthracite and a cubic foot of bituminous coal weighed together 171 lbs. The weight of the latter was 20% less than the weight of the former. What did each weigh?

SUMMARY.

372. **Per Cent** means *hundredths.*

373. **Rate per Cent** means a certain number of hundredths.

The *rate* is the numerator of a fraction of which the denominator is always 100.

374. The **Base** is the number of which the per cent is computed. The Base is always considered as 100 per cent.

375. The **Percentage** is that part of the base which is indicated by the rate per cent.

The word **Percentage** also denotes that part of Arithmetic in which the standard of comparison is 100 per cent.

376. The **Amount** is the sum of the base and the percentage.

377. The **Difference** is the base less the percentage.

378. 1. The *Base* is the *Multiplicand.*

2. The *Rate per cent* is the *Multiplier.*

3. The *Percentage* is the *Product* of which the *Base* and the *Rate per cent* are the factors.

Percentage = Base × Rate Per Cent.

4. The *Amount* is the *Product* of the *Base* and *100 per cent plus the Rate.*

Amount=Base × (100%+Rate.)

5. The *Difference* is the Product of the *Base* and 100% *minus the Rate.*

Difference=Base × (100%−Rate.)

PROFIT AND LOSS.

379. **Cost** is the sum paid for an article.

380. **Price** is the sum asked or received for an article.

381. **Profit,** or **Gain,** is the excess of the *price* above the *cost.*

382. **Loss** is the excess of the *cost* above the *price.*

383. The cost is always considered the *base.* The gain or loss is a certain *percentage* of the base.

The price is either *amount* or *difference,* according as it is either greater or less than the cost.

ORAL.

1. Potatoes that cost 80 cents a bushel are sold at a gain of 25%. What is the gain? What is the price?

SOLUTION. The gain is 25% of 80 cents, which is 20 cents. The price is the cost, 80 cents, + the gain, 20 cents, = $1.

2. How much was the profit on a cow which was bought for $60, and sold at a gain of 10%? What was the price?

3. When oats that cost 40 cents a bushel are sold for 50 cents, what is the profit on a bushel? What is the gain per cent?

SOLUTION. The profit is the excess of the price over the cost, 10 cents. The gain per cent is the profit, divided by the cost, 10÷40=25%.

4. Cattle were purchased at an average of $30 a head, and sold at an average of $40 a head. What was the profit? What was the gain per cent?

5. Cloth that cost 60 cents was sold for 50 cents. What was the loss per cent?

6. For how much must I sell a horse that cost me $200 in order to gain 20%?

7. Coal costing $3 a ton was sold for $5 a ton. What was the gain per cent?

8. When $\frac{1}{5}$ of the cost of an article is gain ; what per cent is gain?
9. When $\frac{1}{5}$ of the price of an article is gain, what per cent is gain?

Suggestions. If $\frac{1}{5}$ of the price is gain, the cost is the other $\frac{4}{5}$ of the price. $\frac{1}{5}$ of the price is therefore $\frac{1}{4}$ of the cost. The cost is the base.

10. When $\frac{1}{4}$ of the price is gain, what per cent is gain ?
11. When $\frac{1}{3}$ of the price is gain, what per cent is gain ?
12. An ax was sold for $1.00, at a gain of 20 cents. What was the cost? What was the gain per cent ?
13. Hay was sold at $8 a ton, at a gain of $33\frac{1}{3}$%. What was the cost ?
14. Hay was sold for $8 a ton, at a loss of $33\frac{1}{3}$%. What was the cost ?
15. If land is bought for $30 an acre, and is sold at a profit of $15 an acre, what is the gain per cent ?
16. If land is bought for $30 an acre, and is sold at a loss of $15 an acre, what is the loss per cent ?
17. When the gain is 100%, what is the price? When the loss is 100%, what is the price?
18. By selling wood at $4 a cord, 20% is lost. What was the cost ?
19. By selling goods at 15% below cost, I lost $45. What was the cost ?
20. I bought two cows at $60 each, and sold them so as to gain 25% on one of them and lost 20% on the other. Did I gain or lose, and how much?
21. I sold two cows at $60 each, gaining 20% on one, and losing 20% on the other. What was the cost of each?

WRITTEN.

22. A man bought 500 tons of coal at $2.75 a ton, and sold it so as to make 60%. What did he sell it for?

23. A house and lot were bought for $12000, and sold for $13500. What was the gain per cent ?

24. By selling a farm at a loss of $750, the owner lost 15%. For how much did he sell ?

25. A vessel was sold for $8550, at a loss of 25%. What was the cost of the vessel ?

26. Apples were bought at $2.75 a barrel, and sold at a loss of 20%. The total loss was $8.80. How many bbls. were sold ?

27. A flour dealer lost 30% by selling flour at $3.50 a bbl. What per cent would he have gained had he sold it at $5.50 a bbl ?

28. A packer sold $72000 worth of pork, $\frac{3}{8}$ of it at a gain of 15%, $\frac{1}{2}$ of it at a gain of 20%, and the remainder at a loss of 32%. Did he gain or lose, and how much.

29. A manufacturer sold two engines for $1650 each. On one he gained 25%, and on the other he lost 25%. Did he gain or lose, and how much ?

COMMISSION AND BROKERAGE.

384. **Commission** is a compensation paid for buying or selling property, collecting debts, or transacting similar business, for another.

Brokerage is commission paid to a broker.

385. The person or firm for whom the business is transacted is called the **Principal.**

386. The person or firm transacting business for another, and receiving commission therefor, is called an **Agent, Factor, Broker,** or **Commission Merchant.**

387. Commission is usually computed at a certain rate per cent of the money expended or collected, which is the *Base.*

Thus, when a real estate agent sells a house for its owner for $6000, he retains 3% (more or less, as is the custom, or as may be agreed upon) of the $6000, which is $180, and pays his principal the *net proceeds*, $6000—$180, =$5820.

388. The **Net Proceeds** of a sale or collection is the sum of money due the principal after the commission and other charges have been deducted.

Net proceeds corresponds to *Difference*.

389. The **Entire Cost** of a purchase is the price paid, plus the commission and other expenses.

Entire cost corresponds to *Amount* when commission and other expenses are a percentage of the cost.

ORAL.

1. An agent collected a debt of $200, charging a commission of 5%. What was his commission. What were the net proceeds?
2. A broker was paid ½% for exchanging $1000 western for eastern money. How much was the brokerage? The net proceeds?
3. My agent charged me 6% for purchasing corn, for which he paid $1200. What was the entire cost of the corn?
4. A collector charged $25 for collecting a debt of $500. What is his rate of commission?
5. A principal received $1920 from his agent, whose charges were 4%. What was the amount of the sale?

SUGGESTION. The sum remitted by the agent was 96% of the sale.

6. A commission merchant remitted $3600 to his principal, charging him 10% commission. What was the amount of sale.
7. My horse cost me $309, including a commission of 3%. What did my agent pay for the horse?

SUGGESTION. For every $1.00 the agent paid for the horse he charged me 3 cents. The entire cost of the horse is therefore 103% of the sum paid by the agent.

8. The entire cost of some wheat was $1040, including 4% commission. What was the price paid for the wheat ?
9. A dealer sends to his agent $9360, with which to purchase wheat at 90 cents a bushel. How many bushels can be bought, allowing the agent 4% commission?
10. If $56 are paid for selling oats for $800, what is the rate of commission? What are the net proceeds?

WRITTEN.

11. I sold, as agent, 2475 bushels of potatoes, for 60 cents a bushel. What was my commission at 6% ?
12. A lawyer collected $1600, and remitted $1520, retaining the difference as his commission. What rate per cent did he charge?
13. The entire cost of a drove of cattle was $12073.40 ; the agent's commission was 6%. What did the agent pay for the cattle?
14. An agent was paid 5% for selling 640 acres of land at $18 an acre. How much was his commission?
15. My collector charges 4% for collecting bills. In March I paid him $385.60 commission. What sum had he collected that month?
16. A commission of 8% was charged on the sale of 14 wagons at $60 each. What were the net proceeds?
17. An agent sold 250 crates of berries at $2 a crate, charging 7½% commission. The other charges were $18 for freight, and $6 for cartage. How much did the owner receive?

18. What sum of money must be sent to an agent with which to purchase 400 bbls of flour at $4.75 a bbl., to pay 5 cents a bbl. storage, and a commission of 2%?
19. A broker purchased 320 R. R. bonds of $100 each, at 25% discount, charging $\frac{1}{2}$% brokerage on the par value of the bonds. What was his brokerage? What was the cost of the bonds to the purchaser?
20. A house was sold on a commission of 6%, the owner receiving $7990. For how much was the house sold? What was the agent's commission?

Insurance.

390. **Insurance** is indemnity against damage or loss.

391. The **Premium** is the sum paid for insurance. The premium is usually a certain per cent of the amount insured. This per cent is called the *rate of insurance.*

392. The **Policy** is the written or printed statement of the contract between the insurer and the insured.

393. 1. The *Amount Insured* is the *Base.*
2. The *Rate of Insurance* is the *Rate.*
3. The *Premium* is the *Percentage.*

Oral.

1. A house is insured against fire for $6000 at $\frac{3}{4}$%. What is the premium?
2. A house worth $12000 is insured for $\frac{3}{4}$ of its value at 1%. What is the premium?
3. The premium for insuring a mill for $10000 is $250. What is the rate of insurance?
4. A stock of goods was insured for $3000, the premium being $60. What was the rate of insurance?

5. The rate of insurance is $1\frac{1}{2}\%$; the premium is \$30. What is the amount insured?
6. The rate of insurance is $\frac{3}{4}\%$; the premium is \$150. What is the amount insured?

WRITTEN.

7. What is the premium for insuring a store worth \$50,000, for $\frac{4}{5}$ of its value, at $1\frac{1}{4}\%$, and a stock of goods worth \$200,000 for $\frac{3}{4}$ its value, at $3\frac{1}{2}\%$?
8. A vessel worth \$30,000 was insured for $\frac{2}{3}$ of her value at 3%. If the vessel is lost, and the insurance paid, what is the total loss of the owner, including the premium paid?
9. A mill was insured for $\frac{1}{2}$ its value; the premium paid was \$250; the rate of insurance 5%. What was the value of the mill?
10. A factory valued at \$14000 is insured for half its value, the premium paid being \$490. What is the rate of insurance?

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11. A man insures his life for \$2500 at the rate of \$22.50 for every thousand dollars of insurance. What is his annual premium? How much does he pay in five years?
12. A man insures his life for \$4000, paying \$13.09 semi-annually for every thousand dollars of insurance. If he should die after 10 years, how much more will his heirs receive than the amount of premium paid?
13. A man pays \$175 a year for a life policy of \$4000, which is to be paid in twenty years. How much will he receive, at the end of twenty years, more than he has paid? If he should die in three years from the date of the policy, how much in excess of the premium paid will his heirs receive?

TAXES.

394. A **Tax** is a sum of money levied upon a person, or upon his property, or business. Taxes are levied by a government, and for public purposes.

395. A **Poll Tax** is a tax upon the person, and is generally assessed, if at all, on legal voters only.

396. A **Property Tax** is a tax on property, and is usually a certain per cent of the assessed valuation of the property taxed.

397. Property is of two kinds: *Real Estate*, and *Personal Property*

398. **Real Estate** (or *Realty*) is immovable property, as lands and houses.

399. **Personal Property** (or *Personalty*) is movable property, such as furniture, clothing, money, etc.

400. The assessed value of the property to be taxed is the *Base.*
The rate of tax is the *Rate.*
The amount of tax to be raised or paid is the *Percentage.*

NOTE. If a poll tax is paid, allowance must be made for it, as is shown in the following example

PROBLEMS.

1. The property of a certain county is assessed at \$30,000,000, and the polls, estimated at 1200 in number, are taxed \$1.00 each. A court house costing \$151,200 is to be erected, for which a tax is to be levied. What is the rate of taxation? How much does Mr. Ames pay, who is assessed \$15000 for real estate, \$1250 for personal property, and who pays tax on 1 poll ?

SOLUTION. The amount of poll tax=1200×\$1.=\$1200. The sum to be levied on property=\$151,200−\$1,200=\$150,000.
The rate of taxation=\$150,000÷\$30,000,000=.005, or 5 mills on a dollar
Mr. Ames' property tax=\$16250×.005=\$81.25.
Mr. Ames' total tax=\$81.25+\$1.=\$82.25.

2. The taxable property of a certain village is assessed at $2,000,000. If a school house costing $22,000 is to be built by tax, what would be the total tax of Mr. Cunningham, who owns houses and lands assessed at $8,000, and who pays for 2 polls at $1.50 each?
3. What is the tax on $17,500 realty and $2500 personal property, when the rate is 15 mills on a dollar?
4. What is the rate of taxation when property assessed at $3250 pays a tax of $52?
5. A tax of $35,305.35 is levied on realty worth $1,549,750, and personalty worth $743,640, after deducting $1.35 on each of 670 polls. A citizen pays $168 tax, including 2 polls. What is his property assessed?

Duties, or Customs.

401. **Duties,** or **Customs,** are taxes levied by a government upon imported goods.

In some countries duties are assessed upon exported goods also.

402. Duties are generally assessed at a certain rate per cent of the cost of the goods, as shown by the invoice.

403. A **Tariff** is a tabulated list of the rates of duties to be collected.

404. The value of the goods is the *Base.*

The rate of tax is the *Rate.*

The duty to be paid is the *Percentage.*

Problems.

1. Find the amount of duty on the following invoice:

50 hogsheads	Molasses,	63 gal.	at 25c.
25 do.	Sugar,	400 lbs.	at 5c.
100 boxes	Raisins,	20 "	at 8c.

The duties are: on Molasses, 20%; on Sugar, 30%; on Raisins, 10%.

2. What is the duty on 18 casks of wine, each cask containing 56 gallons, invoiced at $1.25 a gallon, at $33\frac{1}{3}$%, allowing $12\frac{1}{2}$% for leakage.
3. What is the duty on 60 chests of tea, averaging 80 lbs. each, invoiced at $62\frac{1}{2}$ cents a pound, duty 20%?
4. What is the invoice value of coffee on which $375 duty, at $12\frac{1}{2}$%, is paid ?

SIMPLE INTEREST.

405. **Interest** is money paid for the use of money.

406. The **Principal** is the money for the use of which interest is paid.

407. The **Rate of Interest** is a certain rate per cent of the principal.

408. The **Amount** is the sum of principal and interest.

The rate of interest is the number of cents charged for the use of *one dollar for one year*. The expression, "money is worth 6 per cent," means that a sum equal to 6 per cent of the principal is charged for the use of the principal for one year.

409. The **Time** is the period for which interest is to be paid.

410. **Simple Interest** is interest on the principal only.

411. **Compound Interest** is interest on a principal and unpaid interest.

412. Legal interest is the rate of interest fixed by the law of a State.

413. The computing of interest is, like Commission, Insurance, etc., an application of the principles of Percentage, but with the added element of *Time*.

If interest were always computed for the same time, as for one year, or one month, the time could be disregarded.

The time for which interest is reckoned may be considered as a factor of the rate. If the rate is 6 per cent for one year, it may be considered as $2 \times .06$, or .12, for two years: as $\frac{1}{2} \times .06$, or .03, for six months, etc.

414. The quantities used in Interest and in Percentage correspond as follows:

The *Principal* is the *Base.*

The *Rate*×*Time* is the *Rate per cent.*

The *Interest* is the *Percentage.*

The *Amount of Principal and Interest* is the *Amount.*

415. To find the interest for one or more years:

I. Multiply the principal by the rate expressed as a decimal: then multiply this product by the number of years. Or,

II. Multiply the principal by the product of the rate, expressed as a decimal, and the number of years.

Interest=Principal×$\overline{\text{Rate} \times \text{Time}}$.

ORAL.

1. What is the interest of $100 for 3 years at 5%?

SOLUTION I. The interest of $100 for 1 year at 5%=$100×.05=$5. For 3 years, the interest is 3 times $5, which is $15.

SOLUTION II. The interest of $1.00 for 3 years at 5% is 3×5 cents, which is 15 cents. The interest of $100 is 100 times 15 cents, which is $15.

Find the interest

	of	*for*	*at*		*of*	*for*	*at*
2.	$100	2 yr.	4%.	7.	$150	3 yr.	2%.
3.	$200	1 yr.	8%.	8.	$250	2 yr.	5%.
4.	$300	2 yr.	4%.	9.	$100	4 yr.	6%.
5.	$400	2 yr.	5%.	10.	$100	2 yr.	7½%.
6.	$500	4 yr.	5%.	11.	$300	3 yr.	8%.

WRITTEN.

12.	$375	3 yr.	6%.	18.	$2500	3 yr.	7%.
13.	$428	2 yr.	6½%.	19.	$3642	4 yr.	6%.
14.	$596	3 yr.	4½%.	20.	$4864	3 yr.	4⅓%.

15.	$284	4 yr.	3¼%.	21.	$9260	4 yr.	3¾%.
16.	$685	5 yr.	8%.	22.	$8790	3 yr.	5⅕%.
17.	$850	6 yr.	7½%.	23.	$7850	5 yr.	6¼%.

Note. It is sufficiently accurate to calculate to the nearest cent. In the result, 5 or more mills are usually counted as 1 cent.

416. 1. Find the amount of $350 for 4 yr. at 6%.

Solution I. The interest of $350 for the given time and rate=$350 ×.24=$84. The amount=$350+$84=$434.

Solution II. The amount of $1.00 for the given time and rate is $1.24. The amount of $350 for the same time and rate is 350×$1.24 =$434.

Amount=Principal×(100%+$\overline{\text{Rate}\times\text{Time}}$).

Find the amount

	of	*for*	*at*		*of*	*for*	*at*
2.	$564	3 yr.	6%.	7.	$840	5 yr.	10%.
3.	$496	5 yr.	4%.	8.	$550	3 yr.	7%.
4.	$864	4 yr.	5%.	9.	$960	2 yr.	3½%.
5.	$795	3 yr.	3%.	10.	$645	3 yr.	4½%.
6.	$650	5 yr.	2%.	11.	$780	6 yr.	5½%.

417. 1. Find the interest of $450 for 2 yr. 9 mo. at 4%.

Solution. 2 yr. 9 mo.=2¾ yr. $450×.04×2¾=$49.50.

Find the interest

	of	*for*	*at*		*of*	*for*	*at*
2.	$282	1 yr. 6 mo.	6%.	7.	$960.50	3 yr. 5 mo.	4½%
3.	$540	2 yr. 3 mo.	5%.	8.	$864.75	2 yr. 11 mo.	5%.
4.	$480	3 yr. 4 mo.	7%.	9.	$747.80	1 yr. 1 mo.	6%.
5.	$630	4 yr. 8 mo.	4%.	10.	$564.64	5 yr. 10 mo.	4%.
6.	$840	2 yr. 7 mo.	3%.	11.	$636.48	2 yr. 9 mo.	3½%

Find the amount

	of	*for*	*at*
12.	$1256	2 yr. 4 mo.	7%.
13.	$3064	3 yr. 5 mo.	9%.
14.	$2480.32	4 yr. 6 mo.	5%.
15.	$4950.96	5 yr. 11 mo.	6%.
16.	$8496.50	2 yr. 8 mo.	4%.

418. In computing interest for a time expressed in months and days, it is usual to assume the year as equal to 12 months of 30 days each. Therefore, the interest of any principal for 1 day is $\frac{1}{30}$ of the interest for 1 month, or $\frac{1}{360}$ of the interest for 1 year. Interest thus found is greater than the true interest. (See Art. 426.)

1. Find the interest of $360 for 18 days at 6%.

SOLUTION I. Interest for 1 yr. =$ 360×.06=$21.60.
" " 1 mo. =$21.60÷12=$ 1.80.
" " 1 day =$ 1.80÷30=$ .06.
" " 18 days =$ .06×18=$ 1.08.

SOLUTION II. Since 18 days=$\frac{18}{30}$=.6 of 1 month, the interest for 18 days = the interest of 1 mo., which is $1.80,×.6. $1.80×.6=$1.08.

Find the interest

	of	*for*	*at*		*of*	*for*	*at*
2.	$540	21 da.	6%.	8.	$876	8 da.	5%.
3.	$690	24 da.	6%.	9.	$297	5 da.	7%.
4.	$480	15 da.	5%.	10.	$322	11 da.	8%.
5.	$720	12 da.	$4\frac{1}{2}$%.	11.	$451	16 da.	9%.
6.	$840	9 da.	4%.	12.	$564	13 da.	10%.
7.	$960	6 da.	3%.	13.	$781	14 da.	4%.

419. 1. Find the interest of $274.36 for 2 yr. 7 mo. 17 da. at 5%.

PROCESS.

$274.36
.05
12)13.7180
30)1.1431+
.0381+
$13.718× 2=$27.436
1.1431× 7= 8.001
.0381×17= .647
$36.084

ANALYSIS. The interest of $274.36 for 1 yr. at 5% is $13.718. This sum divided by 12, is the interest for 1 mo., $1.1431. $\frac{1}{30}$ of the interest of 1 mo. is the interest for 1 day, that is, $.0381. Twice the interest for 1 yr.+7 times the interest for 1 mo.+17 times the interest for 1 da. is the interest required.

Find the interest

	of	*for*	*at*
2.	$395.50	2 yr. 5 mo. 16 da.	4%.
3.	$847.36	3 yr. 4 mo. 10 da.	6%.
4.	$592.48	4 yr. 7 mo. 11 da.	5%.
5.	$796.80	2 yr. 3 mo. 5 da.	8%.
6.	$252.64	3 yr. 9 mo. 19 da.	7%.

420. Cancellation may frequently be employed to advantage, especially when the principal is divisible by 2, 3, 5, and their multiples.

1. Find the interest of $240 for 3 yr. 7 mo. at 8%.

PROCESS.

$$\begin{array}{r|r} & 20 \\ \cancel{12} & \$\cancel{240} \\ & .08 \\ & 43 \\ \hline & \$68.80 \end{array}$$

SOLUTION. Write the principal, the rate and the number of months, as factors of a dividend, and the number of months in a year, 12, as a divisor. Then proceed as in Cancellation, remembering that one factor, the rate, is hundredths.

Find the interest

	of	*for*	*at*		*of*	*for*	*at*
2.	$720	2 yr. 3 mo.	8%.	5.	$200	2 yr. 8 mo.	7%.
3.	$360	3 yr. 2 mo.	5%.	6.	$300	1 yr. 7 mo.	9%.
4.	$480	1 yr. 9 mo.	6%.	7.	$400	4 yr. 5 mo.	6%.

8. Find the interest of $480 for 18 days at 3½%.

PROCESS.

$$\begin{array}{r|r} 12 & \$480 \\ 2 & .07 \\ 30 & 18 \\ \hline & \$.84 \end{array}$$

SOLUTION. $480×.07÷2=int. for 1 yr. at 3½%. The division of this product by 12×30=int. for 1 day. The multiplication of this quotient by 18=the int. for 18 days.

Find the interest

	of	*for*	*at*
9.	$600	20 da.	8%.
10.	$450	22 da.	10%.
11.	$840	7 mo. 21 da.	6%.
12.	$630	2 mo. 3 da.	7%.

THE SIX PER CENT METHOD.

421. The interest of $1.00 at 6% is

For 1 yr. 6 cents, or .06 of the principal.
For 1 mo. 5 mills, or .005 of the principal.
For 6 da. 1 mill, or .001 of the principal.

1. Find the interest of $240 for 2 yr. 7 mo. 19 da. at 6%.

SOLUTION. Interest for 2 yr. =$240×.12.
" " 7 mo. = 240×.035.
" " 19 da. = 240×.003⅙.
Int. for 2 yr. 7 mo. 19 da.=240×.158⅙=$37.96. Hence,

422. To find interest by the 6 per cent method:

Multiply the principal by .06 times the number of years, plus .005 times the number of months, plus ⅙ of .001 times the number of days.

1. What is the multiplier, by the 6 per cent method, when the time is 3 yr. 9 mo. 22 days?

SOLUTION. 6 hundredths times the number of yrs.=.06×3=	.018
5 thousandths times the number of mos.=.005×9= . .	.045
⅙ of one thousandth times the number of days=.022÷6= .	.003⅔
The multiplier is	.228⅔

Find the multiplier by 6% method at 6% for

2. 2 yr. 6 mo. 18 da.
3. 3 yr. 8 mo. 24 da.
4. 4 yr. 10 mo. 15 da.
5. 5 yr. 3 mo. 21 da.
6. 6 yr. 9 mo. 27 da.
7. 3 yr. 5 mo. 12 da.
8. 2 yr. 7 mo. 20 da.
9. 4 yr. 11 mo. 16 da.
10. 7 yr. 1 mo. 5 da.
11. 8 yr. 2 mo. 7 da.

423. Find the interest by 6% method at 6%

	of	*for*
1.	$360	1 yr. 6 mo. 27 da.
2.	$960	3 yr. 8 mo. 24 da.
3.	$825	4 yr. 9 mo. 20 da.
4.	$750	2 yr. 3 mo. 12 da.
5.	$127.35	3 yr. 5 mo. 18 da.

	of	*for*
6.	$292 40	5 yr. 7 mo. 17 da.
7.	$438.72	4 yr. 5 mo. 23 da.
8.	$694.60	2 yr. 11 mo. 6 da.
9.	$327.75	3 yr. 4 mo. 11 da.
10.	$240.80	9 yr. 3 mo. 15 da.
11.	$523.46	7 yr. 8 mo. 29 da.
12.	$630.50	5 yr. 9 mo. 21 da
13.	$860.24	6 yr. 8 mo. 19 da.
14.	$942.33	3 yr. 6 mo. 13 da.

15. Find the interest of $1260 for 3 mo. 23 days.

Process.

$1260.
6)1.26.
21
113
$23.73

Analysis. Divide the principal by 1000, by moving the decimal point three places to the left. The quotient, $1.26, is the interest for 6 days. Multiply this quotient by $\frac{1}{6}$ of the number of days. In practice, divide either factor by 6. In this example, $1.26 is divided by 6, while the number of days, 113, is not. Therefore, multiply 21 cents, the interest for 1 day, by 113, the number of days. Hence,

424. To find the interest for a short period, at 6 per cent,
Divide the principal by 1000 and multiply the quotient by $\frac{1}{6}$ the number of days.

Find the interest at 6%

	of	*for*		*of*	*for*
1.	$1500	2 mo. 16 da.	6.	$482.40	96 da.
2.	$2400	3 mo. 8 da.	7.	$372.60	84 da.
3.	$1620	1 mo. 10 da.	8.	$620.40	75 da.
4.	$2550	1 mo. 20 da.	9.	$856.25	33 da.
5.	$1980	2 mo. 6 da.	10.	$274.90	63 da.

Note. The exercise of ingenuity will frequently avoid lengthy operations.

Find the interest at 6%

	of	*for*		*of*	*for*
11.	$540	8 days.	15.	$860	5 days.
12.	$370	9 days.	16.	$650	4 days.
13.	$720	7 days.	17.	$390	3 days.
14.	$940	10 days.	18.	$460	11 days.

425. The six per cent method may be used to find interest at any other rate, by first finding the interest at 6%, and then taking such part of this interest as the given rate is of 6 per cent.

Thus, to find interest at 5%: first find the interest at 6%, and then deduct one sixth of it.

1. Find the interest of $390 for 1 yr. 8 mo. 18 da. at 4½%.

SOLUTION. The interest of $390 for the given time at 6% is $390×.106, which is $41.34. Since 4½% is ¾ of 6%, the interest sought is ¾ of $41.34, which is $30.905.

2. What is the interest of $186.45 for 1 yr. 6 mo. 24 da. at 5%?
3. What is the interest of $320.96 for 2 yr. 9 mo. 27 da. at 7%?
4. What is the interest of $784.50 for 7 mo. 15 da. at 4%?
5. What is the amount of $680.75 for 1 yr. 11 mo. 10 da. at 7½%?
6. What is the amount of $1000 for 2 yr. 5 mo. 8 da. at 7%?
7. Find the amount of $480.90 for 3 yr. 9 mo. 10 da. at 8%.
8. Find the interest of $324 from March 25, to Dec. 15 of the same year at 5%. (30 days to a month.)
9. Find the interest of $1256 from Sept. 5, 1890, to Jan. 13, 1892, at 3½%.
10. Find the amount of $75.49 from Dec. 20, 1891, to April 5, 1894, at 5%.

Find the interest, by any method,

	of	*from*	*to*	*at*
11.	$679.08	Feb. 10, 1890,	Dec. 7, 1891,	6%.
12.	$845.16,	Aug. 13, 1891	Nov. 17, 1893,	5%.
13.	$729.50	May 15, 1892,	Oct. 19, 1894,	4½%.
14.	$340.25	Dec. 11, 1889,	July 5, 1892,	7%.
15.	$659.80	Sept. 4, 1893,	Jan. 20, 1894,	8%.

Find the amount

	of	*from*	*to*	*at*
16.	$580	Mar. 10, 1890,	Aug. 30, 1892,	5%.
17.	$922	Sept. 5, 1891,	Jan. 21, 1895,	4%.
18.	$896	June 9, 1891,	Oct. 24, 1892,	4½%.
19.	$37.56	Oct. 14, 1890,	Dec. 29, 1890,	6%.
20.	$92.66	Nov. 28, 1893,	June 6, 1896,	5½%.

EXACT INTEREST.

426. Exact Interest is found by ascertaining the actual number of days in the given period, and making the interest for each day $\frac{1}{365}$ of the interest for 1 year.

This is the only accurate method; and it is used by the United States Government.

1. Find the interest of a $1000 U. S. bond bearing 4% interest, from Feb. 1, 1891, to June 10, 1891.

SOLUTION. The exact time required is 129 days. The interest of $1000 for 1 yr. at 4% is $40. $\frac{\$40 \times 129}{365} = \$14.137$.

2. Find the exact interest on $500 from June 1, 1890, to Dec. 20, 1890, at 5%.

3. Find the exact interest on $1500 from May 5, 1892, to Aug. 7, 1892, at 6%.

PRINCIPAL, INTEREST, OR AMOUNT, AND TIME GIVEN, TO FIND THE RATE

427. 1. At what rate will \$450 yield \$27 interest in 1 yr. 6 mo?

SOLUTION. Int. of \$450 for 1 yr. 6 mo. at 1%=\$6.75. \$27÷\$6.75=4%.

2. The amount of \$1500 for 2 yr. 8 mo. 20 days is \$1745. What is the rate of interest ?

SOLUTION. The interest is \$1745 – \$1500=\$245.
Int of \$1500 for 2 yr. 8 mo. 20 da.=\$40⅚. \$245÷\$40⅚=245÷$\frac{245}{6}$=6%.
Hence,

428. When Principal, Interest and Time are given, to find the Rate :

Divide the given interest by the interest of the principal for the given time at 1 per cent.

1. At what rate will \$900 yield \$168 interest in 2 yr. 8 mo.?
2. At what rate will \$600 yield \$165.60 interest in 3 yr. 5 mo. 12 da.?
3. At what rate will \$750 amount to \$795 in 1 yr. 4 mo.?
4. At what rate will \$700 amount to \$763 in 1 yr. 6 mo.?
5. At what rate will \$960 amount to \$1084 in 1 yr. 3 mo. 15 days?
6. I wish to loan \$3000 for 60 days so as to receive from it \$40 interest. At what rate must I loan it ?
7. An owner of a U. S. Bond of \$10,000 receives from it \$175 interest every six months. What interest does the bond bear?

INTEREST, OR AMOUNT, RATE AND TIME GIVEN, TO FIND THE PRINCIPAL.

429. 1. What principal will yield \$56.25 interest in 2 yr. 6 mo. at 5%?

SOLUTION. The interest of \$1.00 for 2 yr. 6 mo. at 5% is \$.125. \$56.25 ÷\$.125=450: that is, \$450.

2. What principal will amount to $301.70 in 1 yr. 3 mo. 15 days at 6%?

SOLUTION. The amount of $1.00 for 1 yr. 3 mo. 15 da. at 6% is $1.0775. $301.70÷$1.0775=280: that is, $280.

430. When Interest, Rate, and Time are given, to find the Principal:

Divide the given interest by the interest of $1.00 for the given time at the given rate per cent.

431. When Amount, Rate and Time are given, to find the Principal:

Divide the given amount by the amount of $1.00 for the given time at the given rate per cent.

1. What principal will yield $82.50 interest in 2½ yr. at 6%?
2. What principal will yield $26.25 interest in 90 days at 7%?
3. What principal will amount to $973.75 in 3 yr. 9 mo. at 5%?
4. What principal will amount to $797.00 in 1 yr. 6 mo. 24 da. at 4%?
5. The interest on a note drawing 7% was $19.60 for 4 mo. What was the principal?
6. What sum must I invest in 5½ per cent bonds, in order to receive $343.75 interest every 3 mos.?
7. My savings bank deposit draws 4% annually, and pays me $60 interest every 6 mo. How much have I on deposit?

PRINCIPAL, RATE AND INTEREST GIVEN, TO FIND THE TIME.

432. In what time will $300 yield $52.50 interest at 5%?

SOLUTION. The interest of $300 for 1 yr. at 5% is $15. $52.50÷$15=3.5: that is 3.5 years, or 3 yr. 6 mo. Hence,

433. When Principal, Rate and Interest are given, to find the Time:

Divide the given interest by the interest on the principal at the given rate per cent for one year.

1. In what time will $400 yield $120 interest at 6%?
2. In what time will $275 yield $19.25 interest at 4%?
3. In what time will $600 amount to $636 at 4½%?
4. How long must $842.50 be on interest at 8% in order to amount to $1011 ?
5. In how many years will $500 double itself at 5%?

MISCELLANEOUS PROBLEMS.

434. 1. If an iron ore yields 65% pure iron, how many lbs. of iron may be obtained from 6¼ tons of ore?

2. Of the 1,981,000 miles of telegraph wire in use in the world in 1889, the United States had 39.7% How many miles of wire had the U. S.?

3. A circular field contains 5000 acres ; a square field encircled in it contains 3927 acres. What per cent is the square of the circle ?

4. The salary of the Governor of Iowa, $4,000, is 133⅓% of the salary of the Governor of Kansas. What is the latter's salary?

5. The specific gravity of hard coal is 1.80; that of cork is .24. What per cent is each specific gravity of the other?

6. The wealth of the U. S. in 1888 was 59378 millions of dollars, which was 137½% of the wealth of the U. S. in 1880. What was the wealth in 1880 ?

7. A man 6 ft. high should weigh 180 lbs., which is 125% of the proper weight of a man 5½ ft. high. How many lbs should the latter weigh?

8. What was the total profit on 280 tons of hay, bought at $6 a ton, and sold at a gain of 12% ; on 320 bbls.

of flour, bought at \$4.75 a barrel, and sold at a gain of 6% ; and on 4000 bu. oats, bought at 25c. and sold at $27\frac{1}{2}$c. a bushel ?

9. A mill was offered for sale for \$32000, which was $14\frac{2}{7}$% more than it cost; but it was sold at a loss of 5%. For how much was it sold ?

10. A man refused an offer of \$7364 for his house, since at that price he would have lost $12\frac{1}{2}$%. Later he sold the house for \$10099.20. Did he gain or lose, and what per cent?

11. Mr. C. bought 25 acres of land for \$5625, and sold 80% of it for what he had paid for the whole. What was his gain per cent ?

12. 1200 tons of rail road iron was purchased for \$30 a ton, and sold, through an agent, for \$38 a ton. The agent's commission was $2\frac{1}{2}$%, and other expenses were \$500. What was the gain on the transaction ? What was the gain per cent ?

13. An agent sold fruit, and purchased flour, after deducting his commissions, $4\frac{1}{2}$% for selling, and $2\frac{1}{2}$% for buying. The flour cost \$3820; for what did the fruit sell ? What was each commission?

14. A merchant sold 5800 bbls. of flour at \$5 a bbl., and remitted \$27985. What rate of commission did he charge?

15. A mill worth \$150,000 was insured annually at $7\frac{1}{2}$%. What was the annual premium? In how many years would the sum of the premiums equal the value of the mill ?

16. A building was insured for \$4800 for 5 years. The first year the premium was $1\frac{1}{4}$%; the other four years

$\frac{7}{8}\%$. During the fifth year, the building was burned. What was the total loss to the company?

17. For how much can I insure my residence by paying a premium of $122.50 a year, the rate of insurance being $1\frac{3}{4}\%$?

18. The assessed valuation of the taxable property of a county is $1,596,000; the levy is $12\frac{1}{2}$ mills on a dollar; the cost of collection is $1\frac{1}{2}\%$ of the amount collected. What amount will the county receive?

19. A tax of $70,000 is levied on property valued at $2,800,000. What is the rate? What is the assessed value of a man's property who pays a tax of $204.75, including 5 polls at $1.50 each?

20. In 1889, furs valued at $5,329,588 were imported into the U. S. What was the duty at 20%?

21. A merchant imported 48 pieces of broadcloth, each piece containing 40 yards, valued at $2.50 a yard. The duty was 25%, insurance 4%, freight and other charges $288. For how much per yard must he sell the cloth in order to gain 50% on its entire cost ?

22. The principal items of the interest bearing debt of the U. S., Dec. 1, 1889, were $124,413,000, $4\frac{1}{2}\%$ bonds, $639,531,100 4% bonds, and $64,623,512 Pacific R. R. 6% bonds. What was the annual interest on the total amount ?

23. A man receives every 6 months $1125 interest from $4\frac{1}{2}\%$ bonds. What amount of such bonds has he?

24. Find the interest of $963.50 for 3 yr. 8 mo. 18 days at $7\frac{1}{2}\%$.

25. Find the interest of $225 for 63 days at 6%.

26. Find the exact interest of $800 from July 1, to Sept. 20, at $7\frac{1}{2}\%$.

27. If I borrow $7500 at 6%, and loan it at 8%, what is my annual gain?

28. A man paying $600 a year rent for a house valued at $8000, borrows that sum at 6%, and buys the house, which he insures at $\frac{3}{4}$ of its value at $\frac{3}{4}$%. He also pays $50 a year for repairs. Does he gain or lose by buying the house, and how much ?

29. I received $159.04 interest at 7%, for 1 yr. 9 mo. 9 da. What was the principal ?

30. The amount of a debt, including interest at 5$\frac{1}{2}$% for 90 days, was $2189.70. What was the principal ?

31. In what time will $480, at 5%, yield $64 interest ?

32. In what time will $700, at 6%, amount to $756 ?

33. At what rate will $2880, in 1 yr. 1 mo. 1 da. yield $156.40 ?

34. At what rate will $3000 amount to $3156 in 1 yr. 3 mo 18 da. ?

35. A man borrows at 6% money with which to purchase 600 acres of land at $30 an acre. He held the land 3 years, paying 25 cents an acre each year as taxes, and then sold two thirds of his purchase at $50 an acre, and the balance at $60 an acre. What was his total gain ?

36. A manufacturer is making 20% on a capital of $24000, but sells his business at an advance of 12$\frac{1}{2}$%, and invests his money at 8%. How much does he lose every year.

37. One man owes another $1893.61, due in 2 yr. 3 mos. 10 d. The debtor wishes to pay now a sum that will amount to the debt when it shall become due, money being worth 5%. How much shall the debtor pay?

See Art. 431.

MISCELLANEOUS EXERCISES IN PERCENTAGE.

To be used at the discretion of the teacher.

435. *A.* Find Commission, or Premium, or Gain or Loss, on numbers in first column, at 1%, $\frac{1}{4}$%, $\frac{1}{2}$%, $\frac{3}{4}$%, $1\frac{1}{2}$%, $2\frac{2}{5}$%, $6\frac{1}{4}$%, $8\frac{1}{3}$%.

B. Find Interest on each of the following principals for the time indicated, at 4%, $4\frac{1}{2}$%, 5%, 6%, 7%, $7\frac{1}{2}$%, 8%, 9%, 10%.

1.	$458.75	1y.	6m.	15d.	**16.**	$428.40	1y.	9m.	27d.
2.	$680.50	2y.	9m.	18d.	**17.**	$600.00	2y.	5m.	6d.
3.	$320.82	3y.	2m.	9d.	**18.**	$325.15	3y.	7m.	
4.	$500.00	1y.	9m.	21d.	**19.**	$216.80	2y.	3m.	3d.
5.	$753.375		8m.	20d.	**20.**	$824.60		11m.	15d.
6.	$873.40	5y.	2m.	3d.	**21.**	$324.20	4y.	6m.	18d.
7.	$157.25	3y.	5m.	12d.	**22.**	$ 80.50	1y.	1m.	5d.
8.	$ 32.30	1y.	4m.	27d.	**23.**	$192.60			29d.
9.	$750.00	3y.	8m.		**24.**	$455.24	2y.	0m.	17d.
10.	$324.125	2y.	5m.	6d.	**25.**	$128.125	1y.	3m.	22d.
11.	$862.17	2y.	9m.	17d.	**26.**	$840.75	3y.	5m.	13d.
12.	$920.09	1y.	10m.	3d.	**27.**	$224.375	2y.	9m.	21d.
13.	$382.11	3y.	6m.	19d.	**28.**	$800.00	7y.	7m.	7d.
14.	$809.05	2y.	7m.	21d.	**29.**	$925.06	10y.	6m.	
15.	$395.60	1y.	10m.	12d.	**30.**	$708.20	2y.	8m.	24d.

C. Find numbers of which the numbers below are 1%, $1\frac{1}{2}$%, 2%, $2\frac{1}{4}$%, $2\frac{1}{2}$%, 3%, 4%, 5%, $12\frac{1}{2}$%, $16\frac{2}{3}$%.

D. Find what rate per cent each of the following numbers is of the succeeding number.

1. 40
2. 60
3. 64
4. 84
5. 96
6. 132
7. 240
8. 276
9. 300
10. 312
11. 342
12. 426
13. 405
14. 414
15. 510
16. 525
17. 620
18. 720
19. 382
20. 160
21. 501
22. 921
23. 308
24. 427
25. 390
26. 570
27. 891
28. 927
29. 800
30. 639
31. 711
32. 425
33. 632
34. 375
35. 900
36. 432
37. 723
38. 522
39. 735
40. 640
41. 832
42. 729
43. 324
44. 108
45. 216
46. 432
47. 864
48. 288

PROMISSORY NOTES.

436.

$\$61\frac{50}{100}$. CHICAGO, *Jan. 31*, 1891.

Sixty days after date I promise to pay to the order of *John Roe*, — *Sixty one* $\frac{50}{100}$ — Dollars, with interest at *six* per cent per annum, for value received.

Samuel Orr

437. A **Promissory Note** is a written promise to pay, on demand, or at a certain time, a specified sum of money, for value received.

438. The **Maker** of the note is the person who promises to pay.

439. The **Payee** is the person to whom the promise of payment is made.

In the above note, Samuel Orr is the *Maker*, John Roe is the *Payee*.

440. The **Face** of the note is the sum named in the note.

The face of an interest bearing note is the amount due at maturity.

441. A **Negotiable Note** is a note made payable to bearer, or to the order of the payee.

A note payable to the order of the payee is *negotiated*, or transferred, by the signature of the payee across the back of the note.

The writing of one's name on the back of a note is *indorsing* it, and, as a general rule, renders the indorser liable for its payment.

442. A note **matures**, or becomes payable, on the third day after the time specified in the note. The three days thus added to "the time the note has to run," are called **days of grace.**

The note in Art. 436 is *nominally* due April 1, 1891, but is legally due April 4, 1891, unless April 4 is Sabbath or a legal holiday, in which case it is legally due April 3, 1891. The maturity of a note is usually indicated by a combination of the two dates, thus: April 1/4, 1891.

443. 1. A note payable "with interest" at a rate named, draws interest from date till paid at the rate named, unless such rate is illegal.

2. A note payable "with interest" without specifying any rate, draws legal interest from date till paid.

3. A note drawn without interest draws legal interest from its maturity till it is paid.

444. Find the sum due at maturity of each of the following notes:

1. NEW YORK, March 1, 1891.

$300. Ninety days after date we promise to pay to the order of Charles Johnson, Three hundred Dollars, with interest at six per cent per annum, value received.

ROSS & STEWART.

2. BOSTON, April 5, 1892.

$450. Six months after date, we jointly and severally, promise to pay to the order of David Woods, Four hundred fifty Dollars, for value received.

JAMES BARKER,
HENRY FREEMAN.

The above is called a *joint and several note,* each maker being responsible for its payment.

3. ST. LOUIS, June 10, 1891.

$500. Thirty days after date I promise to pay to Robert Williams, Five hundred Dollars, with interest at five per cent per annum, for value received.

THOMAS PALMER.

The above note is *non-negotiable,* since it is made payable to Robert Williams only, and cannot be transferred to any other person.

445. Find the amount due at the time of payment of each of the following notes :

1. CHICAGO, April 6, 1891.

$50. On demand we promise to pay to Sill & Company, Fifty Dollars, with interest at six per cent per annum, value received. COOK & STONE.

Paid July 10, 1891.

2. MILWAUKEE, Sept. 1, 1891.

$400. Three months after date I promise to pay to the order of John C. Wells, Four hundred Dollars, for value received. DAVID HALLEY.

Paid Jan. 20, 1892. Legal interest in Wisconsin is 7 per cent.

3. BELOIT, Wis., July 8, 1891.

$250. Thirty days after date I promise to pay to Mrs. Mary C. Rose, or order, Two hundred fifty Dollars, for value received. CLEMENT BARR.

Paid November 17, 1891.

PARTIAL PAYMENTS.

446. Partial Payments are payments in part of a note or debt. A record of each payment, with date, is *indorsed* on the back of a note, and is called an **indorsement.**

447. When partial payments are made on an interest bearing note, and the note is paid in full within a year from its date, the following method of finding the amount due at settlement is generally used. This method is called

THE MERCHANTS' RULE.

1. *Find the amount of the note (or debt) from the time when it began to draw interest to the time of settlement.*
2. *Find the amount of each payment from its date to the time of settlement.*
3. *From the amount of the note (or debt) subtract the sum of the amounts of the payments. The remainder is the sum due.*

1. CINCINNATI, January 2, 1891.

$600. On demand I promise to pay to the order of Edward Holmes, Six hundred Dollars, with interest at six per cent per annum, for value received.

RICHARD JENKINS.

Indorsements: March 16, 1891, $200; April 2, 1891, $150; October 11, 1891, $180.

What sum was due November 2, 1891 ?

SOLUTION.	Amount of $600,	10 mos.	6% =			$630.00
	" " $200,	7 mos. 16 da.	6% =	$207.53		
	" " $150,	7 mos.	6% =	$155.25		
	" " $180,	21 da.	6% =	$180.63		$543.41
Balance due Nov. 2, 1891,	. . . .					$ 86.59

2. A debt of $500 was due and unpaid March 1, 1891, and drew 6% interest from that date. The following payments were made on it: April 1, 1891, $50; May 1, 1891, $120 ; June 15, 1891, $80. What was due December 1, 1891 ?

3. What is due July 1, 1894, on a note of $950 dated July 1, 1893, with interest at 8%, on which are the following endorsements? Oct. 1, 1893, $340; Feb. 1, 1894, $425 ; May 1, 1894, $210.

4. Required the balance due Sept. 30, 1893, on a note of

$680, dated Sept. 30, 1892, with interest at 10%, containing the following indorsements: Dec. 30, 1892, $190; Mar. 1, 1893, $265; June 1, 1893, $195.

5. Find the balance at settlement, Dec. 30, 1893, on an account of $850, due Dec. 30, 1892, to which were applied the following partial payments: April 1, 1893, $236; Aug. 1, 1893, $240; Oct. 30, 1893, $225. Interest 9%.
6. What is due Jan. 1, 1894, on a note of $960, dated Sept. 1, 1893, interest at 12%, on which were the following indorsements: Oct. 10, 1893, $256; Nov. 12, 1893, $215; Nov. 28, 1893, $205; Dec. 18, 1893, $128.60?

448. When more than a year has elapsed between the date and the payment of an interest bearing note, on which partial payments have been made, interest is usually computed by a method approved by the Supreme Court of the United States, and therefore called

THE UNITED STATES RULE.

1. *Find the amount of the note (or debt) to the time when the first payment, or sum of the first payments, equals or exceeds the interest due at that time.*
2. *From this amount subtract the payment or sum of the payments made.*
3. *Consider the remainder thus obtained as a new principal, and proceed as directed above.*

1. Find the amount due on the following note, January 8, 1897.

PEORIA, Dec. 8, 1892.

$800. On demand I promise to pay to George Rich, or order, Eight hundred Dollars, with interest at 6 per cent. for value received. CHARLES JACKSON.

Indorsements: Sept. 26, 1893, \$238.40; July 5, 1894, \$127.90; Aug. 2, 1895, \$25.00; Dec. 1, 1895, \$250.00.

SOLUTION. Original principal		\$800.00
Add int. from Dec. 8, 1892, to Sept. 26, 1893, 9 mos. 18 da.		38.40
1st amount		838.40
Deduct 1st payment		238.40
2nd principal		\$600.00
Add Int. from Sept. 26, 1893, to July 5, 1894, 9 mos. 9 da.		27.90
2nd amount		627.90
Deduct 2nd payment		127.90
3rd principal		\$500.00
Add int. from July 5, 1894 to Aug. 2, 1895, 12 mos. 27 da.		32.25
Add int. from Aug. 2, 1895 to Dec. 2, 1895, 4 mos.		10.00
		542.25
Deduct 3rd payment, less than int. due	\$25	
" 4th "	250	275.00
4th principal		\$267.25
Add int. from Dec. 2, 1895, to Jan. 8, 1897, 13 mo. 6 da.		17.64
Amount due, Jan. 8, 1897		\$284.89

2. What is due March 15, 1893, on the following note?

\$1500. ST. LOUIS, March 15, 1890.

On demand, for value received, I promise to pay to the order of John Howe, One thousand five hundred Dollars, with interest at 6 per cent.

SAMUEL FIELDS.

Indorsements are as follows: Sept. 15, 1890, \$400; March 15, 1891, \$500; Sept. 15, 1891, \$300; March 15, 1892, \$90; Sept. 15, 1892, \$300.

3. How much is due on the following note, June 20, 1895?

$\$950\frac{50}{100}$. For value received I promise to pay to the order of Charles Robinson, Nine hundred fifty and $\frac{50}{100}$ Dollars, with interest at 6 per cent.

ALBERT PALMER.

Indorsements are as follows: Oct. 20, 1893, $180; April 20, 1894, $20; Sept. 20, 1894, $360; March 20, 1895, $210. The note is dated June 10, 1893.

4. What is due Sept. 25, 1894, on the following note?

1240\frac{60}{100}$. PHILADELPHIA, June 1, 1892.

Ninety days after date, for value received, I promise to pay to Thomas Hall, One thousand two hundred forty and $\frac{60}{100}$ Dollars, with interest at 6 per cent.

ALFRED GOODRICH.

Indorsements are as follows: Sept. 25, 1892, $340; May 1, 1893, $360; Aug. 25, 1893, $310; Feb. 1, 1894, $80.60.

BANK DISCOUNT.

449. The holder of a promissory note frequently desires to convert it into cash before the note matures. He may, therefore, sell the note to a bank, or other purchaser, for a sum less than the face of the note. The sum for which he sells it is called the *proceeds*. The difference of the face and the proceeds is the *discount*.

450. **Bank Discount** is a deduction from the face value of a promissory note which has been cashed before maturity. Bank Discount is simple interest on the face of a note for the term of discount.

For "face of a note" see Art. 440.

451. The **Proceeds** of a discounted note are its face less the discount.

452. The **Term of Discount** is the time from the day of discount to maturity.

NOTE 1. A note is legally due on the last day of grace. (See Art. 442.)

NOTE 2. When a note is made payable in a certain number of *months*, calendar months are understood. But when a note is made payable in a certain number of *days*, it is usual to count the exact number of days.

Note 3. When a note is discounted at date, the term of discount is the time specified in the note, plus the three days of grace.

Case I.

453. To find the Proceeds of a Note when its Face, Rate of Discount, and Term of Discount are given :

1. *Find the simple interest on the face of the note, at the given rate of discount, from the day of discount to maturity ; this is the bank discount.*
2. *From the face of the note subtract the bank discount; the remainder is the proceeds.*

Remember that the face of an interest-bearing note is the *amount.*

1. Find the proceeds of a 60 day note for $600, discounted at date at 6%.

Solution. The term of discount is 63 days.

Face of note	$600
Int of $600 for 63 da. at 6%	6.30
Proceeds	$593.70

2. Find the proceeds of a 4 months note for $600, bearing 8 per cent interest, discounted at a bank three months after date at 6%.

Solution. Face of note = amt. of $600 for 4 mo. 3 da. at 8% = $616.40

Term of discount is 33 days.

Int. of $616.40 for 33 da. at 6%	3.39
Proceeds	$613.01

Find the maturity, term of discount, and proceeds of the following notes :

3. 450\frac{50}{100}$. Boston, Oct. 1, 1893.

One month after date, value received, I promise to pay to the order of Samuel Kirkwood, Four hundred fifty and $\frac{50}{100}$ Dollars, with interest at 7 per cent.

Joshua Perkins.

Discounted at bank Oct. 12, 1893, at 10%.

4. $\$790\frac{40}{100}$. BROOKLYN, Dec. 25, 1893.
Ninety days after date I promise to pay to the order of James Stewart, Seven hundred ninety and $\frac{40}{100}$ Dollars, with interest at 7 per cent. Value received.
J. C. MOORE.

Discounted at bank Feb. 1, 1894, at 7%.

5. $1000. CINCINNATI, July 1, 1894.
Four months after date, value received, we jointly and severally, promise to pay to John Williams, or order
One thousand Dollars. FRED. JOHNSTON.
ANSON GAY.

Discounted at bank, Aug. 18, 1894, at 6%.

6. Find the proceeds of a note for $675, dated Oct. 1, 1893, payable in 90 days, with interest at $4\frac{1}{2}$%, discounted Oct. 25, 1893, at 6%.

7. A merchant sold 58 yards of silk at $4.60 a yard, and 97 yards of broadcloth at $7.40 per yard, and in payment took a note payable in 60 days ; 15 days thereafter he had the note discounted at bank, at 12%. How much did he realize from the goods, the interest on the note being 10%.

Find the proceeds of the following notes :

	Face.	*Date.*	*Time to Run.*	*Day of Dis*	*Rate of Dis.*
8.	$500	July 1,	3 mo.	Aug. 1	6%.
9.	$750	Sept. 5,	4 mo.	Nov. 1	8%.
10.	$360	Jan. 9,	5 mo.	Feb. 20	7%.
11.	$480	Oct. 7,	60 da.	Oct. 20	9%.
12.	$720	May 15,	90 da.	July 1	6%.
13.	$600	July 20,	30 da.	July 20	7%.
14.	$900	Sept. 1,	21 da.	Sept. 10	6%.
15.	$510	Oct. 10,	60 da.	Oct. 30	8%.

CASE II.

454. To find the Face of a Note, when the Proceeds, Rate of Discount and Term of Discount are given.

Divide the given proceeds by the proceeds of $1.00 at the given rate, for the given term of discount.

1. For what sum must a 60-day note be drawn, that, being discounted at 6%, the proceeds may be $500 ?

SOLUTION. Bank dis. of $1. for 63 da. at 6%=$.0105.
Proceeds of $1.=$1.−$.0105=$.9895.
Face of note required=$500÷$.9895=$505.30+

2. Find the face of a note whose proceeds, when discounted at 8% for 63 days, are $1202.92.
3. Find the face of a 30-day note that will produce $600 when discounted at 7%.
4. Find the face of a 4-month note whose proceeds will be $1500 when discounted at 10%.
5. For what sum must I write a note, payable in 90 days, that when discounted in bank at 8% the proceeds shall be $1526.72 ?
6. I desire to use to-day $680, which I can secure by giving a bankable note payable in 30 days, discounted at 9%. For what sum must I write the note ?
7. I owe to-day a debt of $560.40, which I can meet by giving my note, payable in 45 days, discounted at 7%. For what sum must I write the note so that the proceeds shall be just large enough to discharge the debt?

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8. A gentleman was offered $5480 cash for a house and lot, or a note for $5740, payable in 90 days; he chose the latter and discounted the note in bank at 8½%. Did he gain or lose, and how much?

9. A merchant bought a bill of goods, Nov. 20, 1892, for $5620, giving his note, payable in 3 months, with interest at 5% ; the seller discounted the note Jan. 1, 1893, at 6%. How much did the seller realize in cash for the goods ?

10. A person having a 5-month note for $1500, dated Aug. 20, 1893, gets it discounted, Sept. 30, 1893, at 5½%, and invests the proceeds in hay, at $9 per ton. How many tons of hay did he buy?

TRUE DISCOUNT.

455. When a debtor desires to pay a non-interest bearing debt before it is due, the creditor usually accepts, as full payment, a sum which, if put at interest, would amount to the debt when it becomes due.

Thus, a debt of $618, due in 6 months, may be cancelled by the payment of $600, which sum would, in 6 months at 6 per cent, amount to $618, the face of the debt. In this illustration $600 is the *present worth ;* $18 is the *discount.*

456. The **Present Worth** of a debt, payable at a future time without interest, is the sum which, if put at interest at the current rate, will amount to the debt when it became due.

The present worth is the principal, of which the debt is the amount.

457. **True Discount** is the difference between the face of a debt and its present worth.

True Discount is the interest of the present worth from the time the debt is discounted to the time when the debt becomes due.

458. The following table shows the correspondence of some of the principal quantities considered in this chapter:

TABLE OF CORRESPONDING QUANTITIES.

Percentage.	*Interest.*	*Bank Discount.*	*True Discount.*
Base,	Principal,	Face of Note	Pres. Worth,
Per Cent,	Rate × Time,	Rate × Time,	Rate × Time,
Percentage,	Interest,	Discount,	Discount,
Amount,	Amount,		Debt.
Difference,		Proceeds.	

459. 1. What is the present worth of a debt of $530 due one year from date, if discounted at 6 per cent? What is the discount?

SOLUTION. Amt. of $1. for 1 yr. at 6%=$1.06.
$530÷$1.06 = ... $500.00=present worth.
$530−$500 = ... $ 30.00=discount.

Since the debt is the *amount*, the present worth the *principal*, and the discount the *interest*, the process is exactly the same as finding the principal, when amount, rate and time are given. See Art. 431. Therefore,

460. To find the Present Worth, when the face of a debt, time and rate of discount are given:

Divide the face of the debt by the amount of $1.00 for the given term at the given rate.

1. What is the present worth of a debt of $627 if discounted at 6%, 9 months before it is due?
2. If a debt of $1090.20 be paid 2 yr. 6 mo. before it is due, what deduction shall be allowed, reckoning at 6%?
3. What shall be the discount of a debt of $3768, due Sept. 1, 1894, if paid May 1, 1893, money worth 6%?
4. If I pay a debt of $4800 1 yr. 3 mo. before it is due, what discount shall be allowed me, money being worth 9%?
5. Bought land for $2205, to be paid for in 90 days, and sold it immediately for $2460 cash. How much did I gain, money being worth 8%?

6. A merchant bought goods to the amount of \$4200, $\frac{1}{3}$ payable in 3 mo., $\frac{1}{3}$ in 6 mo. and $\frac{1}{3}$ in 9 mo. Money being worth 9%, what sum would have paid the bill at the time of purchase?
7. Bought a piece of land for \$12360, $\frac{1}{4}$ cash and the rest in 3 annual equal payments, without interest. If money was worth 5%, what sum would have discharged the whole debt at the time of purchase?
8. I am offered for my house and lot \$5400 cash, or \$5565 on 4 mo. Which offer should I accept, money worth 9%?

COMMERCIAL DISCOUNT.

461. Commercial Discount is a deduction from the nominal price of an article, or from the amount of a bill.

Commercial discount is usually a certain percentage of the price, and is often made in consideration of the payment of cash. The nominal price is often called the *list* price.

The list price, minus the discount, is the *net* price.

462. The following bill illustrates commercial discount:

CHICAGO, July 10, 1893.

Robert Bruce,

Bought of James Cole & Co.

2 doz. Mortise Locks, \$8.	16	00		
$\frac{1}{4}$, 5%	4	60	11	40
4 doz. Wood Knobs, \$6.	24	00		
25%, 10%	7	80	16	20
600 lbs. Sash Weights, 4c., . . .	24	00		
30%, 5%, 2%	8	36	15	64
			43	24

From $16, the gross price of the locks, deduct ¼ the amount, $4, leaving $12, of which take 5%, which is 60 cents. From the gross price, $16, take the sum of these discounts, which is $4.60, leaving $11.40 as the net price of the locks. So proceed with each item of the bill.

Find the net amount of each of the following bills:

1. $960, discounts 20% and 10%.
2. $3240, with 20% and 15% off.
3. $1000, discount 25%, 15%, and **10%**.
4. $2400, discount ⅔ and 6%.

COMPOUND INTEREST.

463. When interest is not paid when due, it is frequently added to the principal, and interest computed on this amount. This addition is made at regular intervals of time.

Thus, it is customary for savings banks to pay interest on deposits semi-annually. If the depositor does not withdraw such interest before the next "interest day," (the first day of January or July) the interest is added to the deposit, forming a new principal on which the bank pays interest.

464. **Compound Interest** is interest on a principal and unpaid interest usually added to the principal at regular intervals of time.

1. What is the compound interest of $800 for 3 yr. 6 mo. 24 da. at 6 per cent?

SOLUTION. Principal, ... = $800.00
Int. for 1st year, $800×.06 . . . . . . = 48.00

Amt. for 1 yr. or principal for 2nd yr. . . . . = 848.00
Int. for 2nd yr. $848×.06 . . . . . . = 50.88

Amt. for 2 yr. or principal for 3rd yr. . . . . = 898.88
Int. for 3rd yr. $898.88×.06 . . . . . = 53.93

Amt. for 3 yr. or principal for 6 m. 24 da. of 4th yr. = 952.81
Int. for 6 mo. 24 da. $952.81×.034 . . . . . = 32.40

Amt. for 3 yr. 6 mo. 24 da. . . . . = 985.21
Deduct first principal, ... = 800.00

Compound interest for 3 yr. 6 mo. 24 da. . = $185.21

465. To compute compound interest:

1. *Find the amount of the given principal for the first term of interest.*
2. *Consider this amount as a new principal, and find its amount for the second term; so proceed for the entire time.*
3. *From the last amount deduct the first principal: the remainder is the compound interest.*

NOTE. Unless otherwise stated, interest is compounded annually.

1. Find the compound amount and interest of $1000 for 3 yr. 6 mo. at 6% per annum.
2. What is the compound interest of $800 for 2 yr. 8 mo. 24 da. at 6% per annum?
3. What is the compound interest of $1200 for 2 yr. 4 mo. 12 da. at 8% per annum, interest payable semi-annually?
4. Required the compound interest of $500 for 1 yr. 6 mo. 18 da. at 12% per annum, interest payable quarterly.
5. Find the compound amount of $700 for 1 yr. 8 mo. compounded quarterly at 8% per annum.
6. Find the compound amount of $2000 for 2 yr. 1 mo. 15 da., compounded semi-annually at 6% per annum.
7. At 5%, compound interest, what will $1300 amount to in 3 yr. 4 mo. 24 days?
8. What is the difference between the simple and the compound interest of $5000 for 3 yr. at 8%?
9. Find the difference between the simple interest of $1000 at 6% for 2 years, and the compound interest for same amount and rate and time, compounded semi-annually.

466. The computation of interest, both simple and compound, is greatly abridged by the use of interest tables, which give the interest, or amount, of $1.00 for any desired number of years, months, and days, at different rates per cent. The following is a small portion of a compound interest table:

TABLE

Showing the amount of $1.00, at compound interest, at 4, 5, 6, 7, and 8 per cent, from 1 to 10 years.

YEARS.	4 PER CENT.	5 PER CENT.	6 PER CENT.	7 PER CENT.	8 PER CENT.
1	1.040000	1.050000	1.060000	1.070000	1.080000
2	1.081600	1.102500	1.123600	1.144900	1.166400
3	1.124864	1.157625	1.191016	1.225043	1.259712
4	1.169859	1.215506	1.262477	1.310796	1.360489
5	1.216653	1.276282	1.338226	1.402552	1.469328
6	1.265319	1.340096	1.418519	1.500730	1.586874
7	1.315932	1.407100	1.503630	1.605782	1.713824
8	1.368569	1.477455	1.593848	1.718186	1.850930
9	1.423312	1.551328	1.689479	1.838459	1.999005
10	1.480244	1.628895	1.790848	1.967151	2.158925

1. What is the compound interest of $500 for 4 yr. 6 mo. 18 da. at 6 per cent?

Solution. From the table, the compound amount of $1.00 for 4 yr. at 6% is $1.262477.

The amount of $1.00 for 6 mo. 18 da. is $1.033.

The compound amount of $1.00 for the given time is $1.262477×1.033, which is $1.304138+

The compound amount of $500 is 500×$1.304138=$652.069.

The compound interest is $652.069 - $500=$152.069.

2. What is the compound amount of $600 for 5 yr. at 8 per cent?
3. What is the compound interest of $800 for 6 yr. 8 mo, 24 da. at 6 per cent?
4. Find the compound interest of $300 for 7 yr. 6 mo. at 4 per cent.
5. Find the compound interest of $400, for 2 yr. 6 mo. at 8 per cent.
6. Find the compound interest of $80, for 3 yr. 8 mo. at 6 per cent.
7. What principal will amount to $1028.5296 at compound interest for 5 years at 8 per cent?

SUGGESTION. The compound amount divided by the compound amount of $1.00 equals the original principal.

8. What principal will amount to $393.4302 at compound interest for 10 years at 7 per cent?
9. What principal will produce $526.3728 compound interest in 7 years at 4 per cent?
10. What principal will produce $701.9551 compound interest in 5 years at 5 per cent?

ANNUAL INTEREST.

467. **Annual Interest** is simple interest on the principal, and on each year's unpaid interest until paid.

Annual interest is indicated by inserting in a promissory note the words "with interest payable annually," or, "with annual interest."

1. Find the amount due at maturity of the following note, no interest having been paid:

PHILADELPHIA, March 1, 1894.

$400. Five years after date I promise to pay Robert Clarke, or order, Four hundred Dollars, with interest payable annually, for value received.

JOSEPH MARTIN.

PROCESS. Interest on $400 for 5 yr. at 6% . . . . = $120.00
" " 1st Ann. Int., $24 for 4 yr. 6% = 5.76
" " 2nd " " 24, " 3 yr. 6% = 4.32
" " 3rd " " 24, " 2 yr. 6% = 2.88
" " 4th " " 24, " 1 yr. 6% = 1.44

Total interest, ... $134.40
Principal, ... 400.00

Amount due at maturity, ... $534.40

Each year's interest, $24, unpaid when due, draws simple interest until paid.

Find the amount due at maturity of each of the following notes, interest due annually but not paid :

2. $600, due 3 yr. after date, 8 per cent.
3. $500, due 4 yr. after date, 10 per cent.
4. $300, due 4 yr. 6 mo. 27 da. after date, 10 per cent.
5. $350, due 3 yr. 8 mo. 15 da. after date, 8 per cent.

STOCKS AND BONDS.

468. An **Incorporated Company,** or **Corporation,** consists of several persons authorized by law to transact business as one person.

Banks, Insurance Companies, Colleges, etc., are corporations.

469. A **Charter** is a legal instrument or certificate of incorporation, which creates a corporation and defines its powers.

The charter of a city is granted by a State Legislature ; the charter of a national bank is secured under a general act of Congress

470. The **Capital,** or **Capital Stock,** of a business corporation consists of the money and other property with which it carries on its business.

471. A **Share** of Stock is one of the equal parts into which the capital stock of a corporation is divided.

The nominal value of a share is usually one hundred dollars.

472. A **Stock Certificate** is a document signed by the proper officers of a corporation, specifying the number of shares belonging to the stockholder to whom the certificate is issued.

Such certificates are generally known as *Stocks*. They are bought and sold like any other property.

473. **Dividend** is money paid to the stockholders of a company from the profits of its business.

474. An **Assessment** is a sum of money required of stockholders in order to conduct the business of a company.

475. When a corporation needs more money than its capital, it frequently, if permitted by its charter, borrows money by issuing *bonds*, the payment of which is secured by mortgage on its property.

476. A **Bond** is an interest bearing promissory note of a corporation, or of a government.

The payment of the bonds of a corporation is secured by mortgage of its property. For the payment of the bonds of a government only the faith of the government is pledged. If a government is overthrown, its bonds are worthless.

A bond bears a fixed rate of interest. The dividend paid to stockholders varies with the profits of the business.

477. The **Par Value** of a stock or of a bond is the sum which it promises to pay.

The par value of a $100 bond is $100.

478. The **Market Value** of a stock or of a bond is its selling price.

Stocks are *above par*, or *at a premium*, when their market value exceeds their par value. Stocks are *below par*, or *at a discount*, when their market value is less than their par value.

479. A **Stock Broker** is a person who buys and sells stocks and bonds.

A Stock Broker may buy for himself, or as the agent of another person.

SANTA CLARA COUN
TEACHERS' LIBR
No.

UNITED STATES SECURITIES.

480. The bonds issued by the Government of the United States are called *United States Securities*, or *Government Securities.* They are of two classes: Registered Bonds and Coupon Bonds.

481. A **Registered Bond** is a bond whose ownership is recorded in the Treasury Department at Washington.

A Registered Bond is negotiable only by notifying the proper officer of the Treasury Department that the transfer has been made. Interest on a registered bond is collected only on the written order of the owner.

482. A **Coupon Bond** is a bond made payable to bearer.

Coupons are certificates of interest attached to a bond. There are as many coupons attached to a bond as there are payments of interest to be made. Thus, a bond payable in ten years, with interest payable quarterly, has, originally, forty coupons. Interest on a coupon bond is paid on the presentation of coupons when due.

Bonds are usually designated by the name of the corporation issuing them, and the rate of interest which they bear.

Thus, "U. S. 4's," means U. S. bonds bearing 4% interest per annum.

483. The principles of percentage apply to all calculations concerning stocks and bonds. The *par value* is the *base*, of which *brokerage*, *dividends*, *assessments*, *premiums* and *discounts* are *percentages*. Market value is either *amount*, or *difference* according as such market value is above or below par.

"U. S. 4's" pay 4 per cent of their par value, or $4 on every $100 bond, every year. But U. S. 4's, when purchased at 104, yield their purchaser only $4 on every $104 invested that is, about 3.7%.

PROBLEMS.

ORAL.

1. What annual income is paid by $1000 U. S. 4's?
2. What semi-annual dividend is paid by $4000 Union Pacific 6's?

3. What sum of money is paid to the holder of 5 shares of railroad stock, $100 a share, on which a half yearly dividend of 3% is declared ?

4. A certain stock pays regularly a quarterly dividend of 2%. What is the annual income from $5000 of this stock ?

5. A broker bought 10 shares stock ($100 a share) at 96, and sold at par. How many dollars did he gain?

6. A broker bought stock at 102, and sold at 98. How many dollars did he lose on 100 shares ?

7. How much money does a man pay for 50 shares stock, at 90 ?

8. How much is paid for 20 shares stock at 105 ?

9. A stockholder owning 30 shares stock is assessed 7%. How many dollars does he pay?

10. At 80, how many shares of stock can I buy for $4800?

11. At 120, how many shares of stock can I buy for $9600 ?

12. Find cost of 100 shares of stock at 80, brokerage ⅛%.

SOLUTION. The broker paid $8000, to which he adds ⅛ of 1% of the *par value* of the stock, which is ⅛% of $10000, or $12½. The purchaser pays $8012.50.

13. Find the cost of 40 shares stock at 110, brokerage ⅛%.

Written.

14. Find the cost of $5000 N. Y. Central Railroad stock at 112, brokerage ⅛.

SOLUTION. 112%+⅛%=112⅛%: 112⅛% of $5000=$5625.

15. Find the cost of $3000 Illinois Central Railroad stock at 102, brokerage ⅛.

16. Mr. A. bought $8000 U. S. 4's at 108, brokerage ⅛%; how much did he pay?

17. How much would a seller receive for $4000 stock at 92, brokerage $\frac{1}{4}$?

SOLUTION. The seller receives the market price less the brokerage, 92% $-\frac{1}{4}\%=91\frac{3}{4}\%$. $91\frac{3}{4}\%$ of $4000=$3670.

18. What does the owner receive for 60 shares bank stock at $98\frac{1}{2}$, brokerage $\frac{1}{4}$?

19. A dealer bought 78 shares stock at 52, and sold it at 63, paying $\frac{1}{8}$ brokerage for each transaction. What was his gain?

20. How many shares of stock ($100 each) may be bought for $7460.625 at $108\frac{1}{8}$, including brokerage.

SOLUTION. Since $108\frac{1}{8}$ buys $1.00 of stock, the amount purchased is $7460,625 $\div 1.08\frac{1}{8}=$ $6900; or 69 shares of $100 each.

21. How many shares of stock at 83, brokerage $\frac{1}{4}$, may be bonght for $6243.75 ?

22. What is the par value of U. S. bonds that may be purchased at $112\frac{1}{2}$, brokerage $\frac{1}{8}$, for $10812 ?

23. What is the market price of C. B. & Q. Railroad stock when $5800 stock costs $7264.50, including brokerage $\frac{1}{4}$?

SOLUTION I. The cost of $1.00 of stock is $7264.50 $\div$ 5800 $=$ $$1.25\frac{1}{4}$. Deducting brokerage, the market price is $1.25 ; or,

SOLUTION II. The brokerage is $\frac{1}{4}\%$ of $5800, which is $14.50. $7264.50 $-$ $14.50 $=$ $7250. $=$ market price. $7250 $\div$ $5800 $=1.25$.

24. What is the market price of C. & N. W. Railroad stock, when $6000 stock is bought for $6487.50, including $\frac{1}{8}$ brokerage?

25. How many shares railroad stock, at 91 and brokerage $\frac{1}{8}$, may be bought for $16949.25 ?

26. What is the par value of U. S. 4's that may be bought for $3152.50, at $121\frac{1}{4}$?

27. What is the price of stock when $17458 will purchase $23200 stock?

28. What income is yielded by $8000 5% stock?

29. A certain bank stock pays $2\frac{1}{4}\%$ quarterly. How much does a stockholder receive annually from $1600 of this stock?

30. How much 4% stock must be bought to yield an annual income of $1280 ?

Suggestion. 4% is the rate; $1280 is the percentage; the base is required.

31. A person receives semi-annually $360 from 6% stock. What is the face, or par value, of the stock?

32. A railroad company pays a semi-annual dividend of $2\frac{1}{2}\%$. How many shares does a stockholder own who who receives $1500 a year?

33. A holder of $7200 railroad stock receives in one year $360 dividend. What rate per cent does the stock pay ?

34. At what rate per cent per annum will $1080 bank stock yield a quarterly dividend of $32.40 ?

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35. Find the annual income from $12000 invested in 7% stocks at 120.

Solution. $12000÷1.20=$10000, the par value of stock purchased. $10000×.07=$700, the annual income.

36. Find the annual income from $6750 invested in 4% stock at 90.

37. What income will be received from the purchase of Tennessee 6's at 85 for $10200 ?

38 Which is better, and how much, to invest $10800 in stock at 90, which pays 7%, or in bonds at 80, which pay 6%?

39. When 6% stock is purchased at 120, what rate per cent does the investment pay?

Solution. $120 is paid for $100 stock yielding $6 a year. $6.÷$120 =.05 .

40. When 5% stock is bought at 120, what rate per cent does the investment pay ?

41. When 4% stock is bought at 80, what rate per cent does the investment pay ?

42. If 9% stock is worth 150, what rate of interest will be received from its purchase?

---

43. I wish an income of $400 from a 5% stock which I can buy for 102. How much money must I invest in it ?

SOLUTION. $400 is 5% of $8000, the par value of the stock to be bought. 102% of $8000 is $8160, the market value of the stock.

44. How much money must I pay for 6% stock at 108, in order to receive from it an annual income of $1800 ?

45. A bank declared a semi-annual dividend of $2\frac{1}{2}$%, which paid a stockholder $530 on stock which he had bought at 107. How much had he paid for his stock?

46. How much did I pay for 6% at 75, which yields me me $1500 annually ?

47. A 5% stock was sold at a price that it yielded the bearer only 4%. What did he pay for it ?

SOLUTION. $100 stock pays $5, which is 4% of the price of $100 stock. $5.÷.04=$125.

48. For how much must a man buy 5% stock in order to receive 6% on his investment ?

49. What is paid for 6% stock which yields 8% on the investment ?

50. What is paid for 8% which yields 6% on the investment ?

51. How much premium is paid on 10% stock that yields $7\frac{1}{2}$% on the investment ?

52. At what price is 4% stock bought when it yields an income equal to 6% stock bought at par ?

EXCHANGE.

484. Form of a Time Draft.

$500. *Milwaukee, Feb. 10, 1891.*

Ten days after sight pay to the order of

Samuel Richards,——Five Hundred——

dollars, and charge to account of

To Wm. Drexel, *Oliver Morton.*

Philadelphia.

485. A **Bill of Exchange,** or **Draft,** is an order to pay money, made in one place, payable in another.

486. The **Drawer,** or **Maker,** of the draft is the person, or firm, who signs it.

487. The **Drawee** is the person, or firm, to whom the draft is addressed

488. The **Payee** is the person, or firm, to whom the draft is made payable.

In the above draft, Oliver Morton is the *Drawer*, William Drexel is the *Drawee*, Samuel Richards is the Payee.

489. A **Sight Draft** is a draft payable upon presentation.

A sight draft reads, "*At sight*, pay to the order," etc.

490. A **Time Draft** is payable a specified number of days after its date, or after its presentation to the drawee.

491. The drawee signifies his *acceptance* of the draft, that is, he agrees to pay it, by writing the word *Accepted*, and his signature, across the face of the draft.

The draft thus accepted is called an **Acceptance.**

Three days of grace are generally allowed in the payment of a time draft.

492. **Exchange** is the method of making payments by drafts.

In **Domestic Exchange** drafts are drawn and paid in the same country.

493. Exchange is *at par* when the cost of a draft equals its face value.

Case I.

494. The Face of the Draft and the Rate of Exchange being given, to find the Cost of the Draft.

1. Find the cost of a sight draft on New York for $1200, exchange being 1% premium.

Solution. The cost of $1.00 exchange is $1.01. The cost of $1200 is $1200×1.01=$1212.

2. Find the cost of a sight draft on Philadelphia for $1860, exchange $\frac{3}{4}$ premium.

3. What is the cost of a sight draft for $2250 on Chicago, drawn at New Orleans when exchange is $1\frac{1}{8}\%$ discount?

4. Find the cost of a draft for $1200, payable in New York 30 days after sight, exchange 1% premium, interest 6%.

Solution. $1.00,+1% premium . . . . = $1.01
Deduct interest for 33 days at 6% . . = .0055
Cost of $1.00 . . . . = $1.0045
Cost of $1200=$1200×1.0045 . . . . = $1205.40

5. Find the cost of a draft for $975 on Baltimore, payable 30 days after sight, exchange $\frac{3}{4}\%$ premium, interest 6%.

6. Find cost of a draft on Albany for $1500, 24 days after sight, exchange $\frac{3}{4}\%$ discount, interest 6%.

7. Find cost of a draft for $1800, payable in 60 days, exchange at par, interest 8%.

CASE II.

495. The Cost and Rate of Exchange being given, to find the Face of a Draft.

1. Find the face of a sight draft drawn at St. Louis on Boston, costing $1206, exchange $\frac{1}{2}$% premium.

SOLUTION. The cost of $1.00 exchange is $1.005. The face of a draft costing $1206 is $1206÷1.005, which is $1200.

2. Find the face of a sight draft costing $3680, when exchange is $\frac{3}{8}$% premium.

3. Find the face of a sight draft costing $2750, when exchange is $\frac{3}{4}$% discount.

4. Find the face of a draft, payable 30 days after sight, costing $803.60, exchange 1% premium, interest 6%.

SOLUTION. $1.00+1% premium . . . . = $1.01
Deduct interest 33 da, 6% . . . . . = .0055
Cost of $1.00 . . . . . . = $1.0045
Face of draft costing $803 60=$803.60÷1.0045 - = $800

5. Find the face of a draft payable 60 days after sight, costing $982.75, exchange $\frac{1}{2}$% discount, interest 7%.

6. What is the face of a 30 day draft which I can buy for $1014.50, at 2% premium, interest 6%?

7. What is the face of a 30 day draft purchased for $3000, when the rate of exchange is $\frac{1}{8}$% premium, and interest 6%?

8. What is the face of a 60 day draft purchased for $900, when the rate of exchange is 1$\frac{1}{2}$% premium, and interest 8%?

CHAPTER XIII.

RATIO AND PROPORTION.

RATIO.

Article 496. Numbers, or quantities of the same kind, may be compared in two ways

I. By finding their *difference ;*

II. By finding the *quotient* of either divided by the other.

497. The *difference* of two numbers is their **Arithmetical Ratio.**

498. The *quotient* of one or two numbers divided by the other is their **Geometrical Ratio.**

The **Arithmetical Ratio** of 12 to 4, is 12—4, which is 8.
The **Geometrical Ratio** 12 to 4 is 12÷4, which is 3.

499. By *Ratio*, *geometrical* ratio is generally meant, and is so understood in this book.

500. Ratio may be expressed in three ways :

1. By the sign of division· as 15÷3.
2. As a fraction; as $\frac{15}{3}$.
3. By the colon; as 15:3, which expression is read, *15 is to 3.*

501. The two numbers necessary to form a ratio are called **Terms.** The terms, when written to indicate a ratio, are called a **Couplet.**

The *first* term is the **Antecedent.**

The *second* term is the **Consequent.**

502. Ratio is the quotient of the antecedent divided by the consequent. 15:3=15÷3=5.

Such a ratio is called a simple ratio.

EXAMPLES.

ORAL.

503. Find the ratios expressed by the following couplets:

1. 6 : 3; 12 : 4; 18 : 6; 50 : 10; 35 : 7; 60 : 12.
2. 72 : 9; 56 : 7; 63 : 9; 49 : 7; 84 : 12; 96 : 8.
3. 24 : 6; 75 : 25; 39 : 13; 48 : 6; 60 : 15; 80 : 20.
4. 25 : $12\frac{1}{2}$; $12\frac{1}{2}$: $6\frac{1}{4}$; 100 : $33\frac{1}{3}$; $87\frac{1}{2}$: $12\frac{1}{2}$; 50 : $6\frac{1}{4}$.
5. 15 : 7.5; 75 : 7.5; 125 : $12\frac{1}{2}$; 10 : .1; 18 : 4.5.

A ratio is a fraction when the antecedent is less than the consequent.

6. 4 : 8; 5 : 10; 5 : 15; 8 : 24; 9 : 36; 12 : 60.
7. 15 : 45; 20 : 80; 36 : 72; 40 : 120; $3\frac{1}{3}$: 10.
8. 12 : 48; 48 : 12; 16 : 8; 16 : 80; $4\frac{1}{2}$: 45.

504. It is evident that

1. The **Antecedent** is the *Dividend*, or *Numerator*.
2. The **Consequent** is the *Divisor*, or *Denominator*.
3. The **Ratio** is the *Quotient*, or *value of the fraction*.

Therefore the principles which apply to division and to fractions apply to ratio also, viz:

I. *Multiplication of the antecedent*, or *Division of the consequent*, *by any number* } *multiplies the ratio by that number.*

II. *Division of the antecedent*, or *multiplication of the consequent*, *by any number*, } *divides the ratio by that number.*

III. *Multiplication* or *division of* BOTH *antecedent and consequent by the same number* } *does not change the ratio.*

EXAMPLES.

WRITTEN.

505. Find the ratios expressed by the following couplets:

1. 625 : 25; 963 : 321; 840 : 24; 296 : 37.
2. 804 : 67; 965 : 6755; 750 : 187.5; 3.5 : 17.5.
3. 1648 yds. : 206 yds. ;7 95 lbs. : 15 lbs.; 876 in. : 24 in.
4. 8 ft. : 6 in.

   8 ft. = 96 in.; 96 : 6 = 16, *Ans.*, or
   6 in. = $\frac{1}{2}$ ft.; 8 ft. : $\frac{1}{2}$ ft. = $8 \div \frac{1}{2} = 16$, *Ans.*

5. 2 mi. : 2640 ft.; 7 weeks : $3\frac{1}{2}$ days; 6 doz. : 2 gross.
6. 10 tons : 200 lbs.; 5 yds. 2 ft. 6 in. : 1 ft. 9 in.

506. Couplets, like fractions, may be reduced to higher or lower terms. A couplet is expressed in its *lowest terms* when its antecedent and consequent are prime to each other.

A ratio, like a quotient, and like the value of a fraction, depends on the *relative* value of its terms.

507. Reduce the following couplets to lowest terms:

1. 625 : 175; 840 : 210; 125 : 1000; 69 : 115.
2. 65 : 130; 750 : 12.5; 540 : 45; 67 : 335.

508. Reduce $\frac{3}{4} : \frac{5}{8}$ to a couplet whose terms shall be integers.

SOLUTION. Change the terms to equivalent fractions having a common denominator: $\frac{6}{8}$ and $\frac{5}{8}$. Multiply each term by the common denominator, by removing the denominators: $\frac{6}{8} : \frac{5}{8} = 6 : 5$.

In like manner reduce

2. $\frac{1}{2} : \frac{3}{4}$; $\frac{3}{5} : \frac{7}{10}$; $\frac{2}{3} : \frac{5}{7}$; $\frac{5}{9} : \frac{3}{8}$; $\frac{4}{5} : \frac{6}{11}$.

509. 1. Find the ratio of $3\frac{1}{2}$ to $4\frac{1}{3}$.

SOLUTION. $3\frac{1}{2} : 4\frac{1}{3} = 3\frac{1}{2} \div 4\frac{1}{3} = \frac{7}{2} \div \frac{13}{4} = \frac{14}{13} = 1\frac{1}{13}$. *Ans.*

2. Find the ratios $2\frac{1}{5} : 3\frac{1}{3}$; $5\frac{3}{4} : 6\frac{1}{2}$; $8\frac{1}{5} : 12\frac{1}{2}$.

3. Find the ratio of 2.5 to 1.25.

SOLUTION. 2.5 : 1.25 = 250 : 125 = $\frac{1}{2}$. *Ans.*

4. Find the ratios 8.4 : 16.8; 6.25 : 2.5; 1.25 : 37.5.

510. Since a ratio is the quotient of an antecedent divided by its consequent, it follows that

1. *The antecedent = the consequent multiplied by the ratio.*
2. *The consequent = the antecedent divided by the ratio.*

1. Find the antecedents to the following consequents, when the ratio is 5 : 8, 12, 6, 16, 30, 25.
2. Find the consequents to the following antecedents, when the ratio is 6 : 12, 18, 36, 54, 48, 108.

PROPORTION.

511. The couplets 6:3 and 8:4 have each the same ratio, 2. These couplets may be formed into a **proportion** by connecting them with the sign of equality, thus: 6:3 =8 : 4, or by using the double colon, thus: 6:3 :: 8:4.

512. A **Simple Proportion** is an equality of two simple ratios.

513. A proportion consists of two couplets, that is, of four terms.

As parts of couplets,
The first and third terms are **antecedents**;
The second and fourth terms are **consequents.**
As parts of a proportion,
The first and fourth terms are **extremes**;
The second and third terms are **means.**

Thus, in the proportion 3:9:. 4:12, 3 and 4 are the *antecedents*, 9 and 12 are the *consequents*; 3 and 12 are the *extremes*, 9 and 4 are the *means*.

514. The last term of a proportion is called a **fourth proportional.**

515. A number which, in any proportion, is used for both means, is **a mean proportional** to the other two numbers.

Thus, in 3:6::6 12, 6 is the *mean proportional* to 3 and 12.

NOTE. A mean proportional is always the square root of the product of the extremes.

516. If any three terms of a proportion be given, the fourth term can be found. Two of the three given terms form a couplet; the third given term is either the antecedent or the consequent of a couplet having the same ratio as the given couplet.

517. In the proportion $8:4::6:3$, let any three terms be given, and let the missing term be represented by x. The proportion will then read either

(1.) $x : 4 :: 6 : 3$, or
(2.) $8 : x :: 6 : 3$, or
(3.) $8 : 4 :: x : 3$, or
(4.) $8 : 4 :: 6 : x$, according as the first, second, third, or fourth term is missing.

Solution I. Since a proportion is an equality of ratios, the ratio $x : 4$, in proportion (1) is equal to the ratio $6:3$, that is, is 2. Therefore the antecedent x equals its consequent 4, multiplied by the ratio 2. (Art. 510.) In proportion (2) x is one half of its antecedent, 8, since the consequent 3 is one half of its antecedent 6.

In a similar manner the value of the missing third or fourth term may be found. Hence,

518. When three terms of a proportion are given, to find the missing term;

I. *Find the ratio of the given couplet; if the other given term be an antecedent, divide it by this ratio; if it be a consequent, multiply it by this ratio.*

Find the missing terms in the following proportions :

1. $15 : 5 :: 12 : x$.

Solution. Because 5 is one third of 15, x is one third of 12, that is, 4.

2. $24 : 8 :: 18 : x$.
3. $36 : 6 :: 24 : x$.
4. $48 : x :: 12 : 2$.
5. $35 : x :: 21 : 3$.
6. $63 : 7 :: x : 5$.
7. $54 : 9 :: x : 10$.
8. $x : 8 :: 27 : 3$.
9. $x : 25 :: 16 : 4$.
10. $\frac{1}{3} : \frac{2}{3} :: 4 : x$.
11. $\frac{6}{8} : \frac{3}{8} :: \frac{6}{7} : x$.
12. $\frac{1}{4} : \frac{1}{8} :: \frac{2}{5} : x$.
13. $8 : 2 :: \frac{4}{9} : x$.
14. $3 : 9 :: \frac{2}{11} : x$.
15. $\frac{4}{5} : 4 :: x : 2$.
16. $\frac{5}{6} : x :: \frac{1}{3} : \frac{1}{5}$.
17. $x : \frac{3}{8} :: \frac{1}{9} : \frac{1}{3}$.

519. 8 : 3 :: 24 : 9.

Solution II. An inspection of any proportion will show that the ratio between the antecedents is the same as the ratio between the consequents. Thus, in 8:3::24:9, the first antecedent is one third of the second, as the first consequent is one third of the second. Hence,

II. *Make x bear the same ratio to its corresponding term (antecedent or consequent) that exists between the other two terms taken in the same order.*

Find the missing terms :

1. $15 : x :: 5 : 9$.
2. $40 : 32 :: x : 8$.
3. $x : 3 :: 35 : 21$.
4. $36 : 16 :: x : 4$.
5. $25 : 45 :: 5 : x$.
6. $81 : 72 :: 9 : x$.

520. In every proportion, *the product of the means equals the product of the extremes.*

Solution III. Since 5:8::10:16, $5\times16=8\times10$. If the first term be missing, the proportion reads x:8::10:16; then $16x=8\times10$, and $x=\frac{8\times10}{16}=5$. In a similar manner the missing term may be found, whichever it may be. Hence,

III. *If the means are given, divide their product by the given extreme.*

If the extremes are given, divide their product by the given mean.

Find the missing terms:

1. $3 : 5 :: 9 : x$
2. $8 : 16 :: 10 : x$.
3. $10 : 4 :: x : 36$.
4. $9 : x :: 7 : 10$.
5. $4 : 11 :: 5 : x$.
6. $18 : 72 :: x : 8$.
7. $35 : 10 :: 21 : x$.
8. $27 : 18 :: x : 36$.
9. $x : 81 :: 3 : 54$.
10. $x : 17 : : 5 : 85$.
11. $4\frac{1}{2} : 9 :: x : 75$.
12. $3.5 : x :: 12.5 : 50$.
13. $19 : 95 :: \frac{1}{5} : x$.
14. $\frac{4}{9} : \frac{5}{18} :: 16 : x$.
15. $6 : 40 :: x : 6\frac{2}{3}$.
16. Find a fourth proportional to 8, 5 and 24.
17. Find a fourth proportional to 5, 15 and 15.

PROBLEMS.

521. 1. If 7 tons of coal cost 42 dollars, what is the cost of 9 tons at the same rate?

ANALYSIS. The cost of the 9 tons should bear the same ratio to the cost of the 7 tons, that the 9 tons bear to the 7 tons. Represent by x the number of dollars paid for the 9 tons; then 7 tons : 9 tons :: 42 dollars : x dollars. By Rule III, $x=\frac{9\times42}{7}=\$54$. *Ans.*

NOTE. Observe that in every problem in proportion two of the quantities given are always of the same denomination. These two quantities form a couplet; the other given quantity and the quantity sought form the other couplet; the ratios of the two couplets are, of course, equal. In the above problem, the two amounts of coal form a couplet; the sum of money named ($42) and the sum sought form the second couplet.

2. If 6 barrels of flour cost $33, what is the value of 11 barrels at the same rate?

3. If 30 bushels of wheat cost $37.50, what is the cost of 45 bushels at the same price.

4. If I pay a carpenter $30 for 12 days' work, how much should I pay him for 28 days' work?

5. If 90 acres of land produce 2250 bushels of wheat, how many bushels will 32 acres produce?

6. If 15 horses consume 40 bushels of oats in a certain time, how many bushels will 24 horses consume in the same time?

7. If an upright stick 3 feet long cast a shadow 2 feet long, what is the height of an upright pole casting a shadow 40 feet long at the same hour?

8. If a barrel of flour (196 lbs.) is worth $7.35, what are 100 lbs. worth?

9. If 75 acres of land are worth $375, what are 47 acres worth at the same rate?

10. If 39° of a great circle equal $2697\frac{1}{2}$ miles, how many miles in the equator?

11. If $\frac{2}{3}$ of a farm is worth $2400, what is $\frac{5}{8}$ of it worth?

SUGGESTION. $\frac{2}{3}:\frac{5}{8}::\frac{16}{24}:\frac{15}{24}::16:15$.

12. If $\frac{3}{8}$ of a yard of cloth cost $1.50, what is the value of 38 yards of cloth of the same quality?

13. If $2\frac{1}{6}$ yards of silk cost $13.39, what will $12\frac{1}{4}$ yards cost at the same rate?

14. If $19\frac{1}{2}$ yards calico cost $3.90, what will $16\frac{1}{4}$ yards cost at the same rate?

15. If the interest on $2000 for a certain time is $90, what is the interest on $3600 for the same time and at the same rate?

16. If the interest on $800 for 18 months is $72, what is the interest on the same principal and at the same rate for 22 months?

17. If 800 lbs. of coal are needed to run an engine 10 hours, how many tons of 2000 lbs. will be needed to run the same engine 48 hours?

18. If 36 men can build 240 linear feet of wall in 8 days, how many linear feet of wall can the same men build in 22 days.

19. If 196 lbs. of flour make 240 loaves of bread, how many loaves may be made of 320 lbs. of flour?

20. If a column of mercury 28.42 inches in height weigh .84 of a pound, what is the weight of a column of the same diameter 31.5 inches high?

21. If a church spire 160 feet high be represented by a drawing 12 inches high, what length of line should, on the same scale, represent a tower 120 feet high?

INVERSE PROPORTION.

522. In the problems already given, a change in any term has been accompanied by a *like* change in its corresponding term. For instance, in problem 1, page 257, the greater the quantity of coal, the greater the cost. Such problems are examples of *Direct Proportion.*

523. But in this problem,

1. If a certain quantity of coal last 4 families 6 months, how long will it last 8 similar families?

It is evident that the coal would last a *greater* number of families a *shorter* time: that is,

1st no. families:2d no. families::2nd no. mos.:1st no. mos.

$$4 : 8 :: x : 6,$$

and $x=3$ months. The second ratio, when compared with the first is called an inverse ratio; and a proportion containing such a ratio is called an **Inverse Proportion.** *Time* is a frequent, but not a necessary element in problems involving an inverse ratio.

2. If 20 horses consume a certain quantity of oats in 8 weeks, in how many weeks will 48 horses consume the same quantity of oats ?

3. If a regiment of 840 men has food for 60 days, how many days should the same food last a garrison of 1260 men ?

4. If a ship's company of 80 men be provisioned for 24 days, and 20 of the company be discharged, how many days should the provisions support the remaining men ?

5. If 18 men can build a wall 420 feet long in 30 days, in how many days can 27 men build such a wall ?

6. If 80 yards of cloth, $\frac{3}{4}$ yds. wide, will make a certain

number of suits, how many yards $\frac{7}{8}$ yds. wide will be needed to make the garments?

7. If a pipe which discharges 4 gallons a minute fills a cistern in 2 hours, in how many minutes may the cistern be filled by a pipe discharging 9 gallons a minute?

COMPOUND PROPORTION.

524. A ratio whose terms are the products of the corresponding terms of two or more ratios, may be called a *compound ratio*, when the terms are written as *factors*.

The simple ratios 2 : 3 and 4 : 5 may form the compound ratio $2\times4 : 3\times5$, the antecedents of the simple ratios being the factors of the antecedent of the compound ratio; and the consequents of the simple ratio forming the consequent of the compound ratio.

When the indicated multiplications are performed, the compound ratio becomes a simple ratio. Thus, the compound ratio $2\times4 : 3\times5$ becomes the simple ratio 8 : 15.

525. A **Compound Proportion** is the equality of two ratios, either or both ratios being compound.

EXAMPLES.

Find the missing term:

1. $3 : x :: \left\{\begin{matrix} 4 \times 5 \\ 3 \times 7 \end{matrix}\right\} : \left\{\begin{matrix} 8 \times 9 \\ 5 \times 6 \end{matrix}\right.$.

SOLUTION. (3rd method, Art. 520.) Make x equal to the product of the antecedents divided by the given consequent, thus:

$x = \frac{3\times8\times9\times5\times6}{4\times5\times3\times7} = 15\frac{3}{7}$. *Ans.*

2. $5 : x :: \left\{\begin{matrix} 4 \times 9 \\ 8 \times 5 \end{matrix}\right\} : \left\{\begin{matrix} 3 \times 12 \\ 6 \times 2 \end{matrix}\right.$.

3. $x : 24 :: \left\{\begin{matrix} 27 \times 48 \\ 14 \times 60 \end{matrix}\right\} : \left\{\begin{matrix} 18 \times 36 \\ 42 \times 15 \end{matrix}\right. .$

4. $7 : 9 :: \left\{\begin{matrix} x \times 11 \\ 4 \times 17 \end{matrix}\right\} : \begin{matrix} 68 \times 33 \\ 24 \times 27 \end{matrix} .$

PROBLEMS.

526. 1. If 8 men in 6 days build 30 feet of wall, how many men could, in 4 days, build 50 feet of the same wall?

SOLUTION. (3rd method, Art. 520.) 8 men working 6 days : x men working 4 days :: 30 : 50 : or, $8 \times 6 : x \times 4 :: 30 : 50$: therefore, $x = \frac{8 \times 6 \times 50}{4 \times 30} = 20$ men. *Ans.*

2. If 12 men in 9 days mow 36 acres of grass, how many men would be needed to mow 48 acres in 6 days?

3. If 12 men in 8 days of 10 hours each build a wall 40 feet long, 6 feet high, 2 feet thick, how many feet of wall 8 feet high, 3 feet thick, could 15 men build in 20 days of 8 hours each?

SUGGESTION. There is the same ratio between the total number of hours work of each "gang" of men, as represented by the product of $12 \times 8 \times 10$, for the first gang and the product of $15 \times 20 \times 8$ for the second, that there is between the total number of cubic feet in the wall built by each party respectively.

4. If 14 men in 12 days of 8 hours each, build 105 feet of wall 10 feet high, 2 feet thick, in how many days of 9 hours each would 8 men build 450 feet of wall 18 inches thick, 9 feet high?

5. If 54 men can in 10 days excavate a cellar 100 feet long, 50 feet wide, 10 feet deep, how many men are required to excavate a cellar 150 feet long, 40 feet wide, 12½ feet deep, in 18 days?

6. If a block of marble 20 feet long, 4 feet wide, 8 inches thick, weighs 1200 lbs., what is the weight of a block 12 feet long, 3 feet wide, 16 inches thick?

Suggestion. The labor of re-writing for cancellation may be avoided by writing the extremes on one side of a vertical line and the means on the other side, without the formality of a proportion.

extr's.	*means.*
20	12
4	3
$\frac{2}{3}$	$\frac{4}{5}$
x	1200

$x =$

7. If 800 reams of paper are required for 2500 volumes of a book of 160 octavo pages, how many reams are required for 5000 volumes of a book of 320 pages, each page twice the size of an octavo page?
8. If 80 yards of cloth, $\frac{3}{4}$ yd. wide, cost $160, what is the cost of 66 yards of the same quality, $\frac{5}{4}$ yd. wide?
9. How many bricks in a wall 96 feet long, 8 feet high, 16 inches thick, if there are 147 bricks in a part of the wall 2 feet long, 3 feet high?
10. If 4050 bricks, 8 inches long by 4 inches wide, be needed to pave a certain area, how many tile 9 inches square are needed to pave an area one half the surface of the first?

PROPORTIONAL PARTS.

527. The division of a quantity into two or more parts having a certain ratio to one another is a division into **Proportional Parts.**

1. Divide $15 between A and B so that A's share shall be to B's share as 2 is to 3.

Solution. There will be 5 equal parts ($2+3=5$), of which A will have two and B will have three. That is, A will have $\frac{2}{5}$ of $15, which is $6, and B will have $\frac{3}{5}$ of $15, or $9.

2. Divide $72 in the ratio of 5 to 7.
3. Divide $64 into three parts, in the ratio of 4, 5, and 7.
4 Divide $140 among A, B, and C, in such a manner that A shall have twice as much as B, and B twice as much as C.

5. Divide $3000 among A, B, and C, so that A shall have four times as much as B, and C shall have twice as much as both A and B.

6. Divide 36 into two parts, in the ratio of $\frac{1}{3}$ to $\frac{2}{3}$.

7. Divide 45 into two parts, in the ratio of $\frac{3}{4}$ and $\frac{3}{8}$.

8. Divide $16 between A and B, so that B shall have $\frac{3}{5}$ as much as A.

9. Divide $70 between C and D, so that C shall have $2\frac{1}{2}$ times as much as D.

10. A certain kind of brass is composed of 2 parts copper to 1 part zinc. How many ounces of copper in 6 pounds of such brass?

11. Water is composed of hydrogen and oxygen, in the ratio of 2 to 16, by weight, How many pounds of oxygen in 36 pounds of water?

PARTNERSHIP.

528. 1. A and B are in partnership for one year; A's capital was 3 times B's; in what ratio should the gain or loss be divided? How much should each partner have of gain of $4800?

2. C and D gain $3900 in 6 months; D's capital is $\frac{4}{9}$ of C's; how much of the gain should each have?

3. A and B form a partnership for 1 year, but B withdraws his capital after 6 months. If each furnished an equal amount of capital, in what ratio should the gain be divided at the end of the year?

4. Mr. Barr invested $12000 in a stock farm, and after one year took Mr. Day into partnership, the latter furnishing $8000. Two years later, the profits, $6500, were divided. How much should each partner have?

Solution. Mr. Barr's capital was used 3 years; Mr. Day's was used 2 years. The use of one dollar for 3 years is equivalent to the use of three dollars for 1 year.

Mr. Barr's capital, $12000, for 3 years, . = $36000 for 1 year.
Mr. Day's capital, $8000, for 2 years, . = 16000 " 1 "

The firm's capital was equal to . . $52000 for 1 year.
Mr. Barr's share of the gain was . . $\frac{36}{52}$, or $\frac{9}{13}$, of $6500, or $4500
Mr. Day's " " " " " . $\frac{16}{52}$, or $\frac{4}{13}$, of $6500, or $2000

5. A and B formed a partnership. A furnished $15000 for 2 years; B furnished $12000 for 3 years. They gain $11880. What was the share of each?

6. Two brothers engaged in trade, the elder furnishing $5400 for 9 months, the younger $4500 for 12 months. They gained $1710. What was the share of each?

7. Mr. Smith failed in business, with assets of $9600, liabilities $12000. He owed A $2500, B $9000, C the remainder. What did each creditor receive?

8. D, E and F rented a pasture in common for $66. D pastured 8 cows for 5 months, E 12 cows for 6 months, F 16 cows for 4 months. How much should each pay?

9. A and B formed a partnership, January 1, 1890, for 4 years. A furnished $25000, B $20000. On January 1, 1892, C is admitted to the firm, with a capital of $15000. Dec. 31, 1893, the firm had gained $16800. What was the share of each?

MISCELLANEOUS PROBLEMS IN PROPORTION.

529. 1. If 36 men can build a boat in 58 days, in how many days can 29 men build it?

2. In how many minutes will a cistern holding 225 gallons be filled by a pipe that fills in $7\frac{1}{2}$ minutes a tank holding 150 gallons?

3. When a shadow 6 ft. 8 in. is cast by an upright post 5 ft. 10 in. high, what is the height of a spire casting a shadow 216 feet ?

4. In walking 3600 feet, a man takes steps of 2 ft. 6 in. How many feet would he walk with the same number of steps, if each step was 2 ft. $4\frac{1}{2}$ in. ?

5. If a cistern $17\frac{1}{2}$ ft. long, $5\frac{1}{4}$ ft. wide, and 13 ft. deep, hold 273 barrels, how many barrels does a cistern hold that is 16 ft. long, 7 ft. wide, and $7\frac{1}{2}$ ft. deep ?

6. If 12 men, working 9 hours a day for $15\frac{5}{9}$ days, were able to finish $\frac{2}{3}$ of certain work, how many men may be discharged that the work may be completed in 15 days more, at 7 hours work a day ?

7. If 248 men, in $6\frac{3}{5}$ days of 10 hours each, dig a trench of 7 degrees of hardness, $232\frac{1}{2}$ yds. long, $3\frac{2}{3}$ yds. wide and $2\frac{1}{3}$ yds. deep, in how many days of 9 hours each can 155 men dig a trench of 4 degrees of hardness, $387\frac{1}{2}$ yds. long, $10\frac{1}{2}$ yds. wide, and $1\frac{3}{4}$ yds. deep?

8. If 24 boys can pick 18 cases of berries, each containing 12 boxes, in 1 hour, how many such boys could pick 6 cases, each containing 12 such boxes, in 30 minutes ?

9. If a block of granite 8 ft. long, $5\frac{1}{2}$ ft. wide, $2\frac{1}{4}$ ft. thick, weigh 16,335 lbs., what is the weight of a granite column 20 ft. high, $2\frac{1}{2}$ ft. square?

10. Divide $960 between A and B so that A shall have 5 times as much money as B.

11. C's money is $1\frac{1}{3}$ times as much as D's. They have together $315; how many dollars has each?

12. Divide 462 into four parts that shall be in the ratio of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$.

CHAPTER XIV.

INVOLUTION AND EVOLUTION.

INVOLUTION.

Article 530. A **Power** of a number is the product of that number used as a factor a certain number of times.

Thus, 4 is a power of 2, since $4=2\times2$. So 16 is a power of 2, since $16=2\times2\times2\times2$.

531. Powers are named from the number of times the given number is used as a factor.

The **First Power** of a number is the number itself.

The **Second Power** of a number is the product of that number used twice as a factor.

The **Third Power** of a number is the product of that number used three times as a factor.

Thus: The first power of 4 is . . 4.
The second power of 4 is $4\times4=16$.
The third power of 4 is $4\times4\times4=64$.
The fourth power of 4 is $4\times4\times4\times4=256$.

532. The second power of a number is usually called its *square.*

The third power of a number is frequently called its *cube.*

See the tables of square and cubic measure. Arts. 297 to 300.

533. The **Exponent** of a power shows how many times the given number has been used as a factor.

The exponent is a small figure written to the right of and above the number representing a power. Thus, $5^2=5\times5$: $8^3=8\times8\times8$: $(\frac{2}{3})^2=\frac{2}{3}\times\frac{2}{3}$.

5^2 is read *the second power of* 5, or, 5 *squared.* 8^3 is read *the third power of* 8, or, *the cube of* 8; or, 8 *cubed.*

534. **Involution** is the process of finding the power of a number.

1. Find the fourth power of 12.

SOLUTION. $12^4 = 12 \times 12 \times 12 \times 12 = 20736$.

2. Find the fifth power of .03.

SOLUTION. $.03^5 = .03 \times .03 \times .03 \times .03 \times .03 = .0000000243$.

3. Find the cube of $\frac{3}{5}$.

SOLUTION. $(\frac{3}{5})^3 = \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} = \frac{27}{125}$. Hence,

535. To find a required power of a number:

Use the given number as a factor as many times as there are units in the exponent of the power required.

Find the following required powers:

1. 35^2.
2. 81^3.
3. 45^4.
4. 125^3.
5. 2.5^4.
6. $.125^3$.
7. $.07^4$.
8. $(.5\frac{1}{2})^2$.
9. $(\frac{1}{2})^6$.
10. $(\frac{2}{3})^4$.
11. $(\frac{3}{8})^3$.
12. $(\frac{2}{7})^4$.

EVOLUTION.

536. A **Root** of a number is one of its equal factors.

Thus, since

$4 = 2 \times 2$, and

$8 = 2 \times 2 \times 2$, and

$16 = 2 \times 2 \times 2 \times 2$, 2 is a *root* of 4, of 8, and of 16.

537. A Root is named from the number of times it is used as factor.

Thus, 2 is the *second*, or *square root* of 4; the *third*, or *cube root* of 8; the *fourth* root of 16.

538. A number is its own *first root* as well as its own *first power*.

The *Second*, or *Square Root* of a number is one of its two equal factors.

The *Third, or Cube Root*, of a number is one of its three equal factors, etc.

539. Powers and Roots are correlative.

Thus, 8 is the third power of 2; and 2 is the third root of 8.

540. The sign $\sqrt{}$ is called the **Radical Sign**, and is used to indicate a root.

Thus, $\sqrt{81}$, or $\sqrt[2]{81}$, means the square root of 81, $\sqrt[3]{125}$ means the cube root of 125.

The figure on the left of the radical sign is the **Index** of the root, and shows which root is meant.

The *Index* is usually omitted when the radical indicates the square root.

Instead of the radical sign, a *fractional exponent* is sometimes used.

Thus: $4^{\frac{1}{2}}=\sqrt{4}$; $8^{\frac{1}{3}}=\sqrt[3]{8}$. $8^{\frac{2}{3}}$ means the cube root of the square of 8.

541. A **Perfect Power** is a number whose exact root may be obtained.

Thus, 27 is a *perfect cube*, since its cube root, 3, can be obtained. But 27 is not a *perfect square*.

542. Evolution is the process of obtaining the root of a number.

543. Table of a few roots and powers.

Roots.	1,	2,	3,	4,	5,	6,	7,	8,	9,	10.
Squares.	1,	4,	9,	16,	25,	36,	49,	64,	81,	100.
Cubes.	1,	8,	27,	64,	125,	216,	343,	512,	729,	1000.

This table should be memorized.

SQUARE ROOT.

544. The *square* of any number represented by two or more figures, as 46, may be thus obtained:

46=40 +6, that is, 4 tens plus 6 units. Multiply 46 by itself.

$$\begin{array}{l}
40+6 \\
\underline{40+6\qquad\qquad} \\
\overline{40}^2+40\times6 \\
\underline{\qquad 40\times6+6^2} \\
\overline{40}^2+2(40\times6)+6^2 \\
\;t^2+2(\;t\times u)+u^2
\end{array}
\qquad\qquad
\begin{array}{r}
\text{That is, } \overline{40}^2=1600 \\
2(40\times6)=\;\;480 \\
\underline{6^2=\;\;\;36} \\
\overline{46}^2=2116
\end{array}$$

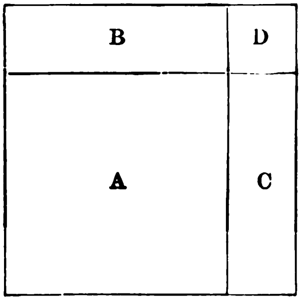
Every perfect square consists of three parts, as here shown, viz.:

1. *The square of the tens ; plus*
2. *Twice the product of the tens and units; plus*
3. *The square of the units.*

Or, more briefly, using t for *tens*, and u for *units* :

$(t+u)^2=t^2+2(t\times u)+u^2$.

This is illustrated by the following diagram, in which the square A is



the square of the tens, ($\overline{40}^2$), each of of the two equal rectangles B and C, is the product of the tens and units, (40×6) and the square D is the square of the units (6^2). The entire square is therefore equal to $\overline{40}^2+2(40\times6)+6^2$, or, in general, $t^2+2(t\times u)+u^2$.

In finding the square root of such a number, it is desirable to retrace the steps taken above. Thus,

1. Find the square root of 2116.

Process.

21·16(46

16

86)516

516

Analysis. 1. Separate the number into periods of two figures each, beginning with the units figure.

2. Find the greatest square in the left hand period. The greatest square in 21 is 16; its root, 4, is the tens' figure of the root sought.

3. From the left hand period subtract the square 16, and annex the right hand period.

4. The number thus formed, 516, contains the other two parts of the square, viz.: $2(t\times u)+u^2$. Divide 51 of the 516 by 2 t, that is, by 8. The quotient, 6, is probably the units' figure of the root.

5. Find the product of $2t+u^2$, that is, of 86, multiplied by u, that is, by 6. This product, which is $2(t\times u)+u^2$, is 516. Therefore the square root of 2116 is 46.

2. Find the square root of 2481.

Process.

24·81 (49.44+

16

89)881

801

1784)8000

7136

17884)86400

Analysis. This number is not a perfect square, but the process is the same as before.

1. Subtract 16, the greatest square in 24 from 24, writing 4, the square root of 16, as the first figure of the root, and annex the next period, 81, to the remainder 8, making 881.

2. Double the root, 4, making 8; 8 is contained 9 times in the 88 of 881; therefore 9 is probably the second figure of the root.

3. Write the 9 as the second figure of the root, and as the second figure in the trial divisor, 89.

4. Find the product of 89×9, which is 801. The *integral* part of the root of 2481 is therefore 49.

To find the decimal part, annex a period of two ciphers, and proceed as before.

NOTES. 1. The number whose root is to be found is divided into periods of two figures each, since, as a general rule, every figure in a square root corresponds to two figures in the square.

2. When the first figure of a root is found, double the number of units is taken as a trial divisor, since the remainder which is to be divided by it, is *twice* the tens, multiplied by the units, plus the square of the units. And

3. Because the remainder, $2(t \times u)+u^2$, contains also the square of the units, u^2, the units' figure is not considered as a part of the trial dividend.

Thus, in Example 2, the trial dividend is 88, not 881.

The quotient of a trial dividend by a trial divisor never exceeds 9.

545. To extract a square root of a number :

1. *Separate the number into periods of two figures, each, beginning with units.*
2. *Find the greatest square in the left hand period, and place its square root to the right. as the first figure of the root.*
3. *Deduct the square of the root found from the first period, and to the remainder annex the next period, for a trial dividend.*
4. *Double the root already found, for a trial divisor ; by it divide the trial dividend, excluding the right hand figure ; the quotient will be the probable second figure of the root.*
5. *Annex the second figure of the root to the trial divisor, for a complete divisor ; multiply this divisor by the second figure of the root, and subtract the product from the trial dividend.*
6. *To the remainder annex the next period, for the next trial dividend. Double the root already found as a*

new trial divisor, and proceed as before until every period has been used.

NOTES. 1. If a zero occur in the root, annex a zero to the trial divisor, annex the next period to the trial dividend, and proceed as before.

2. If there is a remainder after the unit figure of the root has been found, annex to the remainder a period of two zeros, and continue the process. The following figures in the root are decimals.

546. The extraction of the square root of a number is illustrated by the following method of finding the side of a square whose area is given:

1. Find the length of the side of a square whose area is 2116 square feet.

Fig. 1.

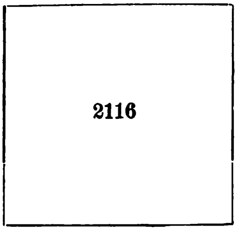


Fig. 2.

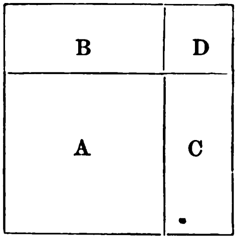
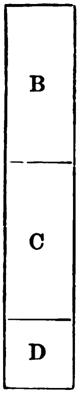


Fig. 3.



SOLUTION. The side of the square is between 40 feet and 50 feet, since 40 feet is the side of a square containing only 1600 square feet, and 50 feet is the side of a square containing 2500 square feet.

Remove the square A, whose side is 40 ft., and whose area is 1600 sq. ft. There remains the surface B + C + D, which is equal to 2116 sq. ft. − 1600 sq. ft. = 516 sq. ft.

The surface, 516 sq. ft., is composed of the two equal rectangles and the square D. The length of the two rectangles is 40 + 40 = 80. The length of the entire rectangle, B + C + D, is 40 + 40 + one side of the square D, which is probably 6, since 516 ÷ 80 = 6 +. Assuming the side of D to be 6, the length of the rectangle B + C + D is 86. The area 516 ÷ 86 = 6, which is the width of the rectangle B + C + D, and one side of the square D.

The side of the original square is therefore 40 ft. + 6 ft. = 46 ft.

547. Find the square root of each of the following numbers :

1. 576.
2. 729.
3. 2809.
4. 4356.
5. 9216.
6. 9604.
7. 12321.
8. 15625.
9. 315844.
10. 674041.
11. 938961.
12. 2050624.

13. Find the square root of $\frac{121}{169}$.

Solution. When each of the terms of a fraction is a perfect square, extract the root of each. $\sqrt{\frac{121}{169}}=\frac{\sqrt{121}}{\sqrt{169}}=\frac{11}{13}.$

Find the square roots of

14. $\frac{225}{1600}$.
15. $\frac{289}{961}$.
16. $\frac{576}{625}$.
17. $\frac{441}{484}$.
18. $\frac{196}{729}$.
19. $\frac{900}{2500}$.

Find the value of each of the following expressions, to two decimal places :

20. $\sqrt{306}$.
21. $\sqrt{511}$.
22. $\sqrt{2108}$.
23. $\sqrt{3216}$.
24. $\sqrt{4526}$.
25. $\sqrt{5221}$.

548. 1. Find the square root of 134.5.

```
   1)1·34.50(11.6—
     1
    ____
 21) 34
     21
    ____
226) 1350
     1356
```

Solution. Separate into periods of two figures each, beginning with the decimal point, thus : 1·34.50, annexing a zero, if desirable. Then proceed according to the rule:

2. Find the square root of .06256.

Suggestion. Separate into periods of two figures each, beginning with the decimal point, thus: .06·25·60.

Perform the following examples :

3. $590.49^{\frac{1}{2}}$.
4. $4.6656^{\frac{1}{2}}$.
5. $.2209^{\frac{1}{2}}$.
6 $.015625^{\frac{1}{2}}$.
7. $.065536^{\frac{1}{2}}$.
8. $53.1441^{\frac{1}{2}}$.
9. $1.679616^{\frac{1}{2}}$.
10. $11.7649^{\frac{1}{2}}$.
11. $2621.44^{\frac{1}{2}}$.
12. $5.764801^{\frac{1}{2}}$.
13. $40.9600^{\frac{1}{2}}$.
14. $.43046721^{\frac{1}{2}}$.

15. Find the square root of $60\frac{1}{16}$.

Solution. $\sqrt{60\frac{1}{16}}=\sqrt{60.0625}=7.75=7\frac{3}{4}$.

16. Find the square root of $\frac{3}{4}$.

SOLUTION. $\sqrt{\frac{3}{4}}=\sqrt{.75}=.0866+$.

17. Find the square root of $7\frac{1}{9}$.

SOLUTION. $\sqrt{7\frac{1}{9}}=\sqrt{\frac{64}{9}}=\frac{8}{3}=2\frac{2}{3}$.

18. Find the square root of $1\frac{64}{98}$.

SOLUTION. $\sqrt{1\frac{64}{98}}=\sqrt{1\frac{32}{49}}=\sqrt{\frac{81}{49}}=\frac{9}{7}=1\frac{2}{7}$.

Find the square roots of

19. $30\frac{1}{4}$.
20. $27\frac{9}{16}$.
21. $12\frac{24}{25}$.
22. $\frac{5}{8}$.
23. $\frac{7}{12}$.
24. $756\frac{1}{4}$.
25. $37\frac{36}{49}$.
26. $981\frac{7}{9}$.
27. $67\frac{18}{75}$.

Find the value of

28. $\sqrt{\frac{6}{7}}$ to 4 decimal places.
29. $\sqrt{\frac{4}{5}}$ to 5 “ “
30. $\sqrt{.5\frac{1}{2}}$ to 4 “ “
31. $\sqrt{1.6\frac{1}{2}}$ to 3 “ “
32. $\sqrt{73}$ to 3 dec. places.
33. $\sqrt{8^3}$ to 3 “ “
34. $\sqrt{\frac{1}{2}+\frac{1}{16}}$.
35. $\sqrt{\frac{3}{5}-\frac{6}{25}}$.

PROBLEMS.

549. 1. A rectangular field measures 624 rods long and 156 rods wide. What is the length of one side of a square field of the same area?

2. How much more will it cost to fence the rectangular field 624 rods long and 156 rods wide, than a square field of the same area, at $3.75 a rod ?

3. What is the length of one side of a square floor containing 1225 sq. ft. ?

4. A rectangular floor containing 1875 sq. yds. is 3 times as long as it is wide. What are its dimensions ?

SUGGESTION. Draw a diagram representing the field divided into 3 equal fields.

5. Find the mean proportional (Art. 515) between 6 and 24.

SOLUTION. The mean proportional of two numbers is the square root of their product. $\sqrt{6\times 24}=\sqrt{144}=12$.

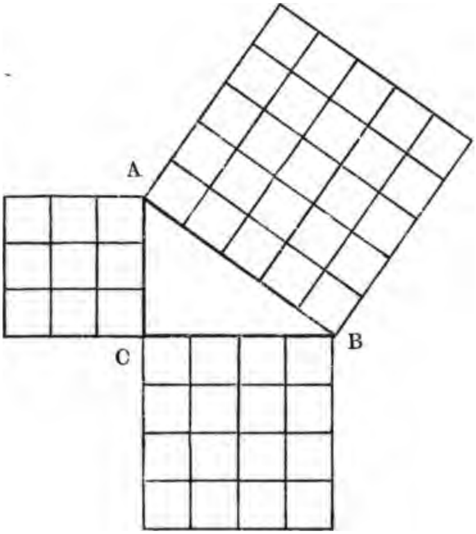
6. Find the mean proportional between 5 and 125; between 49 and 196; between 425 and 17.
7. What is the ratio of the product of the square roots of two quantities to the square root of their product? Illustrate by an example.
8. How many rods on one side of a square field containing 10 acres?
9. How many rods on one side of a square mile of land?

PROPERTY OF THE RIGHT-ANGLED TRIANGLE.

550. It is proved in Geometry that

In every right-angled triangle the square of the hypothenuse is equal to the sum of the squares of the base and perpendicular.

For definitions, see Arts. 577, 578.



The accompanying figure illustrates this property.

Represent the hypothenuse, A B, by H, the base, B C, by B, the perpendicular; A C, by P. Then,

$H^2 = B^2 + P^2$; **that is,**

$5^2 = 4^2 + 3^2$; **or,**

$25 = 16 + 9.$

Since $H^2 = B^2 + P^2$,

$H = \sqrt{B^2 + P^2}.$

1. The base of a right-angled triangle is 6, the perpendicular is 8; what is the hypothenuse?

Solution. $H^2=6^2+8^2=36+64=100$. $H=\sqrt{100}=10$.

2. The base is 12, the perpendicular is 16. Find the hypothenuse.
3. The base is 15, the perpendicular is 20. Find the hypothenuse.
4. The hypothenuse is 10, the base is 6. What is the perpendicular?

Solution. Since $H^2=B^2+P^2$, $P^2=H^2-B^2$. That is, $P^2=\overline{10}^2-6^2=100-36=64$. Since $P^2=64$, $P=\sqrt{64}=8$.

5. The hypothenuse is 10, the perpendicular is 8. What is the base?

Solution. Since $H^2=B^2+P^2$, $B^2=H^2-P^2$. That is, $B^2=\overline{10}^2-8^2=100-64=36$. Since $B^2=36$, $B=\sqrt{36}=6$. In general,

551. I. *The hypothenuse of any right-angled triangle is equal to the square root of the sum of the squares of the other two sides.*

$H=\sqrt{B^2+P^2}$.

II. *Either side about the right angle of a right-angled triangle is equal to the square root of the difference of the other two sides.*

$B=\sqrt{H^2-P^2}$; and $P=\sqrt{H^2-B^2}$.

1. What is the base, when the hypothenuse is 50, and the perpendicular 40 ?
2 What is the perpendicular when the hypothenuse is 30 and the base 18 ?
3. Base is 7, hypothenuse 9. What is the perpendicular?
4. Hypothenuse is 11, perpendicular is 5. What is the base?
5. How far from the base of a perpendicular telegraph pole 60 ft. high could a kite string 100 ft. long reach, if the kite were lodged on the top of the pole?
6. A ladder 20 ft. long leans against the side of a house.

How far from the ground will the upper end of the ladder be if the foot is 3 feet from the wall ?

7. What is the diagonal (Art. 585.) of a blackboard 10 ft. long, 4 ft. wide?

8. What is the diagonal of a slate 6 inches by 10 inches?

9. What is the diagonal of a square whose side is 1?

SOLUTION. Diagonal$=\sqrt{1^2+1^2}=\sqrt{2}$. Therefore,

The ratio of the side of a square to its diagonal is $1 : \sqrt{2}$.

10. What is the diagonal of a square 5 inches on a side?

11. What is the largest square that can be cut from a circular piece of paper 4 inches in diameter?

12. What is the largest square beam that can be cut from a log 20 inches in diameter?

13. Find the distance from the upper N. E. corner to the lower S.W. corner of a room 16 ft. by 12 ft. by 10 ft. This distance is called the *diagonal* of the room.

SOLUTION. $D=\sqrt{16^2+12^2+10^2}=\sqrt{500}=22.3+$ft.

The diagonal of any rectangular solid is equal to the square root of the sum of the squares of its three dimensions.

14. Find the diagonal of a room 20 ft. by 16 ft. by 12 ft.

15. Find the diagonal of a box 17 in. by 9 in. by 6 in., inside measurement.

16. Find the largest cube that can be cut from a globe 18 in. in diameter.

17. Find diagonal, in rods, of a square acre.

18. A steamer goes east $18\frac{1}{2}$ miles an hour; a sloop goes north 4 miles an hour. How far apart are they in 5 hours?

19. A cistern holding 2400 gallons is 10 ft. deep. The length and width are equal ; what are they?

20. Find the side, in rods, of a square field equal in area to three fields containing respectively 10 acres, 4 acres, and 2 acres 144 sq. rd.

CUBE ROOT.

552. The cube of any number represented by two or more figures, as 65, may be thus obtained:

65=60+5, that is, 6 tens plus 5 units.

$$\begin{array}{l}
60+5 \\
\underline{60+5} \\
\overline{60}^2+60\times5 \\
\qquad \underline{60\times5+5^2.} \\
\overline{60}^2\times2(60\times5)\times5^2 \\
\underline{60\ +5} \\
\overline{60}^3+3(\overline{60}^2\times5)+(60\times5^2) \\
\qquad \underline{(60^2\times5)+2(60\times5^2)+5^2} \\
\overline{60}^3+3(\overline{60}^2\times5)+3(60\times5^2)+5^3 \\
t^3+3(t^2\times u)+3(t\times u^2)+u^3
\end{array}$$

That is,

$$\begin{array}{lcr}
\overline{60}^3 & = & 216000 \\
3(\overline{60}^2\times5) & = & 54000 \\
3(60\times5^2) & = & 4500 \\
5^3 & = & 125 \\
\hline
\ldots & = & 274625
\end{array}$$

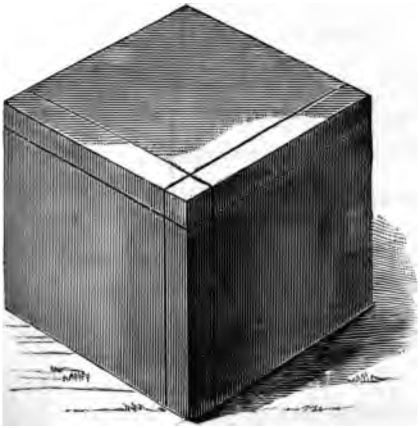
Every perfect cube consists of four parts, as shown above, viz:

1. *The cube of the tens.*
2. *Three times the product of the square of the tens by the units.*
3. *Three times the product of the tens by the square of the units.*
4. *The cube of the units.*

Or, more briefly, using t for tens and u for units,

$$(t+u)^3=t^3+3(t^2\times u)+3(t\times u^2)+u^3.$$

This is illustrated by the following figures, in which the entire cube (the cube of 65) is composed of



1. A cube whose edge is 60, $=\overline{60}^3 \quad . \quad . \quad =216000$
2. 3 rectangular blocks, each $60\times60\times5=3(\overline{60}^2\times5) \ = \ 54000$
3. 3 rectangular blocks, each $60\times5\times5=3(60\times5^2) \ = \ 4500$
4. A cube whose edge is 5 $=5^3 \quad . \quad . \quad . \quad = \ 125$

$$\overline{65}^3 \quad . \quad . \quad . \quad =274625$$

Fig. 2.

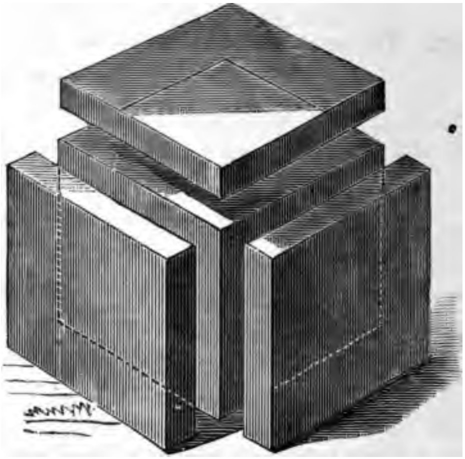
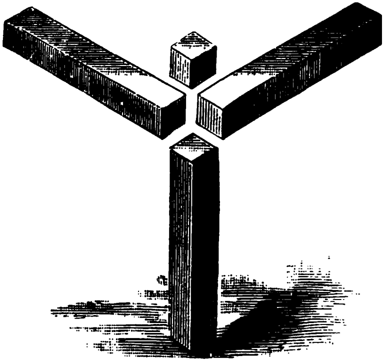


Fig. 3.



In finding the cube root of such a number it is desirable to retrace the steps here taken.

1. Find the cube root of 274625.

Process.

$$\begin{aligned}
t^3+3t^2u+3tu^2+u^3 &= 274{\cdot}625\ (65 \\
t^3 &= \underline{216} = 6^3 \\
3t^2u+3tu^2+u^3 &= 58625 = 58625 \div (3 \times 60^2)\text{ or, }10800 = 5 \\
3t^2u &= \underline{54000} = 3(60^2 \times 5) \\
3tu^2+u^3 &= 4625 \\
3tu^2 &= \underline{4500} = 3(60 \times 5^2) \\
u^3 &= 125 \\
u^3 &= \underline{125} = 5^3
\end{aligned}$$

Analysis. 1. Separate the number into periods of three figures each. Since there are two periods, the root will contain two figures. 2. The first period considered as integral thousands is 274, the greatest cube in which is 216 thousands, the cube root of which is 6 tens, or 60. 3. From the entire cube deduct the cube of the 6 tens = 216000, and there remains 58625, equal to $3t^2u + 3tu^2 + u^3$. 4. $3t^2u$ is hence contained in 58635, and is a product of two factors, one of which is $3t^2$; and 58625 divided by $3t^2$ or $3 \times 60^2 = 10800$ will give 5 for the probable next figure of the root. 5. $3t^2u$ or $10800 \times 5 = 54000$, which deducted from 58624 leaves 4625, equal to $3tu^2$. $3tu^2$ or $3(60 \times 3^2) = 4500$, deducted from 4625 leaves 125, the cube of the 5 units.

Note. To find the cube root of a number of more than two periods, find the root of the greatest cube in the first two periods as already analyzed, and proceed as before with the next period to find the next order of the root, and in like manner until no period remains.

2. What is the cube root of 34012224 ?

Process.

$$
\begin{array}{rl}
34\cdot012\cdot224 & (324 \\
\underline{27} & = 3^3 \\
7012 & \div (3\times30^2) \text{ or } 2700 = 2 \\
\underline{5400} & = 3\,(30^2\times2) \\
1612 & \\
\underline{360} & = 3\,(30\times2^2) \\
1252 & \\
\underline{8} & = 2^3 \\
1244224 & \div (3\times320^2) \text{ or } 307200 = 4 \\
\underline{1228800} & = 3\,(320^2\times4) \\
15424 & \\
\underline{15360} & = 3(320\times4^2) \\
64 & \\
\underline{64} & = 4^3
\end{array}
$$

Solution. The greatest cube in the left-hand period is 27, the cube root of which is 3. From this period deduct the cube of 3, and to the remainder, 7, annex the second period, 012, for a trial dividend = 7012, and employ 3 times the square of this first figure of the root considered as tens = 2700, for a trial divisor. The quotient, 2, will be the probable second figure of the root, considered as units. As in the first step we considered the first period without regard to the others, so now we shall consider the first two periods without regard to the third. From the trial dividend, or that which remains after the cube of the tens is deducted, deduct $3t^2u + 3tu^2 + u^3 = 5400 + 360 + 8$, and there remains of the first two periods 1244.

Now considering the three periods together: to the remainder 1244 annex the third period, 224, for another trial dividend = 1244224, and, as before, employ 3 times the square of the root as far as found, considered as tens = 307200, for another divisor. The quotient, 4, will be the probable third order of the root, considered as units. From the second trial dividend, or that which remains after the cube of the 32 tens is deducted, deduct $3t^2u + 3tu^2 + u^3 = 1228800 + 15360 + 64$, and there being no remainder we conclude that 324 is the cube root of 34012224. Hence, to extract the cube root of a number:

553. 1. *Separate the number into periods of three figures each, beginning with units.*

2. *Find the greatest cube in the first period, and write its cube root to the right, as the first figure of the root.*

3. *Subtract the cube of this root from the first period, and annex to the remainder the second period, for a trial dividend.*

4. *Use three times the square of the first figure of the root, considered as tens, for a trial divisor, and write the quotient, considered as units, as the second figure of the root.*

5. *Subtract from the trial dividend three times the square of the first figure of the root, considered as tens, multiplied by the second figure plus three times the first figure, as tens, multiplied by the square of the second figure, plus the cube of the second figure; and to the remainder annex the third period, if there be one, for another trial dividend.*

6. *Again use three times the square of the root as far as found, considered as tens, for another trial divisor, and write the quotient, considered as units, as the third figure of the root.*

7. *Deduct from the second trial dividend three times the*

square of the first two figures of the root, considered as tens, multiplied by the third figure, plus three times the first two figures, considered as tens, multiplied by the square of the third figure, plus the cube of the third figure, and to the remainder, if any, annex another period, if any, and proceed as before.

554. The roots of decimals are found in the same manner as roots of integers, taking care to separate the power, into periods of three figures each, beginning at units, and pointing as many decimal orders in the root as there are decimal periods in the power.

The roots of common fractions are found by reducing the fraction to its lowest terms, and extracting the root of the numerator for the numerator of the root, and the root of the denominator for the denominator of the root; or, by reducing the common fraction to a decimal, and extracting its root.

555. Find the cube root of each of the following numbers :

1. 3375.
2. 29791.
3. 54872.
4. 110592.
5. 300763.
6. 912673.
7. 753571.
8. 804357.
9. 941.192.
10. 2406.104.
11. 69.426531.
12. 8365.427.

Find the cube root of the following numbers to two decimal orders :

13. 3.
14. 7.
15. 50.
16. 87.

Find the value of each of the following expressions :

17. $\sqrt[3]{\frac{125}{512}}$.
18. $\sqrt[3]{\frac{216}{343}}$.
19. $\sqrt[3]{\frac{125}{729}}$.
20. $\sqrt[3]{15\frac{5}{8}}$.
21. $\sqrt[3]{5\frac{23}{64}}$.
22. $\sqrt[3]{5\frac{104}{125}}$.
23. $\sqrt[3]{.000512}$.
24. $\sqrt[3]{.000729}$.
25. $\sqrt[3]{.003375}$.
26. $\sqrt[3]{3.375}$.
27. $\sqrt[3]{.000001}$.
28. $\sqrt[3]{.016876}$.

PROBLEMS.

556. 1. Find the depth of a cubical excavation whose volume is 50653 cubic feet.

2. A cubical block of stone contains 29791 cubic inches. What is the area of one side?

3. A room in the form of a cube contains 3375 cubic ft. At $1.75 per square yard, how much will it cost to carpet the floor?

4. A cubical box contains 941192 cubic inches. At 9c. a square foot, how much will it cost to paint the outside surface?

5. The dimensions of a rectangular solid are length, 343, width, 216, height, 729. Find the edge of a cube containing an equal number of cubical units.

6. Find the inside dimensions of a cubical box that shall contain a bushel.

7. Find the value of x in the proportion $27 : 64 :: 216 : x^3$.

8. Find the value of x in the proportion
$\sqrt[2]{.0625} : \sqrt[3]{\frac{27}{64}} :: \sqrt[3]{729} : x^3$.

9. Show that $\sqrt[3]{\frac{216}{512}} = \sqrt[3]{.421875}$.

CHAPTER XV.

MENSURATION.

PLANE FIGURES.

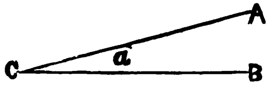
Article 557. A **Line** is that which has length without breadth or thickness.

This is a definition of a *geometrical* line. A line made by a crayon, pencil or pen, is a *physical* line, and has length, breadth, and thickness.

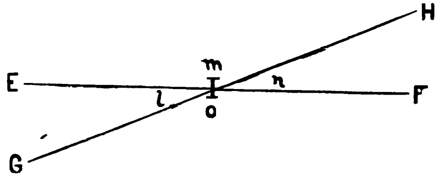
558. A **Straight Line** is a line which does not change its direction.

559. A **Curved Line, or Curve,** is a line which changes its direction at every point.

560. An **Angle** is the divergence of two lines which meet or intersect.

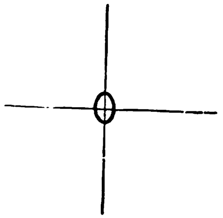


The lines A C and B C meet in the point C ; their divergence is the angle A C B, or C, or *a*. The point C is the vertex of the angle.



The lines E F and G H intersect at the point I, making the angles E I H or *m*, H I F or *n* ; F I G or *o* ; E I G or *l*. The four angles have a *common vertex*, I.

561. A **Right** angle is one of the equal angles formed by the meeting of two lines; or, one of four equal angles formed by the intersection of two lines.



The four angles at the point O are right angles.

562. **Oblique** angles are those angles which are not *right*

angles. Oblique angles are of two kinds, **obtuse and acute.**

An **obtuse** angle is greater than a right angle.

An **acute** angle is less than a right angle.

The angles *m* and *o*, in Art. 560 are obtuse; the angles *l* and *n* are acute.

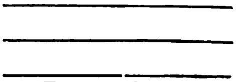
563. A **Surface** is that which has length and breadth, without thickness.

A **Plane** surface, or a **Plane,** is a surface which does not change its direction.

564. A **Curved** surface changes one or both of its directions continuously.

565. A **Quadrilateral** is a plane figure bounded by four straight lines.

566. **Parallel Lines** are lines which lie in the same plane, and are everywhere equally distant.



Parallel Lines.

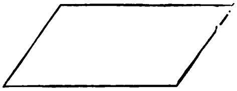
567. A **Parallelogram** is a quadrilateral whose opposite sides are parallel.

The opposite sides of a parallelogram are equal to each other.
The opposite angles of a parallelogram are equal to each other.

568. There are four varieties of the parallelogram, viz.: the *Rhomboid*, the *Rhombus*, the *Rectangle*, and the *Square.*

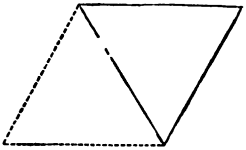
It will be seen that the *Rhomboid* includes the *Rhombus*, and that the *Rectangle* includes the *Square.*

569. A **Rhomboid** is an oblique angled parallelogram.



Rhomboid.

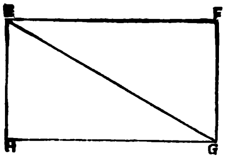
570. A **Rhombus,** or **Rhomb,** is an equilateral, oblique-angled parallelogram.



Rhombus.

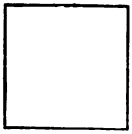
571. A **Rectangle** is a right-angled parallelogram.

A rectangle is sometimes called an *oblong*.



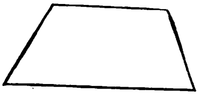
Rectangle.

572. A **Square** is an equilateral rectangle.



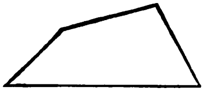
Square.

573. A **Trapezoid** is a quadrilateral which has only two sides parallel.



Trapezoid.

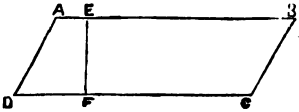
574. A **Trapezium** is a quadrilateral which has no parallel sides.



Trapezium.

Notes. 1. The term quadrilateral includes all four sided plane figures.
2. Quadrilaterals are classified according to the parallelism of their sides.
3. Parallelograms are classified according to the equality of their sides, and according to their angles, whether right or oblique.

575. The **Altitude** of a parallelogram is the perpendicular distance between the opposite sides.



Parallelogram.

E F is the altitude of the parallelogram A B C D.

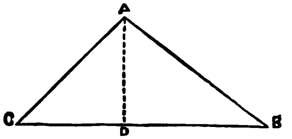
The altitude of a trapezoid is the perpendicular distance between its parallel sides.

It is proved in Geometry that the area of any parallelogram is equal to the area of a rectangle having the same base and the same altitude as the base and altitude of the parallelogram. Therefore,

576. *The area of a parallelogram is the product of its base and altitude.*

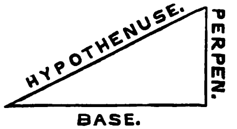
1. Find the area of a parallelogram whose base is 30 ft. and whose altitude is 10 ft.
2. A side of a rhombus is 42 inches; its altitude is 34 inches. Find its area.
3. How many square feet in a square 85 ft. on a side?
4. The area of a rectangle is 285516 sq. yds.; its base is 924 yds. Find its altitude.
5. How many acres in a square field 320 rods on a side?

577. A **Triangle** is a plane figure bounded by three straight lines.



Any side of a triangle may be called its base; the angle opposite its base is called its *vertex*. The altitude of a triangle is the perpendicular distance from its vertex to its base, or to its base produced. A D is the altitude of the triangle A B C.

578. A **right-angled triangle** is a triangle which has a right angle.



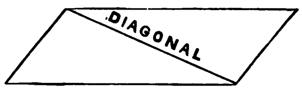
The two sides which include the right angle are called the *Base* and *Perpendicular*. The side opposite the right angle is the *Hypothenuse*.

When standing on its base, the altitude of a right-angled triangle is its perpendicular.

An **obtuse-angled** triangle has one obtuse angle.

An **acute-angled** triangle has all its angles acute.

579. It is proved that every parallelogram may be divided by its diagonal into two equal triangles, each of which has the same base and same altitude as the parallelogram; therefore the area of every parallelogram is *twice* the area of a triangle having the same base and altitude. Hence,



580. *The area of a triangle is one half the product of its base and altitude.*

A parallelogram whose base is 8 feet and altitude 5 feet, has an area of $8\times5=40$ sq. ft.

A triangle whose base is 8 feet and altitude 5 feet, has an area of $\frac{8\times5}{2}=$ 20 sq. ft.

1. What is the area of a parallelogram 16 ft. by 3 ft.?
2. What is the area of a triangle whose base is 16 ft. and altitude 3 ft.?

Find the areas of the following triangles:

3. Base 15, altitude 4.
4. Base 120, altitude 10.
5. Base 48, altitude 20.
6. Base $12\frac{1}{2}$, altitude $6\frac{1}{4}$.

Multiply *half* the base by the altitude.

7. Base 25, altitude 50.

Multiply the base by half the altitude.

8. How many right-angled triangles, each having a base of 4 inches and altitude of 6 inches, can be cut from a rectangular piece of paper 16 inches by 12 inches?

581. A trapezoid may be divided into two triangles, each having the altitude of the trapezoid, their bases being the parallel sides of the trapezoid. Hence,

582. *The area of a trapezoid equals the product of half the sum of its parallel bases, and its altitude.*

If the parallel bases of a trapezoid are respectively 8 and 12 feet, and its altitude 7 feet, the area of a trapezoid is $7\left(\frac{8+12}{2}\right)=70$ sq. ft.

1. What is the area of a trapezoid whose bases are 15 feet and 7 feet, and altitude 9 feet?
2. Bases 10 feet and 20 feet, altitude 3 feet. Area?
3. Bases 13 feet and $6\frac{1}{2}$ feet, altitude 4 feet. Area?
4. The parallel sides of a field are respectively 131 and

243 yds.; the distance between them is 440 yds. How many acres in the field?

5. A plank 12 ft. long is 13 in. wide at one end, and 11 in. wide at the other end. How many sq. ft. does it measure?

583. A **Polygon** is a plane figure bounded by straight lines.

Triangles, squares, etc., are polygons.

584. The **Perimeter** of a polygon is the sum of the lines which bound it.

The perimeter of a triangle is composed of its three sides.

585. The **Diagonal** of a polygon is a straight line connecting two angles which are not adjacent. (The diagonal must lie wholly within the polygon.)

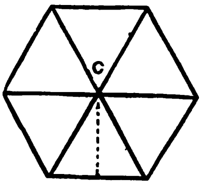
A polygon may be divided into triangles; and its area is the sum of the triangles into which it is divided.

586. A **Regular Polygon** is a polygon which has equal sides and equal angles.

Polygons are named from the number of their sides; as triangles, quadrilaterals, etc. Regular polygons are named as follows

3 sides,	equilateral triangle.	7 sides,	regular heptagon.
4 "	square.	8 "	regular octagon.
5 "	regular pentagon.	9 "	regular nonagon.
6 "	regular hexagon.	10 "	regular decagon, etc.

587. By drawing lines from the center of a polygon to the vertex of each angle, a polygon may be divided into as many triangles as the figure has sides. In a regular polygon, the altitude of each of the triangles is the perpendicular drawn from the center of the polygon to any side. This line is called the **apothem** of the polygon. It is evident that the area of a polygon is equal to the sum of the areas of the triangles into which it is divided. Hence,



588. *The area of a regular polygon is equal to one half the product of the perimeter of its base and its altitude.*

Find the area of the following regular polygons:

1. A pentagon whose side is 40 ft., apothem 28 ft.
2. An octagon whose side is 32 ft., apothem 38 ft.
3. A decagon whose side is 16 ft., apothem 25 ft.
4. A hexagon whose side is 11 ft., apothem 9.5 ft.

589. The six triangles into which a regular hexagon may be divided are equilateral. Hence, if the side is given, the apothem may be found by Art. 550, since the apothem is the perpendicular of a right-angled triangle, of which the other two sides are known.

The side of a regular hexagon is the radius (Art. 593) of the circumscribed circle.

1. Find the apothem of a hexagon whose side is 6.

SOLUTION. The hypothenuse is 6, the base is 3, the perpendicular (that is, the apothem of the polygon) $= \sqrt{36-9} = \sqrt{27} = 5.19+$.

2. Find apothem of a regular hexagon, side 8 ft.; 10 ft.; 12 ft.; 24 ft.
3. Find area of a regular hexagon, side 36 ft.; 40 rods; 16 miles; 60 miles.

590. A **Circle** is a plane figure bounded by a curved line, every point of which is equidistant from a point within, called the **center.**

591. The **Circumference** of a circle is the line that bounds it.

592. The **Diameter** of a circle is a straight line which, passing through the center, connects opposite points in the circumference.

593. The **Radius** of a circle is a straight line which connects the center with any point in the circumference.

A radius of a circle is one half of its diameter.

Diameters of the same circle are equal; radii of the same circle are equal.

594. It is found that if the diameter of a circle is 10000, its circumference is 31416 *nearly;* that is, the *ratio* of the circumference to the diameter is 3.1416—. Hence,

595. *The circumference of a circle equals its diameter multiplied by 3.1416—.*

This ratio, 3.1416, is only *approximate,* but is sufficiently accurate for ordinary calculations. It is represented by the Greek letter π (pronounced *pi.*) Diameter and radius are represented by their initial letters, D and R. Circumference is abbreviated into Cirf.

The circumference $= \pi D$, or, $2 \pi R$.

Diameter $=$ cirf. $\div \pi$.

1. What is the circumference of a circle whose diameter is 8 ?

SOLUTION. $8 \times 3.1416 = 25.1328$.

2. What is the cirf. of a circle whose D is 12 ft.? 18 rods?
3. What is the cirf. of a circle whose R is 5 in.? 20 yds.?
4. What is the diameter of a circle whose cirf. is 8 ft.?

SOLUTION. $8 \div 3.1416 = 2.54+$

5. What is the D of a circle whose cirf. is 15 ft.? 20 ft.?
6. What is the R of a circle whose cirf. is 12 in.? 60 ft.?

596. Area of Circle. A circle may be regarded as a regular polygon of an infinite number of sides. Hence, (Art. 580.)

Area of circle $= \frac{1}{2}$ (circumference $\times$ R) or, substituting for cirf. its value, cirf. $= 2 \pi R$. (Art. 595.)

Area of circle $= \frac{1}{2} \times 2 \pi R \times R = \pi R^2$. Hence, to find area of circle,

597. *Multiply the square of the radius by 3.1416.*

1. Find area of circle whose R is 3.

SOLUTION. $R^2 = 3^2 = 9$; $9 \times 3.1416 = 28.2744$.

2. Find area of circle whose R is 5 ; 8 ; 10 ; 15.
3. Find area of circle whose R is 6 in. ; 7 ft. ; 4 miles.

4. Find R of circle whose area is 600 sq. ft.

SOLUTION. Since Area $= \pi R^2$, $R^2 =$ Area $\div \pi$; and R $= \sqrt{\text{area} \div \pi}$. $\sqrt{\text{Area} \div \pi} = \sqrt{600 \text{ sq. ft.} \div 3.1416} = \sqrt{190.9 \text{ sq. ft.}} = 13\ 7$ ft.

5. Find R of circle whose area is 90 sq. ft.; 150 sq. ft.; 400 sq. ft.

6. Find D of circle whose area is 80 sq. mi.; 750 sq. mi.

7. How many acres in a circular field 100 rods in diameter?

8. How many rods of fence necessary to enclose a circular field containing 1 acre? 2 acres?

9. In the center of a circular field 40 rods in diameter is a circular pond 10 rods in diameter. How many square rods of the field are not covered by water?

Since area of circle $= \pi R^2$ or $3.1416 \times R^2$; and since $R^2 = D^2 \div 4$, the area of circle $= 3.1416 \times (D^2 \div 4) = (3.1416 \div 4) \times D^2 = .7854 \times D^2$. Hence,

To find area of circle,

598. *Multiply the square of the diameter by .7854.*

For rough calculations the circle may be regarded as *three-fourths* of the square of the diameter.

1. Find the area of a circle whose diameter is 10; 12; 14; 16.

2. What is the area of a circle which may be cut from a board 1 foot square? 3 feet square? 15 in. sq.?

3. How many acres in a circular field the diameter of which is 400 rods?

SIMILAR SURFACES.

599. **Homologous lines** are lines having the same relative position.

Thus, the hypothenuses of right-angled triangles are homologous; the diameters of circles are homologous.

600. **Similar figures** are figures whose homologous lines are proportional.

Thus, two right-angled triangles are similar when
The base of 1st : base of 2nd :: hypoth. of 1st : hypoth. of 2nd.

601. The areas of similar figures have the same ratio to each other as the squares of their homologous lines.

Thus, if the bases of two similar triangles are respectively 3 and 5, the areas of the triangles have the ratio of 3^2 to 5^2, or 9 to 25.

602. The homologous lines of similar figures have the same ratio to each other as the square roots of the areas of the similar figures.

Thus, if the areas of two similar rectangles have the ratio of 81 to 49, their bases have the ratio of $\sqrt{81}$ to $\sqrt{49}$, or 9 to 7.

1. A triangle having an area of 32 ft. has a base of 8 ft. What is the area of a similar triangle whose base is 10 ft.?

SOLUTION. Represent the required area by x; then $8^2 : 10^2 :: 32 : x$, that is, $64 : 100 :: 32 : x$. Therefore,

$$x = \frac{100 \times 32}{64} = 50.$$

2. The areas of two similar rectangular fields are respectively 36 acres and 25 acres. The first field is 60 rods long; what is the length of the second?

603. Circles are similar figures; therefore the areas of circles are to each other as the squares of their respective radii, diameters and circumferences.

Thus, if the diameters (radii or circumferences) of two circles are 2 and 3 respectively:
Area of first circle : area of second circle :: $2^2 : 3^2$, or as $4 : 9$.

1. How many circles 4 inches in diameter contain the same area as a circle 8 inches in diameter?

SOLUTION. The circles are to each other as 4^2 to 8^2, or as 16 to 64. $64 \div 16 = 4$, Ans. A briefer method is to find the square of the quotient of one diameter divided by the other, thus: $\left(\frac{8}{4}\right)^2 = 2^2 = 4.$

2. What part of a circle of 20 ft. radius is a circle of 4 ft. radius?

SOLUTION. $\frac{4^2}{20^2} = \frac{16}{400} = \frac{1}{25}$. Ans. Or, $\left(\frac{4}{20}\right)^2 = \left(\frac{1}{5}\right)^2 = \frac{1}{25}$.

3. How many circles 2 inches in diameter are equal in area to a circle 10 inches in diameter? To a circle 10 feet in diameter?
4. What is the ratio of the quantities of water conveyed by a tunnel 7 feet in diameter and a pipe 2 feet in diameter?
5. How many water pipes $\frac{3}{4}$ inch in diameter may be supplied by a tunnel 6 feet in diameter, no allowance being made for difference in velocity of the water?
6. A 20 inch water pipe branches into 3 equal branches. What should be the diameter of each, that the same volume of water may be conveyed?
7. A reservoir is emptied by a $2\frac{1}{2}$ inch orifice in 2 hours. What must be the diameter of the orifice that will empty it in half an hour?
8. How many circles $\frac{1}{2}$ inch in diameter are equal in area to a circle 4.5 inches in diameter?
9. One circle contains 100 times the area of another. What are their relative diameters?

SOLIDS.

603. A **Solid** is a volume bounded by polygons.

The polygons are the *faces* of the solid.

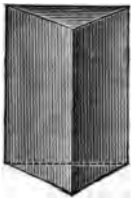
The lines in which the polygons meet are the *edges* of the solid.

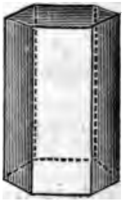
The points in which the edges meet are the *vertices* of the solid

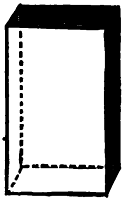
604. A **Prism** is a solid, two of whose faces, called *ends* or *bases*, are equal and parallel polygons, and whose other faces, called *sides*, are parallelograms.

605. The **altitude** of a prism is the perpendicular distance between its bases.

A *right* prism is a prism the edges of whose sides are perpendicular to its bases.


Triangular Prism.


Hexagonal Prism.


Parallel opiped.

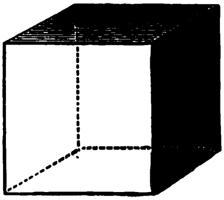
Prisms are named from the polygons which form their bases. A *triangular* prism is a prism whose bases are triangles; a *hexagonal* prism is a prism whose bases are hexagons, etc.

606. A **Parallelopiped** is a prism whose bases are parallelograms.

A parallelopided has six faces, all of which are parallelograms.

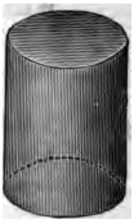
A *rectangular* parallelopiped is a right parallelopiped, all of whose faces are rectangles. A brick is an example.

607. A **Cube** is a rectangular parallelopiped, all of whose faces are squares.


Cube.

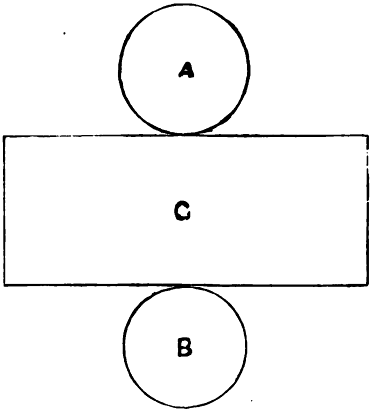
608. A **Cylinder** is a right prism whose bases are circles.

The parallelograms which form the sides of the cylinder have *length* but not width.


Cylinder.

609. The **convex surface** of a prism is the entire **external** surface of the prism, *excluding the bases.*

The convex surface of a right prism is composed of as many rectangles as its base has sides. The altitude of these rectangles is the altitude of the prism; the sum of their bases is the perimeter of either base of the prism. *The convex surface of a cylinder equals a rectangle whose dimensions are the altitude and circumferences of its cylinder.*



The rectangle C represents the convex surface of a cylinder. The circles A and B represent its bases.

610. *The* **convex surface** *of a right prism equals the product of its perimeter and altitude.*

611. *The* **volume** *of any prism equals the product of its base and altitude.*

Since the base of a cylinder is a circle, the convex surface of a cylinder $= 2 \pi R A$ (Art. 595); and the volume of a cylinder $= \pi R^2 A$ (Art. 597.)

1. Find entire surface of a brick 8 inches by 4 inches by 2 inches.
2. Find convex surface of a right prism the perimeter of whose base is 20 inches, altitude 40 inches.
3. Find convex surface of a cylinder the circumference of whose base is 15 inches, altitude 30 inches.
4. Find convex surface of a right octagonal prism whose height is 6 feet and one of the sides of base 6 inches.
5. Find entire surface of a cube whose edge is 4 inches. Find number of cubic inches in this cube.
6. How many sq. yds. in floor, ceiling and walls of a room 12 ft. long, 10 ft. wide, 9 ft. high?
7. How many cu. yds. in the room described in Ex. 6?

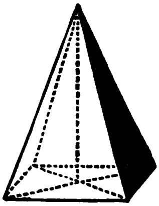
8. Find number of cu. ft. in triangular prism whose base is 64 sq. ft., altitude 16 ft.
9. What is the entire surface of a block of stone in the form of a rectangular parallelopiped 9 ft. by 4 ft. by 3 ft. How many cu. ft. in the stone?
10. How high must a circular vat be to contain 960 gals., if its base has an area of 14 sq. ft.
11. What is the convex surface of a steam boiler 12 ft. long, 3 ft. diameter? Entire surface? Volume in cubic feet?
12. How many tons of iron in a mile of water pipe 22 in. bore, the iron averaging 2½ inches thick and weighing 450 lbs. to the cu. ft.?

612. A **Pyramid** is a solid, whose base is a polygon, and whose sides are triangles meeting in a point called the **vertex.**

The sum of the sides is the *convex surface* of the pyramid.

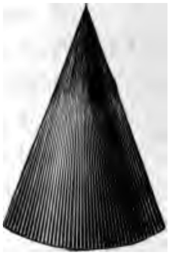
613. A **right** pyramid is a pyramid whose base is a regular polygon, whose sides are equal triangles, and whose vertex is in a line drawn perpendicular to the center of its base.

Pyramids, like prisms, are named from the polygons which form their bases, as triangular pyramids, etc.



614. A **Cone** is a right pyramid whose base is a circle.

As the circumference of a circle may be regarded as composed of an infinite number of straight lines so the convex surface of a cone may be supposed to consist of an infinite number of triangles, whose bases are in the circumference of the base of the cone.



615. The **altitude** of a pyramid or cone is the perpendicular distance from its vertex to its base.

616. The **slant height** of a right pyramid or cone is a line drawn from its vertex perpendicular to the perimeter of its base.

In a right pyramid, the slant height is the altitude of each of the equal triangles which compose the convex surface.

Since the convex surface of a right pyramid or cone is composed of equal triangles, the altitude of each of which is the slant height of the pyramid or cone

617. *The convex surface of a right pyramid or cone is the product of the perimeter of its base and half its slant height.*

It is proved in Geometry that a pyramid is one third of a prism having the same base and altitude; and that a cone is one third of a cylinder having the same base and altitude; therefore,

618. *The volume of a pyramid or cone is one third the product of its base and altitude.*

Convex surface of cone $= \pi \text{D} \times \frac{1}{2} \text{S H}$

Volume of cone $= \frac{1}{3} \pi \text{R}^2 \times \text{A}$.

1. Find the convex surface of a pyramid the perimeter of whose base is 40 inches and whose slant height is 3 feet.
2. Find surface of a square pyramid whose slant height is 60 inches, one side of whose base is 24 inches.
3. Find convex surface of cone whose base is 60 feet in circumference and whose slant height is 40 feet.
4. Find convex surface of a cone whose altitude is 8 feet and diameter of base 12 feet.
5. Find entire surface of a cone the radius of whose base is 10 inches and whose altitude is 50 inches.
6. Find volume of pyramid the area of whose base is 25 sq. ft. and whose altitude is 16 feet.

7. Find volume of square pyramid whose altitude is 8 feet and one side of whose base is 3 feet.
8. Find volume of cone whose altitude is 12 feet and diameter of base 4 ft.
9. The great pyramid of Ghizeh was originally 480 feet high, with a square base 764 feet on each side. How many cu. ft. of masonry in it ?
10. The Pyramid of Rome is a cone 120 feet high, on a base of 95 feet diameter. Find its convex surface and volume.

619. A **Sphere** is a solid, every point of whose surface is equidistant from a point called the **center.**

620. A **diameter** of a sphere is a straight line through the center and terminating in opposite points of its surface.

If a sphere be divided into two equal parts by a plane passing through its center, the section thus made is a **great circle** of the sphere.

621. *The* **surface** *of a sphere equals four of its great circles;* that is,

Surface of sphere $= 4\pi R^2$; or, (since $4 R^2 = D^2$) $= \pi D^2$.

Since the sphere may be considered as composed of an indefinite number of cones having their bases in the surface of the sphere, and their altitude the radius of the sphere :

622. *The volume of a sphere is the product of its surface and one third of its radius ;* or,

Volume of sphere $= \frac{4}{3}\pi R^3$ (that is, $\frac{1}{3} R \times 4\pi R^2$).

The volume of a sphere also equals two thirds the volume of a cylinder whose diameter and altitude are the diameter of the sphere, which gives the same formula: $(\pi R^2 \times 2R \times \frac{2}{3} =) \frac{4}{3}\pi R^3$.

1. Find surface of sphere whose diameter is 4; is 8; is 16.
2. Find volume of sphere whose diameter is 4; is 8; is 16.
3. Find surface and volume of sphere whose circumference is 20.
4. How many square miles on the surface of the earth, assuming the diameter to be 8000 miles.

Similar solids are to each other as the cubes of their like dimensions. Thus, spheres are to each other as the cubes of their diameters or radii

5. How many balls 1 inch in diameter may be made from a ball 6 inches in diameter?

SOLUTION. The balls are to each other as the cubes of their diameters; that is, as $1^3 : 6^3$, or as $1 : 216$. Therefore, 216 balls 1 inch in diameter may be made from one ball 6 inches in diameter.

6. How many bullets $\frac{1}{2}$ inch in diameter may be made from a lead ball 5 inches in diameter.
7. If the moon's diameter is 2000 miles, and the earth's 8000 miles, how many moons equal the earth in volume?
8. If the sun's diameter is 880000 miles, how many planets having the diameter of the earth equal his volume?

MISCELLANEOUS PROBLEMS IN MENSURATION.

1. The sides of a field in the shape of a trapezoid are 72 and 80 rods respectively; the perpendicular distance between them is 40 rods. What is the side of a square field containing the same area?
2. What is the radius of a circle containing the same area as a triangle whose base is 24 feet and altitude 36 feet?

3. How many sq. ft. of lumber necessary to board the gable ends of a barn, each end being 24 ft. wide, 20 ft. high to the eaves, 28 ft. high to the ridge pole?
4. What is the area of a quadrant whose radius is 4 ft.?
5. The slant height of a cone is 24 inches, the diameter of its base 12 in. What is the diameter of a sphere having an equivalent volume?
6. How many square miles on the surface of an asteroid 300 miles in diameter?
7. If an aperture an inch square be plugged with a round plug an inch in diameter, what area will remain open?
8. How many sq. ft. in the exterior surface of a spire in the form of a hexagonal pyramid 100 ft. high, each side of base 4 ft.?
9. A square pyramid 270 ft. high contains 14520 cu. yds. What is the length of one side of base?
10. What are the relative diameters of a 27 lb. cannon ball and a 64 lb. cannon ball?
11. Two square fields have the ratio 3 : 5. The smaller contains $3\frac{3}{4}$ acres. Find length of side of larger field.
12. Find the area of one side of a cubical mound whose volume is equal to the volume of another mound 288 ft. long, 216 ft. broad, 48 ft. high.
13. How many globes of gold, each 5 inches in diameter, weigh as much as one of the same material 15 inches in diameter?
14. A cone 3 feet high contains 1539 cu. ft. What is the volume of a similar cone 5 ft. high?
15. Find the ratio of the surfaces of two spheres whose diameters have the ratio 6 to 7.

CHAPTER XVI.

MISCELLANEOUS EXAMPLES.

1. The sum of two numbers is 2836 ; their difference is 987. What are the numbers?
2. The sum of four numbers is 4987; three of the numbers are 1820, 529, and 2417. What is the fourth number?
3. The product of three numbers is 4375 ; two of the numbers are 7 and 25. What is the third number?
4. At an election the total number of votes cast for two candidates was 8136. The successful candidate was elected by a majority of 590 votes. How many votes did each receive?
5. Find the product of the prime factors of 21, 52 and 95.
6. Find the product of the prime factors common to 84, 126 and 252.
7. At what price per barrel can flour be purchased by different persons using respectively $54, $228, and $138, so that each may purchase an exact number of barrels?
8. What is the least number of years that may be divided into either 42 years, 96 years, or 180 years?
9. What is the smallest number of marbles that can be distributed equally among 9, 21, or 25 boys?
10. A farmer sold 10 loads of wheat, each load containing 16 bags of 3 bushels each, valued at 90c. per bushel. He received in payment woolen cloth, worth $1.50 per yard. How many yards did he receive?

11. How many acres in a farm which consists of 5 fields containing, the first $124\frac{3}{8}$ acres, the second $49\frac{5}{6}$ acres, the third $96\frac{1}{2}$ acres, the fourth $108\frac{1}{3}$ acres, the fifth $82\frac{5}{12}$ acres?
12. If the farm described in Ex. 11 were divided into 3 equal fields, how many acres would each contain?
13. $21\frac{1}{5}$ is one half of what number? Is one third of what number?
14. The sum of two numbers is $126\frac{3}{7}$; the smaller number is one half of the greater. What is the greater number?
15. Find cost of $327\frac{3}{8}$ cords of wood at \$$5\frac{3}{4}$ per cord.
16. Find cost of 1 ton of hay if $28\frac{2}{3}$ tons cost \$$243\frac{2}{3}$.
17. Paid \$$597\frac{3}{4}$ for a lot, and sold it so as to lose $\frac{1}{8}$ of what was paid. For what was the lot sold ?
18. How many yards of muslin worth $8\frac{1}{3}$c. per yard, can be bought for \$7.25 ?
19. How many miles will a vessel sail in 36 hours, averaging $7\frac{5}{9}$ miles per hour?
20. Having a farm worth \$3250, I sold part of it for \$812.50. What was the value of the part sold compared with the value of the farm? Compared with the value of the part that remained ?
21. A man expended \$$59\frac{1}{2}$, which was $\frac{1}{9}$ of his money. How much money had he remaining ?
22. A man expended \$$59\frac{1}{2}$, which was $\frac{2}{9}$ of his money. How much money had he remaining ?.
23. A man owning $\frac{3}{7}$ of a vessel, sold $\frac{1}{2}$ of his share for \$$2910\frac{3}{11}$. What was the value of the vessel, at the same rate of valuation?
24. How many thirds in $\frac{3}{2}$?

25. If I spend $\frac{2}{9}$ of my money, and lose $\frac{1}{5}$ of it, what part have I left ?

26. If I spend $\frac{2}{9}$ of my money, and lose $\frac{1}{5}$ of the remainder, what part have I left ?

27. I spent $\frac{1}{4}$ and $\frac{1}{5}$ of my money, and then had $22. How much had I at first ?

28. If 6 men can do a piece of work in $4\frac{1}{2}$ days, in how many days can 4 men do it ?

29. The difference between $\frac{3}{7}$ and $\frac{3}{8}$ of A's money is $\$9\frac{3}{5}$. How much has he?

30. $\frac{1}{8}$ of a dollar is what part of $37\frac{1}{2}$ cents ?

31. Add 175 units, 175 tenths, 175 hundredths, 175 thousandths.

32. From one hundred subtract $135\frac{1}{8}$ hundredths.

33. Divide the cube of 12.5 by the square of 6.35.

34. What decimal equals $\frac{1}{2}+\frac{1}{5}+\frac{1}{10}$?

35. What common fraction equals $.1+.125+.3\frac{1}{3}$?

36. The product of two factors is .0625. One of the factors is 1.25; what is the other?

37. The divisor 5.125 is 5 times the quotient. What is the dividend ?

38. What is $\frac{1}{8}$ of $\frac{3}{5}$ expressed decimally?

39. Find sum of .3, .36 and .81 expressed as a common fraction.

40. What is .25 of .5 of $32040.96 ?

41. Find cost of laying $\frac{1}{2}$ mile of sidewalk, at $\$.62\frac{1}{2}$ per linear foot.

42. Find cost of laying a sidewalk $\frac{3}{4}$ of a mile long, 6 feet wide at $12\frac{1}{2}$ cents per square foot.

43. Find cost of building a wall 120 feet long, $1\frac{3}{4}$ feet thick, 8 feet high, at 15 cents per cubic foot.

44. How many square feet on the surfaces of a block of stone, 8 feet long, 4 feet wide, 2 feet thick?
45. From a pile of stone measuring 40 feet long, 12 feet high, 20 feet wide, 15 loads are taken, averaging $1\frac{1}{3}$ cubic yards each load. How many cubic feet of stone remain in the pile?
46. What is the difference in square feet between 25 feet square and 25 square feet?
47. How many bushels will a bin contain which is 15 feet long, 12 feet high, 9 feet wide?
48. How many gallons will a vat contain which is 11 feet high, 7 feet long, 6 feet wide?
49. How many quarts in .75 of a bushel?
50. What decimal of a cubic foot are 108 cubic inches?
51. Find cost of 12300 bricks, at $7.50 per M.
52. How many yards of carpeting, $\frac{3}{4}$ yard wide, are required to cover a floor 12 feet by 18 feet?
53. Mr. J. buys the N. $\frac{1}{2}$ of S. E. $\frac{1}{4}$ of Sec. 30, T. 7 N. R. 5 E., etc., at $5.25 per acre. How much does he pay?
54. A ship's chronometer shows a gain of 1 h. 20 m. How many degrees has the vessel sailed? E. or W.
55. What part of a circumference is an arc of 11° 15′?
56. How many perches of stone in a quarry 4950 feet long, 200 feet wide, if the stone is 24 feet deep?
57. Find value, at $3.25 per cord, of a pile of wood 320 feet long, 4 feet wide, 8 feet high.
58. What is the value of 8 lb. 3 oz. 16 pwt. $1\frac{1}{2}$ gr. of gold, at $15.85 an ounce?
59. From .9 of a day subtract $\frac{1}{8}$ of a week.
60. How many barrels of water, 30 gallons each, have fallen on an acre of land, during a rain fall of $1\frac{1}{4}$ inches?

61. The consequent of a couplet is $12\frac{1}{2}$; the ratio is 4: what is the antecedent ?

62. The antecedent is $37\frac{1}{2}$; the consequent is 150: what is the ratio?

63. If 192 tons of coal cost $1536, what will 51 tons cost?

64. If $\frac{5}{8}$ of an acre of land is worth $25, what are $2\frac{1}{5}$ acres worth?

65. If a vessel sails 325.2 miles in 46.5 hours, how many miles will she sail in $39\frac{4}{7}$ hours?

66. If 7 men can mow 13 acres of grass in 2 days, how many acres can 16 men mow in $3\frac{1}{2}$ days?

67. What will be the cost of 12 barrels of sugar, averaging 225 lbs. each, if $23\frac{1}{2}$ lbs. cost $2.585 ?

68. In how many days can 90 men dig a ditch that 45 men can dig in 17 days ?

69. Find the cost of $28\frac{3}{4}$ yards of cloth $\frac{3}{4}$ yards wide, if $39\frac{1}{2}$ yards of the same material $\frac{3}{8}$ wide cost $118.50.

70. If 18 men can cut 90 cords of wood in 5 days, working 8 hours per day, how many cords can 24 men cut in 7 days, working 9 hours per day?

71. Express in fraction in lowest terms the quotient of the G. C. F. of 300, 525, 225 and 375 divided by the L. C. M. of 10, 16, 24 and 48.

72. Boston is 71° 4′ 9″ W. When it is 10 o'clock A. M. in Boston, it is 53 m. $57\frac{3}{5}$ sec. past 8 o'clock A. M. in Chicago. Find longitude of Chicago.

73. By what part of itself must $\frac{23}{25}$ be diminished that the remainder may equal $\frac{4}{15}$.

74. A farm was sold for $8000, which was $\frac{4}{5}$ of its cost. What part of the price was the loss?

75. How many yards of carpet $2\frac{2}{3}$ feet wide are necessary to cover a floor $14\frac{1}{2}$ feet long, $10\frac{2}{3}$ feet wide?

76. A and B can build a house in 18 days; A alone can build it in 24 days: in how many days can B build it?

77. When will the hour and minute hands of a clock be together, between 3 and 4 o'clock?

78. A man bought 3 calves and 7 sheep for $29, and 8 calves and 4 sheep for $48. What was the average cost of each calf? Of each sheep?

79. The difference in the lengths of two logs is 9 feet; the shorter log is $\frac{11}{14}$ the length of the longer. What is the length of each?

80. The time past noon is $\frac{3}{7}$ of the time to midnight. What is the hour?

81. The base of a triangle is 120 feet; its altitude is 60 feet. What is its area?

82. A triangle has the same altitude as a rectangle, but its base is 4 times as long as the base of the rectangle. What is the ratio of their areas?

83. What is the area of a circle whose radius is 16 feet?

84. What is the circumference of a circle whose diameter is 248 feet?

85. I loaned Mr. Wells $280 for 2 years 6 months, at 8% interest. What will he owe me at the expiration of that time?

86. What is the interest of $360 for 6 months 18 days, at 7%?

87. The gain on the sale of a farm was $300, which was $8\frac{1}{3}$% of its cost. What was its cost?

88. I insured my house, valued at $4000, for $\frac{3}{4}$ of its value, $1\frac{1}{4}$%. What was the premium?

89. A collector receives 6% on all collections. He collects $3800; what does he pay his employer?

90. By selling a house for $2200, I lost 8⅓%. What had I paid for the house?

91. Find cost of 3½ acres of land at $1.33⅓ per square rod.

92. If 2.25 tons of hay cost $15.75, what will 5.75 tons cost ?

93. A merchant received $4160 with which to purchase mdse., after deducting his commission of 4%. What was the amount expended ?

94. If the rate of commission is 2% and the commission $112, what is the sale?

95. What is the value of 5 shares of stock of a company at 16⅔% below par, the original value being $25 per share?

96. Find brokerage at ¼% on sale of 200 shares of stock, each share $50, at 80%.

97. When stock originally worth $3000 sells for $3250, what is the per cent premium?

98. Insurance was effected on a store to the amount of $3760, at the rate of $10 on the $1000. Cost of insurance, including $1 for policy?

99. Insured 4000 bbls. of pork, worth $23 a bbl., for ⅘ its value, at ¾%. Find premium.

100. Find duty at 5c. per lb. on 450 boxes raisins weighing 25 lbs. each.

101. Paid $35 city tax, including $7 personal tax. What was the value of my real estate, the tax being 18 mills on the dollar ?

102. If $200 is loaned Oct. 1, 1869, what will pay the loan on the 28th of March following, interest 9%?

103. Invested \$3600 at 5% and received \$4600 principal and interest. For what time was the money invested?

104. At what rate will \$9000 amount to \$9519 in 5 mo. 23 da.?

105. What principal on interest at 8% from Nov. 9, 1890, to March 9, 1894, will amount to \$2201.60?

106. A speculator bought in Chicago 5000 bush. corn at 58c. a bush.; he paid 7c. a bush. freight, and $\frac{1}{2}$c. a bush. insurance to Buffalo, where he sold the corn at 70c. a bush. What was the profit on the transaction?

107. How much greater is the interest of \$750.25 for 2 yr. 4 mo. 15 da. at 8% per annum, than the interest of \$570.52 for $2\frac{2}{3}$ at 9% per annum?

108. A man sold a piece of cloth for \$24 and thereby lost 25%. If he had sold it for \$34 would he have gained or lost, and what per cent?

109. Find the area of a circle whose diameter is the cube root of 314432.

110. Find the area of a regular decagon whose side is .65 and apothem 1.

111. Find the area in square inches of the cross-section of a round bar of iron 3 inches in diameter.

112. A, B and C gain \$4300. What is each man's share of the gain when divided in the ratio of 3 to $2\frac{1}{2}$ to $1\frac{2}{3}$?

APPENDIX.

CHAPTER XVII.

THE METRIC SYSTEM.

623. The **Metric System** of Measures and Weights is a decimal system. The *base*, or principal unit of the system, is the *meter*, a measure of length, equal to 39.37+inches.

The meter, when adopted by the French Government, near the beginning of the present century, was supposed to be one ten-millionth part of a quadrant of the earth's circumference, measured from the equator to the pole.

624. The names of the different units of length, surfaces, volume and weight are formed by prefixing certain Latin and Greek prefixes to the principal unit of each table.

Thus, a *deci*meter is one tenth of a meter; a *deca*meter equals ten meters etc.

625. The prefixes used are as follows: For multiples of the principal units:

Deca = 10: as a decameter = 10 meters.
Hecto = 100: as a hectoliter = 100 liters.
Kilo =1000: as a kilogram =1000 grams.
Myria =10,000: as a myriagram=10,000 grams.

For sub-multiples, or parts of the principal units:

Deci = $\frac{1}{10}$: as a decimeter= $\frac{1}{10}$ of a meter.
Centi = $\frac{1}{100}$: as a centiliter = $\frac{1}{100}$ of a liter.
Milli =$\frac{1}{1000}$: as a milligram=$\frac{1}{1000}$ of a gram.

MEASURES OF LENGTH.

626. The standard unit is the **meter.** (*M.*)

1 Kilometer (*KM.*) = 1000 meters.
1 Hectometer (*HM.*) = 100 meters.
1 Decameter (*DM.*) = 10 meters.
1 Decimeter (*dm.*) = .1 meter.
1 Centimeter (*cm.*) = .01 meter.
1 Millimeter (*mm.*) = .001 meter.

EQUIVALENTS IN U. S. MEASURES.

1 Meter = 39.37+ inches.
1 Decimeter = 3.937 inches.
1 Centimeter = .3937 inches.
1 Millimeter = .03937 inches.

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1 Inch = 2.54 centimeters.
1 Foot = 3.048 decimeters.
1 Yard = 0.9144 meters.
1 Mile = 1.60933 kilometers. ✓

To reduce kilometers to miles, multiply by .62138.

MEASURES OF SURFACE.

627. The unit of surface is the **are** (*A.*) whose side is 10 metres.

100 Centiares (*CA.*) = 1 are (*A.*)
100 Ares 1 hectare (*HA.*)

EQUIVALENTS IN U. S. MEASURES.

1 Centiare = 1.196 square yards.
1 Are = 119.6 square yards.
1 Hectare = 2.471 acres.

628. Surface is also measured by the squares of the linear units, and volume by the cubes of the linear units.

Q is added to the abbreviation to denote square measure ; C is added to denote cubic measure. Thus, MC = 1 cubic meter.

1 sq. meter = 10.7643 sq. feet.
1 sq. centimeter = .155 sq. inch.
1 sq. millimeter = .0015 sq. inch.
1 sq. kilometer = .386116 sq. miles.

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1 square foot = 9.28997 sq. dm.
1 square inch = 6.45137 sq. cm.
1 square yard = .8361 sq. meter.
1 square mile = 2.589895 sq. km.

Measures of Volume and Capacity.

629. The unit of volume is the **stere** (*St.*) a cube whose edge is 1 meter.

10 decistere (*dst.*) = 1 stere (*St.*)
10 steres = 1 decastere (*DSt.*)

Equivalents in U. S. Measures.

1 decistere = 3.53 cu. ft.
1 stere = 1.308 cu. yd.
1 decastere = 13.08 cu. yd.

630. The unit of capacity (dry and liquid measures) is the **liter** (*L.*) a volume equal to 1 cubic decimeter.

1 kiloliter (*K L*) = 1000 liters.
1 hectoliter (*H L.*) = 100 liters.
1 decaliter (*D L.*) = 10 liters.
1 deciliter (*d l.*) = .1 liter.

1 centiliter (*c l.*) = .01 liter.
1 milliliter (*m l.*) = .001 liter.
1 liter = 1 cubic decimeter.
1 liter = 1000 cubic centimeters.
1 kiloliter = 1 stere.

EQUIVALENTS IN U. S. MEASURES.

1 milliliter	=1 cu centimeter	=.061 cu. in.	=.27 fluid dr.
1 centiliter	=10 cu. centimeters	=.6102 cu. in.	=.338 fluid oz.
1 deciliter	=$\frac{1}{10}$ decimeter	=6.1022 cu. in.	=.845 gill wine m.
1 liter	=1 cu. decimeter	=.908 qt. dry m.	=1.0567 qt wine m.
1 decaliter	=10 cu. decimeters	=9.08 qts. dry m.	=2.6417 gal. wine m.
1 hectoliter	=$\frac{1}{10}$ cu. meter	=2 bu. 3 35 pk.d.m.	=26.417 gal. wine m.
1 kiloliter or stere	=1 cu. meter	=.2759 cord	=264.17 gal. wine m.

WEIGHTS.

631. The unit of weight is the **gram** or **gramme** (*g*), equal to the weight of one cubic centimeter of distilled water at the temperature of 4° C.

1 millier (*mil.*) or tonneau (*ton.*) = 1,000,000 grams.
1 quintal (*Ql.*) = 100,000 grams.
1 myriagram (*Mg.*) = 10,000 grams.
1 kilogram or kilo (*Kil.*) = 1000 grams.
1 decigram (*dg.*) = .1 gram.
1 centigram (*cg.*) = .01 gram.
1 milligram (*mg.*) = .001 gram.

EQUIVALENTS IN U. S. MEASURES.

1 milligram = .0154 grains troy.
1 centigram = .1543 grains troy.
1 decigram = 1.5432 grains troy.

1 gram = 15.4327 grains troy.
1 decagram = .35274 ounce av.
1 hectogram = 3.5274 ounce av.
1 kilogram = 2.2046 lbs. av.
1 myriagram = 22.046 lbs. av.
1 quintal = 220.46 lbs. av.
1 millier or ton = 2204.6 lbs. av.

EXAMPLES.

PROCESS.

632. 1. Add 18 meters = 18. meters.
125 millimeters = .125 meters.
125 kilometers = 125000. meters.
492 decimeters = 49.2 meters.
Ans. 125067 325 meters.

2. Add 29 hectometers + 846 meters + 84 centimeters + 8 myriameters.
3. Add 16 MQ+29 KMQ+18 cm Q.
4. Add 86 HMC+924 DMC+48 mm C.
5. From 54 hectares take 296 centiares.
6. Value of my farm of 80 acres, at $20 per hectare ?
7. How many cords of wood in 8579.4855 steres ?
8. What is the weight, in pounds and ounces, of 50 packages, each weighing 38 Kg., 225 g. ?
9. How many bushels in 650 hectoliters of wheat?
10. How many liters in 126 gallons of wine?
11. How many kilograms in 3 barrels of flour?

For calculations in which only approximate accuracy is desired, the following equivalents may be used:

1 meter = 1.1 yd.
5 meters = 1 rod.
1 kilometer = 200 rods.

1 liter = .908 qt. dry m.
1 kilogram = 2.2 lb. av.
1 tonneau = 2200 lb. av.

CHAPTER XVIII.

AVERAGE OR EQUATION OF PAYMENTS.

633. **Average** or **Equation of Payments** is a method of finding the time when several debts due at different times may be paid at once, without loss to either debtor or creditor. The time of such payment is called the *Average* or *Equated Time.*

634. The **Term of Credit** of a debt is the time that elapses between the contraction of the debt and its maturity.

635. The **Average Term of Credit** of several debts is the average of their respective terms of credit.

636. The **Focal Date** of several debts is a date assumed at pleasure, to which all dates are referred, and from which the several terms of credit are reckoned.

NOTE. The focal date assumed is usually the earliest date at which any of the debts become due.

637. Average of payment is based upon the following facts, viz :

1. The interest or use of a certain sum for a given time is equal to the interest of twice the sum for one-half the time, or one-half the sum for twice the time, etc.
2. Money paid before it is due is compensated by an equal sum kept the same length of time after it is due.

638. Average of payments is considered under two cases, as follows :

Case I. That in which all the items are either debits or credits.

Case II. That in which the items are both debits and credits.

CASE I.

639. To find the Average Time of Credit, or Equated Time of Payment, when the terms begin at the same time.

1. A owes B $300 due in 3, months, $400 due in 5 months, and $600 due in 6 months. What is the average time of credit if the whole account be paid at once?

PRODUCT METHOD.

PROCESS.

$300 × 3 = $900 × 1
400 × 5 = 2000 × 1
600 × 6 = 3600 × 1

$1300 × (5) = $6500 × 1
$6500 ÷ $1300 = 5

SOLUTION. The use or interest of $300 for 3 months is equivalent to the use or interest of 3 times $300 = $900 for 1 month; the use of $400 for 5 months is equivalent to the use of 5 times $400 = $2000 for 1 month; and the use of $600 for 6 months is equivalent to the use of 6 times $600 = $3600 for 1 month. Hence the use of the several debts for their respective terms of credit is equivalent to the use of $900 + $2000 + $3600 = $6500 for 1 month, which is equivalent to the use of $1300, the sum of the debts, for as many months as $1300 is contained times in $6500, which are 5 times. Therefore the average term of credit is 5 months.

INTEREST METHOD.

640.

PROCESS.

Interest of $300 for 3 mos. at 6% = $ 4.50
Interest of $400 for 5 mos. at 6% = 10.00
Interest of $600 for 6 mos. at 6% = 18.00

Int. of $1300 for (5) mos. at 6% = $32.50
Int. of $1300 for 1 mo. at 6% = $ 6.50. $32.50 ÷ $6.50 = 5.

SOLUTION. Reckoning interest at 6%, the interest of $300 for 3 months is $4.50; the interest of $400 for 5 months is $10; the interest of $600 for 6 months is $18.00; and the interest of the several items for their respective terms of credit is \$4.50+\$10+\$18=\$32.50. The interest of \$1300, the sum of the debts, at the same rate for 1 month is \$6.50 To yield an interest of \$32.50 will require as many months as \$6.50 are contained times in \$32 50, which are 5 times. Therefore the average term of credit is 5 months.

NOTE. Any rate of interest may be assumed with the same result. Hence,

641. I. *Multiply each debt by its term of credit and divide the sum of the products by the sum of the debts: the quotient is the average term of credit.* Or,

II. *Compute the interest of each debt for its term of credit, and divide the sum of the interests by the interest of the sum of the debts for one period of the same denomination as the several terms of credit: the quotient is the average time of credit.*

The average term of credit added to the date of the contraction of the debts if at the same date, or to the focal date if at different dates, is the average or equated time of payment.

2. On the 1st of January, 1892, a merchant owed for goods \$500 due in 2 months, \$600 due in 4 months, and \$800 due in 8 months. What is the average term of credit, and at what time may the whole debt be paid at once ?

3. September, 1, 1891, my indebtedness was as follows : \$300 due in 4 months, \$500 due in 5 months, \$600 due in 6 months, and \$700 due in 8 months. At what time can I discharge the full obligation without loss to either debtor or creditor ?

4. Bought a farm, Oct. 15, 1889, for \$4800, paying $\frac{1}{3}$ down, $\frac{1}{3}$ in 9 months, and the balance in 1 year. At what time may the whole price be equitably paid?

5. A man purchased a house and lot for \$5600 ; terms,

$2000 cash, $1200 in 1 year, $1800 in 1 year and 8 months, and the balance in 2 years. For what time should a note be drawn to cover the purchase in a single payment ?

CASE II.

642. To find the Average Term of Credit, or Equated Time of Payment, when the Terms of Credit begin and mature at different times.

1. Find the equated time at which the following account may be settled :

CHICAGO, Jan. 1, 1891.

H. H. Dore

To A. Jones, *Dr.*

1890.	May 5.	To Mdse.	on 2 mos.	$1250
	June 10.	To "	on 4 mos.	2460
	Aug. 1.	To "	on 6 mos.	1590
	Sep. 15.	To "	on 4 mos.	2160

SOLUTION. Select, for convenience, the earliest date at which any item becomes due, July 5, as the focal date.

$1250, the debt contracted May 5, is due on that day, and has no term of credit.

$2460, the debt contracted June 10, is due Oct. 10, and has a term of credit of 97 days from the focal date.

$1590, the debt contracted Aug. 1, is due Feb. 1, 1891, and has a term of credit of 211 days from the focal date.

$2160, the debt contracted Sept. 15, is due Jan 15, 1891, and has a term of credit of 194 days from the focal date.

In this manner the example becomes one in Case I.

$1250 having no term of credit, its use or interest is equivalent to nothing; the use of $2460 for 97 days is equivalent to the use of 97 times $2460=$238620 for 1 day; the use of $1590 for 211 days is equivalent to the use of 211 times $1590=$335490 for 1 day; the use of $2160 for 194 days is equivalent to the use of 194 times $2160=$419040 for 1 day; and the use of the several debts for their respective terms of credit is equivalent to the use of $238620+$335490+$419040=$993150 for 1 day, which is equivalent to the use of $7460, the sum of the debts, for as many days as $7460 is contained times in $993150, which is 133+ times. July 5+133 days= Nov. 15, which is the equated time.

PROCESS.

1890. *mo.*		*da.*	*da.*
May 5+2=July 5	$1250×	0=$	0×1
June 10+4=Oct. 10	2460×	97=	238620×1
Aug. 1+6=Feb. 1, 1891.	1590×	211=	335490×1
Sept. 15+4=Jan 15, 1891.	2160×	194=	419040×1
	$7460×	(133)=	$993150×1

$993150÷$7460=133+

Hence,

643. *Select, for convenience, the earliest date at which any debt becomes due, as the focal date. From the focal date compute the term of credit for each item until its maturity, and proceed as in Case I.*

2. Find the equated time for the following bill:

1892. Jan. 1. 36 yards of linen, @ $2.50 on 2 mo. Feb. 1. 40 yards of cambric, @ 60c., on 2 mo. Mar. 1. 60 yards of cloth, @ $8.00, on 3 mo. Apr. 1. 24 yards of velvet, @ $16.00, on 3 mo., May 1. 50 yards of lace, @ $12.50, on 4 mo.

3. 1891, Jan. 20, A owes me $360; Feb. 25, $450; March 10, $575 ; April 15, $690 ; and May 21, $720. At what time may he make an equitable payment of the whole indebtedness ?

4. Purchased the following goods on 6 months' credit: 1891, Jan. 25, $360; Feb. 12, $375; March 10, $450; April 15, $550 ; May 18, $620. At what time may the whole be equitably paid ?

5. A merchant bought goods as follows : 1892, Sept. 10, $500, on 6 mo.; Oct. 15, $440, on 2 mo.; Nov. 20, $560, on 3 mo.; and Dec. 25, $800, on 4 mo. At what time may the whole debt be equitably discharged ?

CHAPTER XIX.

EQUATION OF ACCOUNTS.

644. An **Account** is a record of the items of debit and credit in business transactions.

645. A **Merchandise Balance** is the difference between the debits and credits of an account.

646. A **Cash Balance** is the merchandise balance plus or minus the interest thereon, according to the time of settlement. If settlement is made after the equated time, the *cash balance* is the balance of items plus the interest from the equated time to the time of settlement; but if settlement is made before the equated time, the *cash balance* is the balance of items minus the interest from the time of settlement to the equated time.

647. **Averaging an Account** is a method of finding the equated time for the payment of the balance of an account.

CASE I.

648. To find the Equated Time for the payment of the Merchandise Balance of an account.

1. *Dr.* JAMES THOMPSON IN ACCT. WITH JOHN WILSON. *Cr.*

1890.				1890.			
Jan. 1	To Mdse., 3 mo.	$300	00	June 1	By Cash,	$400	00
Feb. 1	To Mdse., 8 mo.	500	00	July 1	By Cash,	500	00
March 1	To Mdse., 6 mo.	600	00	Aug. 1	By Cash,	600	00
April 1	To Mdse., 5 mo	800	00	Sept. 1	By Cash,	600	00

Find the time when the balance of the account is due.

Equating each side separately, the earliest date at which any item of either side becomes due is selected as the focal date for that side, and the account is equated as follows:

PROCESS.

Due.	*Items.*	*Days.*	*Equiv. Days.*
April 1,	$300 ×	0 =	0×1
Oct. 1,	500 ×	183 =	91500×1
Sept. 1,	600 ×	153 =	91800×1
Sept. 1,	800 ×	153 =	122400×1

$2200×(139)=$305700×1

$305700÷$2200=139.

The equated time for the payment of the debit side is 139 days from April 1, the focal date, which is Aug. 18.

Due.	*Items.*	*Days.*	*Equiv. Days.*
June 1,	$400 ×	0 =	0×1
July 1,	500 ×	30 =	15000×1
Aug. 1,	600 ×	61 =	36600×1
Sept. 1	600 ×	92 =	55200×1

$2100 × (51)= $106800×1

$106800÷2100=51.

The equated time for the payment of the credit side is 51 days from June 1, the focal date, which is July 22.

The account therefore appears as follows :

Dr.	*Cr.*
Due Aug. 18, 1890. $2200.	Due July 22, $2100.

In finding when the merchandise balance, $100, is due, it will be seen that at the time of settlement, which is the maturity of the debit side, the credit side has had the use of $2100 from July 22 to Aug. 18, which is 27 days, which is equivalent to the use of $56700 for 1 day; and to counterbalance this use the payment of the balance, $100, which is on the debit side, may be delayed as many days after settlement as $100 is contained times in $56700, which is 567 times ; and 567 days from Aug. 18, 1890, is March 10, 1892, the equated time for the payment of the balance.

2. Find the merchandise balance of the following account and when it is due:

Dr. GEORGE H. THOMAS. *Cr.*

1890.				1890.			
May 10	To Mdse., . .	$320	00	July 30	By Cash,	$280	00
July 12	To Mdse., .	600	00	Sept. 30	By Cash,	300	00
Sept. 20	To Mdse., . .	250	00	Oct. 25	By Cash,	400	00
Dec. 15	To Mdse., .	300	00	Dec. 30	By Cash,	250	00

PROCESS.

Cr.	*Items.*		*Equiv.*	*Days.*
May 10,	$320×	0=	$ 0	×1
July 12,	600×	63=	37800	×1
Sept. 20,	250×	133=	33250	×1
Dec. 15,	300×	219=	65700	×1

$1470+(93)=$136750×1
$136750÷$1470=93

The equated time for the payment of the debit is 93 days from May 10, the focal date, which is Aug. 11.

	Items.		*Equiv.*	*Days.*
July 30,	$280 ×	0=	$ 0	×1
Sept. 30,	300 ×	62=	18600	×1
Oct. 25,	400 ×	87=	34800	×1
Dec. 30,	250 ×	153=	38250	×1

$1230×(75–)=$91650
$91650÷$1230=75

The equated time for the payment of the credit side is 75 days from July 30, the focal date, which is Oct. 13.

The account therefore appears as follows:

Dr.	*Cr.*
Due Aug. 11, $1470.	Due Oct. 13, . . $1230.

In finding when the merchandise balance, $240, is due, it will be seen that at the time of settlement—the maturity of the credit side—the debit side has had the use of $1470 from Aug. 11 to Oct. 13, which is 63 days, which is equivalent to the use of $92610 for 1 day; and to counterbalance this use the credit should be entitled to the use of the balance, $240, for as many days before settlement as $240 is contained times in $92610, which is 386– times. Oct. 13, 1890–386 days=June 14, 1889, the equated time for the payment of the balance of the account.

NOTE. It is obvious that when the larger side of the account is due at the later date the balance is due after settlement; but when the smaller side is due at the later date the balance is due before settlement. Hence,

649. *Find the equated time for each side separately. Multiply the side of the account which falls due first by the time that elapses between the equated times of the two sides, and divide the product by the balance of the account. Add the quotient to the later date when the larger side of the account falls due at that date, and subtract it when the smaller side falls due at that date.*

CASE II.

650. To find the Cash Balance of an Account.

The cash balance of Problem 1 is \$100 plus the interest thereon—for convenience at 6%—for 567 days, which is \$9.45. \$100 + \$9.45 = \$109.45.

The cash balance of Problem 2 is \$240 minus the interest thereon for 368 days, which is \$14.72. \$240 − \$14.72 = \$225.28. Hence,

651. *To find the cash balance, compute the interest on the merchandise balance for the time that elapses between the time of settlement and the equated time, and add the same to the merchandise balance if settlement is after the equated time, but subtract if time of settlement is before equated time.*

1. What is the merchandise balance in the following account, and when is it due?

Dr. *Cr.*

1890.				1890.			
May 5	To Mdse., . .	\$600	00	June 30	By Cash,	\$400	00
June 10	To Mdse., . .	750	00	July 25	By Cash,	500	00
Aug. 20	To Mdse., . .	460	00	Aug. 18	By Cash,	420	00
Sept. 15	To Mdse., . .	650	00	Oct. 20	By Cash,	300	00

2. Find equated time for payment of balance of account.

Dr. WILLIAM HARRISON. *Cr.*

1890.				1890.			
April 5	To Mdse , .	$590	00	May 25	By Prod'ce	$380	00
June 1	To Mdse., . .	480	00	July 30	By Cash,	640	00
Aug. 10	To Cash, . . .	350	00	Sept. 30	By Mdse.,	240	00
Oct 20	To Mdse., . .	560	00	Dec. 1	By Cash,	440	00

3. Find merchandise balance of the following account and when due.

Dr. ELLIS RUTHERFORD. *Cr.*

1892.				1892.			
Jan. 10	To Mdse. on 20 d.	$540	00	Feb. 25	By Mdse. 30 d.	$580	00
Mar. 15	To Mdse. on 30 d.	450	00	Apr. 30	By Cash, . .	490	00
May 20	To Mdse. on 45 d.	390	00	June 15	By Mdse. 60 d.	520	00
July 30	To Mdse. on 60 d.	420	00	June 30	By Mdse. 60 d.	200	00

4. Find the cash balance of the following account:

Dr. EDGAR SARGENT, JR. *Cr.*

1891.				1891.			
May 20	To Mdse 3 mos.	$ 90	00	June 10	By Cash, . .	$ 66	00
July 30	To Mdse. 5 mos.	125	00	Aug 20	By Mdse. 4 mos.	120	00
Sept. 30	To Mdse. 4 mos.	80	00	Oct. 30	By Mdse. 5 mos.	130	00
Oct. 1	To Mdse. 2 mos.	40	00	Nov. 1	By Mdse. 3 mos.	100	00

5. The merchandise balance of an account, $260, is due June 20, 1893. What is the cash balance Oct. 25, 1893, interest at 8% ?

6. The merchandise balance of an account is $96 at settlement, Sept. 30, 1894. What is the cash balance May 10, 1895, interest at 6% ?

	Dr.	*Cr.*
7.	Due Sept. 1, 1884, $1190	Due July 30, 1883, $1850.

Find cash balances, interest at 7%.

	Dr.	*Cr.*
8.	Due Oct. 20, 1892, $1260	Due Aug. 25, 1892, $940

Find cash balance, interest at 6%.

CHAPTER XX.

ALLIGATION.

652. **Alligation** is the equation of values or quantities in compounding.

It is of two kinds: **Alligation Medial** and **Alligation Alternate.**

653. **Alligation Medial** is the process of finding the value of a compound, the value and the quantity of each ingredient being given.

654. **Alligation Alternate** is the process of finding the *proportion* of different ingredients of given value, necessary to form a compound of a given value.

655. All operations in alligation are based upon the principle that, in compounding, what is gained on one ingredient is lost on another. That is, each ingredient *above* the average value is combined with a sufficient quantity of an ingredient *below* the average, to have the loss on the one balanced by the gain on the other. If *two* ingredients are compounded, it is evident that one must be *above* and the other *below* the average ; if three or more, it is evident that *at least* one must be above or below the required price or quality.

ALLIGATION MEDIAL.

656. 1. Find the value of one pound of mixed tea consisting of 4 lbs. worth 50 cents per lb., 3 lbs. worth 60 cents per lb., and 2 lbs. worth 71 cents per lb.

Solution.

4 lbs. @ 50 cents = \$2.00
3 lbs. @ 60 cents = 1.80
2 lbs. @ 71 cents = 1.42
9 lbs. are worth \$5.22
1 lb. is worth $\frac{\$5.22}{9}$=58 cents.

2. Find the average value of 25 bush. oats worth 21 cents per bush., 100 bush. worth 22 cents per bush., 80 bush. worth 24 cents per bush., and 60 bush. worth 33 cents per bush.

3. What per cent of alcohol in a mixture of 4½ gal. 88% strong, 6 gal. 89⅔% strong, 5 gal. 96% strong, 5½ gal. 98% strong?

4. How many carats fine is a mixture of 4 pwt. of gold 16 carats fine, 6 pwt. 18 carats fine, 8 pwt. 20 carats fine, 2 pwt. pure gold ?

5. Find value per gallon of 30 gal. alcohol at 80 cents 30 gal. at \$1.00, 30 gal. at \$1.20, diluted with 10 gal. water.

ALLIGATION ALTERNATE.

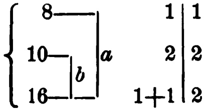
CASE I.

657. To find the Proportion of Different Ingredients of given value or quality to form a compound of a given value or quality.

1. In what proportion may sugars worth 8, 10 and 16 cents per lb., respectively, be mixed to form a mixture worth 12 cents per lb.?

Solution In this example *two* of the proposed ingredients are below the average price and *one* above the average price. Each of the two former — the 8 cent sugar and the 10 cent sugar must therefore be combined with the 16 cent sugar.

(*a*) Combining the 8 cent and the 16 cents, the gain, 4 cents on one lb. of the 8 cent balances the loss, 4 cents, on one lb. of 16 cent. Hence, equal quantities of the 8 cent and the 16 cent sugars may be taken. (*b*) Combining the 10 cent with the 16 cent, the gain on one pound of the 10 cent sugar is 2 cents, while the loss on one lb of the 16 cent sugar is 4 cents. Hence, two lbs. of the 10 cent sugar must be taken for every lb. of the 16 cent sugar taken. Adding the lb. taken in (*b*) to the lb. taken in (*a*), the proportion is 1 lb. of the 8 cent, 2 lb. of the 10 cent, 2 lb. of the 16 cent.



Note I. An *infinite* number of results, all equally correct, may be obtained in every question like the above.

Note II. The correctness of any result may be verified by an operation in Alligation Medial, which may be regarded as a *proof* of Alligation Alternate.

2. In what proportion may teas which cost 80 cents, $1.00, $1.20, and $1.40 per lb. be mixed, that the whole may be sold for $1.54 per lb. at a gain of 40%?

3. In what proportion may alcohols worth, respectively, $1.40, $1.50, $1.60 and $2.00, be mixed to be sold at $1.80 per gal.?

Case II.

658. To find the Proportions of Different Ingredients, of given value or quality, to form a compound of a given value or quality, when the quantity of one or more of the ingredients is given.

1. B has 50 lb. of damaged mustard seed, valued at 40 cents per lb. How many lbs. of good seed worth $1.20 per lb. must be mixed with it to have the mixture worth $1.00 per lb.?

Solution. By Case I, the seeds must be mixed in the proportion of 1 lb. of the damaged to 3 lb. of the good. Since there are 50 lb. of the damaged article, there must be 150 lb. of the other.

2. C has 100 bbls. of pork worth $8 per bbl. and 100 lbs. worth $9 per bbl. How many bbls. worth $6 per bbl. must he buy to make the whole average $7 per bbl.?

Solution. By Alligation Medial, the average value of the pork on hand is $8.50 per bbl. By Case I, the proportion of the pork on hand to that to be purchased is as 2 : 3. Since there are 200 bbls. on hand, he must purchase 300 bbls. Proof by Alligation Medial.

3. How many bushels of corn worth, respectively, 40 cents, 52 cents and 60 cents per bush. may be mixed with 80 bush. corn worth 48 cents per bush. that the whole may average 50 cents per bush.?
4. How much water must be mixed with 4 qts. of alcohol 90%, and 6 qts. 95% to make a mixture 80%?
5. How many shares stock at 78c. must be added to 50 shares worth 84c. and 70 shares worth 60c. to make the whole average 75c.?

Case III.

559. To find the Proportions of the Ingredients when the value or quality and the quantity of the compound are given.

1. How much sugar worth 8 cents, 10 cents, 12 cents and 14 cents per lb. will make a mixture of 400 lb. worth 11 cents per lb.?

Suggestion. Find the proportion of each kind of sugar by Case I. Divide the quantity of the compound (400 lb.) by obtaining proportional parts. Art. 527.

2. How much gold 16 carats fine, 20 carats fine, and 22 carats fine make a ring 18 carats fine weighing 5 pwts.?

CHAPTER XXI.

ARITHMETICAL SERIES.

660. An **Arithmetical Series** is a series of numbers each of which is formed from the number immediately preceding by the addition or subtraction of a constant number.

Thus, 4, 6, 8, 10, is an arithmetical series in which the constant number, or *common difference*, is 2.

In the series 12, 9 6, 3, the common difference is 3.

661. The difference between any two consecutive terms is called the **Common Difference.**

662. An **Ascending Series** is formed by the *addition* of the common difference. As 5, 10, 15, 20, 25, etc.

663. A **Descending Series** is formed by the *subtraction* of the common difference. As 36, 28, 20, 12, etc.

664. The numbers which compose a series are called **Terms;** of these, the first and last are **extremes,** the others, **means.**

665. The four qualities considered in an arithmetical series are:

1. The first term, (a)
2. The last term, (l).
3. The common difference, (d).
4. The number of terms, (n).
5. The sum of the terms, (s).

If three of these be known, either or both of the other two may be found.

666. Since each term (except the first) of an Ascending Series is formed by the addition of the common difference to the preceding term, each term (except the first) may be regarded as being composed of two parts, the first term (a), and the common difference (d), taken a certain number of times; and every ascending series may be thus expressed: .

1st.	2nd.	3rd.	4th.	5th.
a	$a+d$	$a+2d$	$a+3d$.	$a+4d$, etc.

Hence,

I. *The last term of an ascending series equals the first term plus the common difference taken as many times less one as there are terms in the series;* that is, $l=a+(n-1)d$.

II. *The first term of an ascending series equals the last term minus the common difference taken as many times less one as there are terms in the series;* that is, $a=l-(n-1)d$.

By finding the difference of the last term ($a+4d$) and the first term (a), it is seen that the remainder ($4d$) equals the common difference (d) taken one less time than the number of terms ($n-1$). Hence,

III. *The common difference of an ascending series equals the difference of the first and last terms divided by one less than the number of terms;* that is,

$$d=\frac{l-a}{n-1}$$

IV. *The number of terms of an ascending series equals one plus the difference of the first and last terms divided by the common difference;* that is, $n=1+\frac{l-a}{d}$.

Half the sum of the first and last terms is the *average* term of the series, that is, $\frac{a+l}{2}$, which, multiplied by the number of terms is evidently the sum of the terms. Hence,

POUNDS IN A BUSHEL, AVOIRDUPOIS WEIGHT.

As prescribed by statute in the States named below, unless specific agreements to the contrary are made.

COMMODITIES.	*California.*	*Connecticut.*	*Delaware.*	*Illinois.*	*Indiana.*	*Iowa*	*Kentucky.*	*Louisiana.*	*Maine.*	*Massachusetts.*	*Michigan.*	*Minnesota.*	*Missouri.*	*N. Hampshire.*	*New Jersey.*	*New York.*	*Ohio.*	*Oregon.*	*Pennsylvania.*	*Rhode Island.*	*Vermont.*	*Washington T.*	*Wisconsin.*
Barley	50	--	--	48	48	48	48	32	--	46	48	48	48	--	48	48	48	46	47	--	46	45	48
Beans	--	--	--	60	60	60	60	--	--	--	60	--	60	--	--	60	--	--	--	--	--	--	--
BlueGrass Seed	--	--	--	14	14	14	14	--	--	--	--	--	14	--	--	--	--	--	--	--	--	--	--
Buckwheat	40	45	--	42	50	52	52	--	--	46	42	42	52	--	50	48	--	42	48	--	46	42	42
Castor Beans	--	--	--	46	46	46	--	--	--	--	--	--	46	--	--	--	--	--	--	--	--	--	--
Clover Seed	--	--	--	60	60	60	60	--	--	--	60	60	60	--	64	60	60	60	--	--	--	60	60
Dried Apples	--	--	--	24	25	24	--	--	--	--	28	28	24	--	--	--	--	28	--	--	--	28	28
Dried Peaches	--	--	--	33	33	33	--	--	--	--	28	28	33	--	--	--	--	28	--	--	--	28	28
Flax Seed	--	--	--	56	56	56	56	--	--	--	56	--	56	--	55	55	56	--	--	--	--	-	56
Hemp Seed	--	--	--	44	44	44	44	--	--	--	--	--	44	--	--	--	--	--	--	--	--	--	--
Indian Corn	52	56	56	56	56	56	56	56	--	56	56	56	52	--	56	58	56	56	56	--	56	56	56
Indian Corn in Ear	--	--	--	70	68	70	--	--	--	--	--	--	--	--	--	--	--	--	--	-	--	--	--
Indian Corn Meal	--	--	--	48	50	--	50	--	50	50	--	--	--	--	--	--	--	--	--	50	--	--	--
Mineral Coal	--	--	--	80	70	80	--	--	--	--	--	--	80	--	--	--	--	--	--	--	--	--	--
Oats	32	28	--	32	32	35	33⅓	32	30	32	32	32	35	30	30	32	32	34	32	--	32	36	32
Onions	--		--	57	57	57	57	--	--	52	56	--	57	--	--	--	--	--	--	50	--	50	57
Potatoes	--	60	--	60	60	60	60	--	60	--	60	--	60	60	60	60	--	60	--	60	60	60	60
Rye	54	56	--	56	56	56	56	32	--	56	56	56	56	--	56	56	56	56	56	--	56	56	56
Rye Meal	--	--	--	--	--	--	--	--	50	50	--	--	--	--	--	--	--	--	--	50	--	--	--
Salt	--	--	--	--	50	50	50	--	--	--	--	--	50	--	--	56	--	--	--	--	--	--	--
Timothy Seed	--	--	-	45	45	45	45	--	--	--	45	--	45	--	--	44	--	--	--	--	--	--	46
Wheat	60	56	60	60	60	60	60	60	--	60	60	60	60	--	60	60	60	60	60	--	60	60	60

1. In Pennsylvania 80 lb. coarse, 70 lb. ground, or 62 lb. fine salt make 1 bushel; and in Illinois, 50 lb. common, or 55 lb. fine salt make 1 bushel.
2. In Maine 64 lb. of ruta-baga turnips, or of beets make 1 bushel.

Ex. Page 20.

2 31378
3. 116059
4. 838236

---

2. 58.5616
3. 46.5413
4. 425.24408

Page 21.

2. 288931
3. 296938
4. 1912.597
5. 589.18
6. $1395.43

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1 66465
2. 503260

Page 22.

3. 1,617, 025,000
4. $336,439, 727
5. $52,824, 640

Page 25.

3. 5213
4. 2977
5. 7108
6. 4748
7 1385
8. 25163
9. 68095
10. 55967
11. 3287
12. 18716

Page 26.

14. 112.94
15. 10.464
16. 28.8846
17. 83.125
18. 8.375
19. 9.1734
20. $46.25
21. $2.952

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1. 1,116 942
2. 257127 093
3. $50,972, 604
4. $10,622, 985

Page 27.

5. 46,226, 569

Page 31.

2. 5663
3. 31050
4 253680
591920
761040
5. 228240
342360
456480
6. 242800
339920
437040
7. 6386400
4789800
3991500
8. 247.5
9. $108.75
$253.75
$290.00
10. 422.80
507.36
761 04

Page 32.

2. 287708
3. 252504
4. 525252
5. 549516
6. 150289.2
7. 49135.12
8. 3876,094
9. 239.2596
10. 1869670
11. 1010097
12. 1764928
13. 4922368
15. 2923000
16. 1380000
17. 840000
18. 2410000
19. 352512
20. 289788
21. 439432
22. 202860
23. 206016
24. 15000
25. 15000
26. 15000
27. 15000
28 15000
29. 18944
30. 18944
31. 20473
32. 20473

Page 33.

33. 18837
34. 18837

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1. 92628000
2. 1641600
3. 91241856
4. 11772000
5. 45118016

Page 38

1. 234$\frac{33}{86}$
2. 227$\frac{12}{42}$
3. 211$\frac{13}{85}$
4. 109$\frac{2}{59}$
5. 186$\frac{37}{48}$
6. 203$\frac{29}{75}$
7. 293$\frac{75}{84}$
8. 450$\frac{12}{69}$
9. 295$\frac{9}{87}$
10. 445$\frac{20}{92}$
11. 2580$\frac{69}{827}$
12. 2115$\frac{118}{452}$
13. 891.$\frac{786}{759}$
14. 1460$\frac{192}{675}$
15. 1434$\frac{157}{578}$

Page 39.

16. 155.125
17. 373.875
18. 54.125
19. 135.36
20. 248.75
21. 134.472
22. 119.877
23. 43.309
24. 85.239
25 157.586
26. 143.
27 130.
28. 2513.512
29. 683.421
30. 1556.20
31. 172.565
32. 1076.595
33. 523.630
34. 1236.084
35. 359.5
36. 1788.884
37. 753.117
38. 258.586
39. 596.585
40. 1203.193
41. 646.713
42. 442 081
43. 1733.518
44. 430.304
45. 694.627
46. 1423.118
47. 900.610
48. 1198.235
49 1578.86
50. 1068.768
51. 778.938

Page 40.

2. 1411$\frac{1}{3}$
3. 446$\frac{1}{8}$
4. 935
5. 566
6. 2413$\frac{1}{2}$
7 4115
8. 3513$\frac{5}{7}$
9. 8307
10. 2759$\frac{5}{8}$
11. 7455$\frac{4}{8}$
12. 6675
13 7469$\frac{9}{12}$
14. 5813$\frac{9}{12}$
15. 3295$\frac{4}{11}$
16. 4144$\frac{9}{13}$
20. 607
21. 604
22. 908
23. 4007
24. 5008
25. 8009
26. 6090$\frac{3}{8}$
27. 6080$\frac{7}{11}$
28. 5070$\frac{1}{12}$

Page 41.

13. 1439
14. 495$\frac{1524}{17869}$

Page 42.

15. 51
16. 7626190
17. 505
18. 88

Ex. Page 42.

19. 956484

---

2. 140 *r* 56
120 *r* 56
105 *r* 56
93 *r* 86

3. 145 *r* 349
116 *r* 349
83 *r* 249

Page 43.

4. 9 *r* 154
21 *r* 124
10 *r* 394

5. 25 *r* 2079
10 *r* 8079
14 *r* 1079

---

17. 5523
18. 171119
19. 25320
20. 9435
21. 14056

Page 44.

2. 2800
3. 1200
4. 180
5. 1650
6. 2400
7. 790

Page 45.

9. 4000
10. 5400
11. 6250
12. 8050
13. 6180
14. 61733$\frac{1}{3}$
15. 40000
16. 3820
18. 25.38
19. 55.08
20. 51.84
21. 95.36
22. 470.52
23. 437.44
24. 1491.6
25. 3585.6

---

1. 1359962$\frac{1}{2}$
2. 23
3. 560591
4. 32–
5. 462+

Page 49.

1. Ill. 56650
Wis. 56040
2. 9565
3. 9402
4. 286277
5. 20.779
6. 983 ; 809
7. 1559
8. 3
9. $1129

Page 50.

10. 5625
11. 2177280
12. 18
13. 150
14. 7$\frac{111}{231}$
15. 7899 375
16. 165.8+
17. $1,164,
690,590
18. $180.254+
19. $71.885+
20. 1204499
52369+

Page 56

16. 706.18354
17. 758.5389
18. 67.134756
19. 100.880
5486
21. 13.5511
22. 4.92251
23. 373.75217
24. 77.1987
25. .534154
26. 315.091
466
27. $51.025
28. $151.875

Page 60.

1. 1246.08
2. 111.853
3. 2231.55
4. 3405.48
5. 4 09332
6. 23 7654
7. 17.871
8 8.334
9. 3.421365
10. 184.9404
11. .1896675
12. .0003315
576
13. 2.2446423
14. $396.48
15. $3.025
16. $21.15
17. $3.9375
18. $7.8125
19. $54.1875
20. $19.125

Page 61.

2. 370.99
3. 74.314
4. 63.173
5. 3174.2
6. 321.49
7. 30070.
8. 328.
9. 751.3
10. 5226.1
11. 11,927.5
12. 1250700.
13. 600800.
14. 19350000.
15. 805360.
16. 7070800.
17. 5074.1

Page 62.

20. .9910
21. 1.9555
22. 21.6776
23. 14.6207
24. 1.2416
25. 6.5387
27. .014966
28. .007312
29. .645340
30. .020213
31. .000040
32. .000433
33. .000152
34. .000207

Page 64

2. .00105
3. .00105
4. .0318
5. .03003
6. .0007201
7. .40502
8. 125.36
9. 2.648
10. 13.44
11. .01575
12. .0084
13. 63.445
14. 1.5625
15. 8.522
16. 33.331
17. 1.0001
18. .0375
19. 2.307
20. 11.2
21. .15625
22. 39.0625
23. .01499
24. 17.008
25. 4162200.
26. .62
27. .004735

Page 65.

1. 3675.069
2. 8003.255
69
3. 14014.903
4. 1357.63
5. .992
6. 114.75
7. 357 84$\frac{3}{8}$
8. 496.189
9. .004
10. 1.6

Page 66.

11. 10.7$\frac{3}{8}$
12. 367 68
13. $207.
14. 15
15. 375
16. 5400
17. 21900
18. $315.20
19. $2.73
20. $250.
21. 1500
$4387.50
22. $396

Page 70.

11. $24.578
12. $21.975

Ex. Page 70.

13. $67.805
14. $86.825
15. $2253.78
16. $84.15
17. $3.375
18. $7.625
19. $1468.80
20. $48580.
21. $311.475
22. $5028.50
23. $295.50
24. $1050.
25. $50.
26. $150.
27. $1.10
28. $1.22
29. $3.20
30. $7.899+
31. $7.
32. $12.20

Page 71.

3. $107.03
4. $638.56
5. $1093.75
6. $328.125
7. $273.
8. 92750
9. 1,500,000

—

2. $26.7225
3. $27.554
4. $36.4375
5. $169830.

Page 73.

1. $25.82

Page 74.

2. $8.46
3. $6.65
4. $253.15
5. $42.78+
6. $818.78

Page 85.

2. 2.2.3.3
3. 2.2.2.7
4. 3.3.3.3
5. 2.2.2.11
6. 5.23
7. 5.5.13
8. 2.2.107
9. 2.3.5.19
10. 2.2.2.3.59
11. 2.3.417
12. 5.3.3.83
13. 2.2.3.5.67
14. 2.2.2.13.31
15. $2^6.89$
16. 2.2.3.3.5.5.5
17. $2^4.151$
18. 5
19. 7
20. 3.5
21. 11
22. 5
23. 2.2.3
24. 5
25. 3

Page 88.

2. 11
3. 27
4. 42
5. 17
6. 126
7. 42
8. 34
9. 33
11. 7
12. 11
13. 182
14. 272
15. 5
16. 18
17. 87

Page 91.

2. 20160
3. 7686024
4. 1620
5. 12480
6. 28860
7. 8640
8. 360 in.
9. 14400 A.
10. 468 gal.

—

2. 3
3. 5

Page 92.

5. 36
6. 10
7. $10\frac{2}{3}$
8. 14
9. $1\frac{1}{3}$
10. $\frac{24}{35}$
11. 6
12. 1
13. 21
14. 16
15. 36
16. 56
17. 2

Page 97.

16. $\frac{479}{12}$
17. $\frac{21142}{25}$
18. $\frac{6864}{71}$
19. $\frac{7142}{19}$
20. $\frac{8887}{21}$
21. $\frac{16819}{39}$
22. $\frac{15153}{20}$
23. $\frac{26055}{31}$
24. $\frac{57366}{65}$

Page 98.

14. $47\frac{3}{5}$
15. $56\frac{4}{11}$
16. $31\frac{6}{17}$
17. $18\frac{8}{27}$
18. $12\frac{20}{48}$
19. $17\frac{31}{35}$
20. $21\frac{9}{40}$
21. $14\frac{44}{51}$
22. $24\frac{140}{385}$
23. $9\frac{132}{527}$
24. $15\frac{410}{626}$
25. $13\frac{288}{625}$

Page 100.

37. $\frac{14}{27}$
38. $\frac{17}{34}$
39. $\frac{109}{137}$
40. $\frac{11}{13}$
41. $\frac{83}{304}$
42. $\frac{3}{5}$
43. $\frac{14}{107}$
44. $\frac{7}{8}$
45. $\frac{107}{117}$
46. $1\frac{13}{361}$
47. $1\frac{9}{16}$
48. $\frac{4}{5}$
49. $\frac{11}{31}$
50. $\frac{13}{67}$
51. $\frac{13}{17}$
52. $\frac{19}{67}$

Page 102.

16. $\frac{250}{1280}$, $\frac{352}{1280}$
17. $\frac{1750}{2520}$, $\frac{441}{2520}$
18. $\frac{118}{300}$, $\frac{155}{300}$
19. $\frac{2}{14}$, $\frac{3}{14}$, $\frac{3}{14}$, $\frac{6}{14}$
20. $\frac{40}{190}$, $\frac{188}{190}$, $\frac{55}{190}$, $\frac{74}{190}$
21. $\frac{45}{355}$, $\frac{85}{355}$, $\frac{91}{355}$, $\frac{135}{355}$

Page 105.

25. $3\frac{7}{60}$
26. $2\frac{31}{72}$
27. $2\frac{1}{9}$
28. $2\frac{85}{198}$
29. $2\frac{49}{72}$
30. $30\frac{11}{51}$
31. $20\frac{55}{78}$
32. $17\frac{381}{504}$
33. $18\frac{181}{1080}$
34. $26\frac{85}{1856}$
35. 2
36. $2\frac{1}{3}$

Page 106.

22. $14\frac{59}{126}$
23. $3\frac{157}{216}$
24. $12\frac{117}{506}$
25. $4\frac{188}{549}$
26. $15\frac{305}{372}$
27. $9\frac{13}{14}$

Page 107.

28. $10\frac{1}{4}$
29. $26\frac{5}{6}$
30. $80\frac{67}{210}$
31. $22\frac{61}{63}$
32. $13\frac{29}{48}$
33. $45\frac{3}{5}$
34. $20\frac{1}{3}$

Page 109.

30. $15\frac{9}{40}$
31. $14\frac{3}{13}$
32. $26\frac{6}{17}$
33. $41\frac{83}{160}$
34. $28\frac{1}{3}$
35. $45\frac{3}{8}$
36. $16\frac{1}{3}$

Ex. Page 109.
37. 28
38. $59\frac{3}{5}$
39. 75
40. 73
41. $86\frac{3}{4}$
43. $1746\frac{3}{5}$
44. $1164\frac{8}{25}$
45. $3863\frac{1}{4}$
46. $6746\frac{11}{14}$
47. $12924\frac{35}{61}$
48. $20793\frac{7}{26}$

Page 110.
12. 525
13. 1188
14. $135\frac{3}{8}$
15. 204
16. 1062
17. 1278

Page 111.
19. $106\frac{3}{25}$
20. $461\frac{13}{81}$
21. 432
22. $816\frac{9}{26}$
23. $22488\frac{24}{25}$
24. $35863\frac{11}{19}$
25. $62094\frac{37}{67}$
26. 133542

---

21. $12\frac{1}{2}$
22. $16\frac{1}{2}$
23. 1.
24. 2
25. 2

Page 112.
33. $\frac{4}{27}$
34. $\frac{15}{56}$
35. $1.\frac{361}{480}$
36. $4\frac{16}{35}$
37. $3\frac{5}{33}$
38. $22\frac{1}{2}$
39. $16\frac{11}{14}$
40. 350
41. $3\frac{3}{8}$
42. $\frac{3}{16}$

Page 114.
24. $\frac{3}{196}$
25. $\frac{9}{331}$
26. $\frac{7}{190}$
27. $\frac{39}{764}$
28. $\frac{5}{181}$
29. $\frac{71}{990}$
30. $\frac{56}{1083}$
31. $\frac{67}{450}$
32. $\frac{14}{285}$
33. $\frac{3}{40}$
34. $\frac{4}{5}$
35 $\frac{7}{8}$
36. $\frac{3}{7}$
37. $11\frac{1}{16}$
38. $1\frac{2}{9}$

Page 115.
17. $57\frac{3}{5}$
18. $64\frac{3}{7}$
19. $104\frac{8}{11}$
20. $41\frac{5}{8}$
21. $70\frac{5}{6}$
22. 275
23. 10
24. 36
25. 28
26. 36
27. $19\frac{7}{88}$
28. $11\frac{11}{24}$

Page 116.
29. $1\frac{43}{144}$
30. $2\frac{1}{7}$
31 $\frac{7}{8}$
32. $1\frac{1}{5}$
33. $\frac{1}{6}$
34. $1\frac{77}{670}$
35. $2\frac{40}{189}$
36. $2\frac{88}{513}$
37. 2
38. $1\frac{55}{71}$

Page 117.
2. $1\frac{1}{8}$
3. $1\frac{11}{24}$
4. $\frac{18}{35}$
5. $\frac{1}{8}$
6. $1\frac{3}{5}$
7. $\frac{18}{35}$

Page 119.
27. $\frac{4}{625}$
28. $\frac{56}{625}$
29. $\frac{23}{40}$
30. $\frac{1}{80}$
31. $\frac{227}{400}$
32. $\frac{9}{400}$
33. $\frac{56}{125}$
34. $\frac{81}{1250}$
35. $\frac{27}{400}$
36. $\frac{16}{625}$
37. $\frac{15}{16}$
38. $\frac{5}{32}$

---

1. .59375
2. .215
3. .9125
4. .46153+
5. 2.625
6. 4.375
7. .81666+
8 .84210
9. 1.14285+
10. 2.31428+
11. 1.92307+
12. .04615+

Page 121.
1. $\frac{8141}{32}$
2. $\frac{6346}{49}$
3. 3540
4. $374\frac{33}{256}$
5 $\frac{12978}{50}$
6. $\frac{1686}{9}$
7. $\frac{17}{18}$
8. $\frac{17}{42}$
9. $\frac{15}{48}$, $\frac{36}{48}$, $\frac{28}{48}$
10. $\frac{10080}{14112}$,
$\frac{12544}{14112}$, $\frac{7056}{14112}$
11. $1\frac{217}{240}$
12. $70\frac{71}{140}$
13. $28\frac{97}{135}$
14. $46\frac{3}{16}$
15. $215\frac{31}{183}$
16. $135\frac{1}{63}$
17. $31\frac{25}{36}$
$19\frac{31}{36}$
18. $31
19. $5586\frac{11}{24}$

Page 123.
20. $377\frac{23}{48}$
21. 651
22. 623
23. $242\frac{4}{7}$
24. $\$7\frac{4}{5}$
25. $\$2100\frac{3}{4}$
26. $\$1873\frac{4}{5}$
27. $10500
28. 1250 sq. m.
29. $96000
30 210
31. $2889\frac{43}{60}$
32. $\$3795\frac{5}{9}$
33. $281\frac{59}{66}$
34. $99\frac{67}{99}$
35. $31725

Page 125.
36. $\frac{121}{128}$
37. $33\frac{47}{54}$
38. $99\frac{17}{60}$
39. 1
40. $6\frac{2}{3}$
41. $\$20\frac{5}{32}$
42. 200
43. $24\frac{1}{3}$
44. $705\frac{3}{4}$
45. 8
46. $32\frac{3}{4}$
47. $8\frac{3}{7}$
48. $2077\frac{3}{11}$
49. $7028
50. 4023
51. 870
52. 1440

Page 127.
53. 35000
54. $\frac{1}{3}$
55. 11856
56. $\frac{19}{3262}$
57. H. $418
C. $1045
58. $25\frac{53}{99}$
59. $179\frac{15}{77}$
60. $2\frac{2}{3}$
61. $15\frac{321}{480}$
62. 1
63. $7500
$1500
64. 925
65. $1\frac{34}{45}$
66. $3750
$3000
$15000

Page 128.
67. $\$2\frac{1}{6}$
68. $\frac{17}{54}$; $\frac{17}{27}$
69. $\frac{85}{126}$; $\frac{17}{63}$
70. 16.
71. $\frac{47}{126}$; $\frac{65}{126}$
72. 132
73. $1\frac{21}{55}$
74. $\frac{13}{96}$
75. $\$6\frac{7}{12}$
76. 12
77. $\$205\frac{31}{32}$
78. $7161.
79. $1061\frac{1126}{1519}$
80. $\frac{15}{32}$ m.
81. 3.3825

Page 129.
1. 15
2. $22\frac{1}{2}$
3. $5\frac{1}{7}$
4. 30
5. $8\frac{3}{11}$
6. $\frac{20}{49}$

SANTA CLARA COUNTY
TEACHERS' L

Ex. Page 129.

7. $\frac{84}{685}$
8. E & F by $\frac{81}{304}$ of $1.

Page 130.

10. 5 h. 27$\frac{3}{11}$ m. P. M.
9 h. 49$\frac{1}{11}$ m. P. M.
11. 240
12. 120

Page 131.

14. C $5
S $3
15. C $.80
W $1.25
16. 3 P. M.
17. 4 h. 48 m. P. M.
18. 6 P. M.
19. 2 h. 24 m. P. M.
20. 65 ft.
21. $80
22. $72
23. 142
24. E $1355
F $2710
G $1355
H $4420

Page 132.

25. 1st in 6 h.
2nd " 3 h.
3rd " 2 h.
26. 65$\frac{5}{11}$
27. 54$\frac{3}{4}$, 17$\frac{1}{2}$
28. 65
29. 84, 28, 16
30. 218\frac{3}{4}$
156\frac{1}{4}$
$125
31. 80
32. 48 in.
33. 630

Page 149.

101. 33759
102. 77440
103. 228718

Page 150.

104. 1160
105. 5723
106. 35761
107. 572052
108. 10831
109. 2080
110. 660
111. 3960
112. 10890
113. 20736

Page 151.

116. 134 sq. rd. 12 sq. yd. 129.6 sq. in.
117. 182 rd. 2 yd. 7.2 in.
118. 243 A. 32 sq. rd.
119. 19 cu. ft. 1226.88 cu. in.
120. 2 yd. 1 ft. 0$\frac{6}{7}$ in.
121. 13 sq. yd. 4 sq. ft.
122. 76 cu. ft. 1382$\frac{3}{5}$ cu. in.
123. 20 cu. ft. 432 cu. in.

Page 152.

126. 56 m. 80 rd.
127. 13A 118 sq. rd. 6$\frac{1}{4}$ sq. yd.
128. 397 T. 863 lb.
129, 4905 gal. 1 qt. 2 gi.
130. 2 lb. 10 oz. 18 pwt. 7 gr.
131. 5$\frac{29}{32}$
132. 13 lb. 9 oz. 5 dr.
133. 16 oz. 4 dr. 1 sc. 17 gr.
134. 12 bu. 2 pk.
135. 75 cd. 84 cu. ft.
136. 8 cd. 74 cu. ft. 656 cu. in.
137. 43 cu. ft. 1590 cu. in.
138. 1112 P. 24 cu. ft.
139. 6 A. 2 sq. ch. 8 P.
140. 1058 m. 54 ch.
141. 611 ch. 3 rd. 21 l. 2.68 in.
142. 467 rd. 16 l. 7.28 in.
143. 3021 rd. 15 ft. 6 in
144. 78$\frac{1}{8}$ sq. m.
145. 6638 cd. 6 cu. ft.
149. $\frac{281}{4000}$
150. $\frac{125}{198}$
151. $\frac{5}{1024}$
152. $\frac{1}{256}$
153. $\frac{1}{480}$

Page 153.

155. .5
156. .5
157. .25
158. .00208+
159. .0205+
163. $\frac{7}{96}$
164. $\frac{3}{16}$
165. $\frac{1}{640}$
166. .00125
167. .0083$\frac{1}{3}$
168. .000001 13+
169. .00002 34+
172. 118$\frac{86}{175}$
173. 455$\frac{35}{48}$
174. 12$\frac{12}{35}$
175. 98$\frac{7}{16}$
176. 138$\frac{47}{175}$
177. 318$\frac{29}{72}$

Page 157.

2. 39 T. 1 cwt. 88 lb. 6 oz.
3. 29 m. 3 fur. 2 rd. 1 yd. 30.65 in.
4. 64 bu. 3 pk. 1 qt.
5. 32° 56′ 41″
7. 14 cu. yd. 24 cu. ft. 10 cu. in.

Page 158.

8. 18 T. 2 cwt. 50 lb. 14.2 oz.
9. 12 lb. 8 oz. 16 pwt. 9 gr.
10. 19 gal. 1 pt. 2$\frac{1}{4}$ gi.
11. 19 bu. 1 pk. 1 qt. 0.25 pt.
12. 3 yr. 5 mo. 16 d.
13. 90°
14. 80° 51′ 10.95″
15. 1 in.
17. 17 s. 5 d. 1$\frac{1}{2}$ far.
18. 2 m. 6 f. 20 rd. 12 ft. 8$\frac{3}{4}$. in.
19. 10 A. 17 sq. rd. 27 sq. yd. 8 sq. ft. 95.59 sq. in.
21. 116 sq. rd. 13 sq. yd. 140$\frac{3}{8}$ sq. in.
22. 38 gal. 3 qt. 3.2 gi.

Ex. Page 158.

23. 1 m. 177 rd. 2 ft. 5.08 in.

—

2. 124 T. 17 cwt. 98 lb.

Page 159.

3. 231 m. 5 f. 23 rd. 15 ft. 6 in.
4. 11 lb. 4 oz. 15 pwt. 10 gr.
5. 126 bu. 1 pk. 1 qt. $\frac{4}{5}$ pt.
6. 6 cirf. 248° 12′ 45.6″

—

2. 3 lb. 5 oz. 5 pwt. $13\frac{5}{8}$ gr.
3. 1 T. 15 cwt. 87 lb. 9.502 oz.
4. 14 cu. yd. 20 cu. ft. $1050\frac{1}{24}$ cu. in.
5. 1 sq. m. 555 A. 150 sq. rd. $67\frac{1}{5}$ sq. in.

—

1. 21 bu. 3 pk. 4 qt. $\frac{7}{40}$ pt.
2. 3 qt. 1 pt. $3\frac{1}{8}$ gi.
3. 3 gal. 1 qt. 1.6 pt.
4. £297 12s. 8'd. 2 far.

Page 160.

5. 17
6. 36 m. 98 rd. 1 yd.
7. 11 d. 17 h. 36 m.
8. 1 m. 302 rd. 15 ft.

Page 164.

15. 10962 sq. ft. 1218 sq. yd.
16. 25600 sq. rd. 160 A.
17. 12.77+
18. 6 A. −12 sq. ft.
19. 1200
20. 1400
21. 8
22. 8

Page 165.

23. $28800
24. $5.25
26. 224
27. 1620
28. 1728
29. 74
30. $\$48.53\frac{1}{8}$
31. $\$83.14\frac{4}{9}$
32. 45

Page 166.

33. $19.125
34. $\$25.47\frac{7}{9}$
35. $\$27.87\frac{1}{27}$
36. $73.50
37. $122.88

Page 167.

39. $40\frac{1}{3}$; $40\frac{1}{2}$
40. $79.57+
41. $25. $2.
42. Either.
43. 108
44. $2131.20
45. $2038.96

Page 169.

9. 900
10. $66.
11. 2521. 21+
12. 900; 24 300
13. 18000
14. $337.92

Page 170.

15. 56700
16. $1425.60
17. $7901.62
18. $6125\frac{5}{8}$
19. $20845\frac{11}{25}$
20. 200
21. $8317\frac{11}{25}$
22. $10\frac{4}{5}$
23. $1885\frac{1}{11}$
24. $29\frac{1}{11}$

Page 171.

25. $210\frac{35}{64}$
26. $154\frac{2}{7}$
27. $1244\frac{4}{9}$
28. $40\frac{5}{33}$
29. 89600

Page 174.

6. 4 h. 43 m. $50\frac{4}{5}$ s.
7. 12 h. 17 m. 10 s.
8. 7 h 29 m. 47 sec. A.M
9. 55° 45′

Page 175.

10. 3° 11′ W.
11. 16° 23′ E.
12. 83° 3′ W.
13. 1 h. 43 m. $50\frac{4}{5}$ s. A M.
1. 34,151, 400
2. 19,162
3. $\frac{63}{128000}$
4. 2.
5. 1.
6. $\frac{5}{88}$
7. .03125
8. 10374
9. $59\frac{7}{11}$ sq. rd.
10. .275
11. $.625
12. $280.

Page 176.

13. $10.50
14. $\$2.07\frac{8}{35}$
15. $3.
16. $792.
17. $138.60
18. $46.20
19. $5.
20. $12.92
21. $18.6624
22. $50.88
23. $16.80

Page 177.

24. $14.75
25. $96.
26. $134.40
27. $141.75
28. $706.86
29. $64.68
30. $132.
31. 49.14
32. 11762.96+
33. 36480
34. 3.705625

Page 178,

35. 236.99+
36. $57\frac{3}{4}$
37. 6
38. 2504.7
39. 121 lb. 5 oz. 10 pwt. 12 gr.
40. 15 mi. 194 rd. 2 yd. 5 in.
41. $157\frac{1}{4}$
42. 23 hrs. $37\frac{7}{15}$ m.
43. 66 yr. 10 mo. 29 d.
44. 61 yr. 4 mo. 29 d.
45. 7 mo. 8 d.
46. Sept. 21
47. 59 m. $23\frac{3}{5}$ s.

Page 179.

48. 5 h. 9 m. $57\frac{1}{3}$ s.

Ex. Page 179.

49. 35° 38′ 43″ E.
50. 43° 12′ E.

—

1. $78\frac{3}{8}$
2. $97\frac{4}{5}$
3. $370\frac{5}{12}$
4. 3082
5. 80.089+
6. 37.36+ or $37\frac{1663}{4608}$
7. $4488\frac{3}{16}$
8. 396
9. $30\frac{1}{4}$

Page 180.

10. $39\frac{3}{8}$
11. 45
12. 4800
13. 100

—

1. 12—
2. 23—
3. 42—

Page 181.

1. 16
2. $97.20
1. 2016
2. $121.65
3. $95.76

Page 182.

1. 720
2. 980
3. 1350
4. $61.44

—

1. 420

—

1. 45 M·
2. 70+M.

Page 183.

1. 1995
2. 1440
3. $37\frac{1}{2}$
4. $3\frac{3}{4}$

—

1. 7400
2. 960
3. 4270
4. 3245

—

1. 8
2. $3\frac{3}{4}$
3. 176, 312
4. 4160

Page 187.

13. $2791.04
14. 2705.04
15. 338.10
16. 2240
17. $28800.
18. $149.

Page 188.

19. $2880.
20. $35400.
21. A, $3.55
22. $222\frac{2}{9}$

Page 189.

16. 21%

Page 190.

17. 4%
18. 75
19. 6%
20. $1\frac{1}{2}$
21. $27\frac{81}{517}$
22. $121\frac{19}{36}$
23. $82\frac{2}{7}$
24. $14\frac{2}{7}$

Page 192.

19. 15840
20. 7680
21. $30600
22. $15000
23. 70000
24. $9000
25. 4000
26. 1600
27. 384
28. $13000

Page 194.

18. 7000
19. 53500
20. 55500

Page 195.

21. 2426
22. 787.5
23. 900
24. N. 4000 A. 3750
25. 500000
26. 1250
27. 77000
28. 4000 4360
29. N. 75000 S. 77000

Page 197.

15. 8000
16. 1500
17. 1700
18. 1500
19. $2133\frac{1}{3}$
20. 510
21. $2,000, 000
22. 29058+
23. Y. 1600 R G. 1500

Page 198.

24. A. 95 B. 76

Page 200.

22. $2200

Page 201.

23. $12\frac{1}{4}$
24. $5000
25. $11400
26. 16
27. 10
28. $8370 g.
29. $220 l.

Page 203.

11. $89.10
12. 5
13. $11390
14. $576
15. $9640
16. $772.80
17. $438.50

Page 204.

18. $1958.00
19. $160. $24160
20. $8500 $510

Page 205.

7. $5750
8. $10,600
9. $10,000
10. 7%
11. $56.25 $281.25
12. $2952.80
13. $500 $3475

Page 207.

2. $91
3. $300
4. .016
5. $11020.

—

1. $323.50

Page 208.

2. $367.50
3. $600
4. $3000

Page 209.

12. $67.50
13. $55.64
14. $80.46
18. $525.
19. $874.08
20 $656.64

Page 210.

15. $36.92
16 $274.
17. $382.50
21. $1389.
22. $1371.24
23. $2453. 125

—

2. $665.52
3. $595.20
4. $1036.80
5. $866.55
6. $715.
7. $1260.
8. $665.50
9. $1027.20
10. $732.075
11. $1037.40

—

2. $25.88
3. $60.75
4. $112
5. $117.60
6. $65.10
7 $147. 676
8. $126. 108+
9. $48.607

Ex. Page 210.

10. $131.749+
11. $61.26+
12. $1461.14+
13. $4006.18+
14. $3038.392
15. $6708.554+
16. $9402.793+

Page 211.

2. $1.89
3. $2.76
4. $1.00
5. $1.08
6. $.84
7. $.48
8. $.97⅓
9. $.28⅞
10. $.787½
11. $1.804
12. $2.03⅔
13. $1.214+

Page 212.

2. $38.93+
3. $170.884+
4. $136.681+
5. $144.309+
6. $67.251+

—

2. $129.60
3. $57.
4. $50.40
5. $37.33⅓
6. $42.75
7. $106.
9. $2.66⅔
10. $2.75
11. $32.34
12. $7.71 4/7

Page 213.

2. .153
3. .224
4. .2925
5. .3185
6. .4095
7. .207
8. 158⅓
9. .297⅔
10. .425⅚
11. .491⅙

—

1. $34.02
2. $215.04
3. 237.875
4. $102.75
5. $26.48+

Page 214.

6. $98.78+
7. $117.94+
8 $122.25−
9. $66.15+
10. $134.24+
11. $243.32+
12. $219.729−
13. $346.82+
14. $199.93+

—

1. $19.
2. $39.20
3. $10.80
4. $21.25
5. $21.78
6 $7.71+
7. $5.21+
8. $7.755
9. $4.71−
10. $2.88+

Page 215.

11. $.72
12. $.555
13. $.84
14. $1.56⅔
15. $.71⅔
16. $.43⅓
17. $.195
18. $.84⅓

—

2. $14.61+
3. $63.47−
4. $19.61¼
5. $780.02+
6. $1170.72+
7. $626.24−
8. $11.70
9. $59.59
10. $84.14−

Page 216.

11 $74.35+
12. $95.55−
13. $79.69+
14. $61.13+
15. $19.94+
16. $651.69+
17. $1046.57+
18. $951.44
19. $38.03−
20. $105.51+
2. $13.835+
3. $23.178+

Page 217.

1. 7%
2. 8%
3. 4½%
4. 6 %
5. 10%
6. 8%
7. 3½%

Page 218.

1. $550.
2. $1500.
3. $820.
4. $750.
5. $840.
6. $25000.
7. $3000.

Page 219.

1. 5 yr.
2. 1 yr. 9 mo.
3. 1 yr. 4 mo.
4. 2 yr. 6 mo.
5. 20 yr.

—

1. 8125
2. 786457
3. 78 37/50
4. $3000.
5. 750%
   13⅓%
6. 43184 ms.
7. 144
8. $392.80

Page 220.

9. $26600.
10. 20
11. 25
12. $7960.
    22⅓%
13. $4000.
    $180.
    $93.18
14. 3½
15. $11250.
    13⅓ yr.
16. $4698.

Page 221.

17. $7000.
18. $19650.75
19. .025
    $7890
20. $1,065,917.60
21. $5.0625
22. $35,057,239.72
23. $50,000.
24. $268.576
25. $2.3625
26. $13.31 37/73

Page 222.

27. $150.
28. $25 gain.
29. $1280.
30. $2160.
31. 2 yr. 8 mo.
32. 1 yr. 4 mo.
33. 5
34. 4
35. $10810.
36. $2640.
37. $1700.

Page 225.

1. $304.65
2. $450.
3. $502.29+

Page 226.

1. $50.78+

Ex. page 226

2. $403.57+
3. $254.66+

Page 227.

2. $264.10
3. $13.63
4. $61.92

Page 228.

5. $197.31
6. 180.56

Page 229.

2. $32.32
3. $262.12

Page 230.

4. $234.77

Page 231.

3. Nov. 1–4
23d.
$450.49

Page 232.

4. Mar. 25–28.
55d.
$796.08
5. Nov. 1–4
78 d.
$987.
6. $675.
7. $985.80
8. $494.75
9. $738.83+
10 $352.16
11. $474.
12. $714.36
13. $596.15
14. $897.75
15. $505.12+

Page 233.

2. $1220.
3. $603.87+
4. $1553,06
5. $1558.93
6. $685.66
7. $565.68
8 $133.96g.

Page 234.

9. $5642.31
10. 163.73+

Page 235.

1. $600.
2. $142.20
3. $279.11
4. $485.39
5. $298.24

Page 236.

6. $4020.38
7. $11528.90
8. $2.91 : lat.

Page 237.

1. $691.20
2. $2203.20
3. $573.75
4. $752.

Page 238.

1. $226.75
2. $138.43+
3 $245.01+
4. $100.60
5. $798.82+
6. $2267.
899
7. $1535.
01+
8. $98.56
9. $5.50+

Page 240.

2. $881.59+
3. $384.74
4. $102.67+
5. $85.22
6. $19.09+
7. $700.
8. $200.
9. $400.
10. $550.

Page 241.

2. $755.52
3. $730.
4. $462.15
5. $465.31+

Page 244.

15. $3063.75
16. $8650.

Page 245.

18. $5895.
19. $838.50
21. 75
22. $9600.
24. 1.08
25. 186
26. $2600
27. $75\frac{1}{4}$

Page 246.

28. $400.
29. $144.
31. $12000.
32. 300
33. 5%
34. 12%
36. $300.
37. $720.
38. 7's; $30.

Page 247.

40. $4\frac{1}{6}$%
41. 5%
43. 6%
44. $32400.
45. $22684.
46. $18750.
48 $83\frac{1}{3}$
49. 75
50. $133\frac{1}{3}$
51. $33\frac{1}{3}$
52. 150

Page 249.

2. $1873.95
3. $2224.687+
5. $976.95
6. $1482.

Page 250.

7. $1774.80
2. $3666.25
3. $2770.78+
5. $1000.
6. $1000.
7. $3012.804
8. $899.10+

Page 253.

1. 25: 3;
35: 8.
2. 12; $\frac{1}{7}$; 4; $\frac{1}{5}$
3. 8; 53; $36\frac{1}{2}$
5. 4; 14; $\frac{1}{4}$
6. 100; 10

---

1. 25:7; 4:1;
1:8; 3:5
2. 1:2; 60:1;
12:1; 1:5

---

2. 2:3; 6:7;
14: 15; 40:
27; 22 : 15

---

2. $\frac{33}{50}$; $\frac{23}{25}$; $\frac{82}{125}$
4. $\frac{1}{3}$; $2\frac{1}{2}$; $\frac{1}{80}$;

Page 257.

2. $$60\frac{1}{3}$
3. $56.25
4. $70.
5. 800
6. 64.
7. 60
8. $3.75
9. $235

Page 258.

10. 24900
11. $2250
12. $152
13. $75.705
14. $3.25
15. $162
16. $88.
17. $1\frac{22}{35}$
18. 660
19. $391\frac{41}{49}$
20. $93\frac{3}{25}$
21. 9 in.

Page 259.

2. $3\frac{1}{3}$
3. 40
4. 32
5. 20
6. $68\frac{4}{7}$

Page 260.

7. $53\frac{1}{3}$
2. $1\frac{1}{2}$

Page 261.

3. 64
4- 1512

---

2. 24
3. 50
4. 54
5. 45
6. 1080

Ex. Page 262
7. 6400
8. $220.
9. 18816
10. 800
—
2. $30.
$42.
3. $16.
$20.
$28.
4. $80. $40.
$20.

Page 263.
5. $800.
$200.
$2000.
6. 12, 24
7. 30, 15
8. $10. $6
9. $50. $20.
10. 64
11. 32
—
1. $3600.
$1200.
2. $2700.
$1200.
3. 2 : 1

Page 264.
5. A $5400.
B $6480.
6. $810.
$900.
7. A $2000.
B $7200.
C $400.
8. D $15.
E $27.
F. $24.
9. A $8000
B $8400
C $2400
1. 72
2. $11\frac{1}{4}$

Page 265.
3. 189 ft.
4. 3420
5. 192
6. 4
7. 24
8. 16
9. 20625
10. A $800
B $160
11. C $180
D $135
12. 180
120
90
72

Page 267.
1. 1225
2. 531441
3. 4100625
4. 1953125
5. 39.0625
6. .00195
3125
7. .0000 2401
8. .3025
9. $\frac{1}{64}$
10. $\frac{16}{81}$
11. $\frac{27}{512}$
12. $\frac{16}{2401}$

Page 272.
1. 24
2. 27
3. 53
4. 66
5. 96
6. 98
7. 111
8. 125
9. 562
10. 821
11. 969
12. 1432
14. $\frac{15}{40}$
15. $\frac{17}{31}$
16. $\frac{24}{25}$
17. $\frac{21}{22}$
18. $\frac{14}{27}$
19. $\frac{3}{5}$
20. 17.49
21. 22.60
22. 45.91
23. 56.70
24 67.27
25. 72.25
—
2. .250+
3. 24.3
4. 2.16
5. .47
6. .125
7. .256
8. 7.29
9. 1.296
10. 3.43
11. 51.2
12. 2.401
13. 6.4
14. .6561

Page 273.
19. 5.5
20. 5.25
21. 3.6
22. .79+
23. .76+
24. 27.5
25. $6\frac{1}{7}$
26. $.31\frac{1}{3}$
27. $8\frac{1}{6}$
28. .9258
29. 89442
30. .7416
31. 1.284
32. 8.544
33. 22.627
34. $\frac{3}{4}$
35. $\frac{3}{5}$
—
1. 312 rd.
2. $11.70
3. 35 ft.
4 75×25

Page 274.
6. 25; 98; 85
7. Same.
8. 40
9. 320

Page 275.
2. 20
3. 25
—
1. 30
2. 24
3. 5.65+
4. 9.79+
5. 80 ft.
6. 59.92 ft.

Page 276.
7. 10.77+ft.
8. 11.66+in.
10. 7.07+in.
11. 2 82+in.
12. 14.14+in.
14. 28.28+ft.
15. 20.14+in.
16. 10.39+in.
17. 17.8
18. 94.6+mi.
19. 5.66+
20. 52

Page 281.
1. 15
2. 31
3. 38
4. 48
5. 67
6. 97
7. 91
8. 93
9. 9.8
10. 13.4
11. 4.11
12. 2.03
13. 1.43
14. 1.91
15. 3.65
16. 4.43
17. $\frac{5}{8}$
18. $\frac{6}{7}$
19. $\frac{5}{9}$
20. $2\frac{1}{2}$
21. $1\frac{3}{4}$
22. $1\frac{4}{5}$
23. .08
24. .06
25. .15
26. 1.5
27. .01
28. .26−

Page 282.
1. 37
2. 961 sq. in.
3. $43.75
4. $36.015
5. 378
6. 13−in.
7. 8
8. 27
9. [illegible]

Ex. Page 286.
1. 300 sq. ft.
2. 1428 sq. in.
3. 7225
4. 309 yd.
5. 640

Page 287.
1. 48 sq. ft.
2. 24 sq. ft.
3. 30
4. 600
5. 480
6. $39\frac{1}{16}$
7. 625
8. 16

——

1. 99
2. 45
3. 39
4. 17

Page 288.
5. 12

Page 289.
1. 2800
2. 4864
3. 2000
4. $313\frac{1}{2}$

——

2. 6.9+
8.66
10.39+
20.78+
3. 3366.36 sq. ft.
4156.8+ sq. yd.
664.80+ sq. mi.
9352.8 sq. mi.

Page 290.
2. 37.6992− ft.
56.5488 rd.
3. 31.416− in.
125.664− yd.
5. 4.77+ft.
6.36+ft
6. 1.9+in.
9.54+ft.

——

2. 78.54−
201.0624−
314.16−
706.86−
3. 113.09 sq. in.
153.9384− sq. ft.
50.2656− sq. mi.

Page 291.
5. 5.35+ft.
6.9+ft.
11.28+ft.
6. 10.09+ mi.
30.9+mi.
7. 49+
8. 44.8369+
63.3974+
9. 1178.1

——

1. 78.54
113.0976
153.938+
201.0624
2. .7854 sq. ft.
7.0686 sq. ft.
1 sq. ft. 32. 715 sq.in.
3. 785.4

Page 292.
2. 50

Page 293.
3. 25; 3600
4. 49:4
5. 9216
6. 11.54+in.
7. 5 in.
8. 81
9. 10:1

Page 295.
1. 112 sq. in.
2. 800 sq. in.
3. 450 sq. in.
4. 24 sq. ft.
5. 96 sq. in.
64 cu. in.
6. $70\frac{2}{3}$
7. 40

Page 296.
8. 1024
9. 150; 108
10. 9 ft. 2 in.
11. 113.0976 sq. ft.
127.2348 sq. ft.
84.8232 cu. ft.
12. 1587.489

Page 297.
1. 5 sq. ft.
2. 20 sq. ft.
3. 1200 sq. ft.
4. 188.496 sq. ft.
5. 13.3+sq. ft.
6. $133\frac{1}{3}$ cu. ft.

Page 298.
7. 24 cu. ft.
8. 50.26 cu. ft.
9. 93391360 cu. ft.
10. 19257.6153+sq.ft.
283529.4 cu. ft.

Page 299.
1. 50.2656
201.0624
804.2496
2. 33.5102
268.0832
2144.6656
3. 127.07+
134.70
4. 201062400
6. 1000
7. 64
8. 1331000

——

1. 55.13+rd
2. 11.7+ft

Page 300.
3. 1152
4. 12.5864
5. 11.8+in.
6. 282744
7. .2146 sq. in.
8. 1200.72
9. 66 ft.
10. 3 : 4
11. 31.6+rd.
12. 20,736
13. 27
14 7125
15. 36 : 49

Ex. Page 301.
1. $1911\frac{1}{2}$
$924\frac{1}{2}$
2. 221
3. 25
4. 3773
4363
5. 103740
6. 42
7. $6.
8. 100 80 yr.
9. 1575
10. 288 yds.

Page 302.
11. $461\frac{11}{34}$
12. $153\frac{59}{72}$
13. $63\frac{3}{5}$
14. $84\frac{2}{7}$
15. $1882.
40625
16. $\$8\frac{1}{2}$
17. $\$523\frac{1}{32}$
18. 87
19. 272
20. $\frac{1}{4}$; $\frac{1}{5}$
21. $476
22. $\$208\frac{1}{4}$
23. $\$13581\frac{3}{11}$
24. $4\frac{1}{2}$ thirds.

Page 303.
25. $\frac{25}{48}$
26. $\frac{13}{48}$
27. $40
28. $6\frac{1}{4}$ da.

Ex. Page 303

29. $\$179\frac{1}{5}$
30. $\frac{1}{8}$
31. 194.425
32. 98.64875
33. 48+
34. .8
35. $\frac{67}{120}$
36 .05
37. 5.253125
38. .075
39. $\frac{147}{100}$
40. $4005.12
41. $1650.
42. $2970.
43. $252

Page 304.

44. 112 sq. ft.
45. 9060 cu. ft.
46. 600 sq. ft.
47. $1301\frac{11}{14}$
48. 3456
49. 24
50. .0625
51. $92.25
52. 32
53. $420
54. W.
55. $\frac{1}{32}$
56. 960000
57. $260
58. $\$1581.87\frac{61}{64}$
59. 36 min.
60. $1357\frac{5}{7}$
61. 50
62. $\frac{1}{4}$
63. $408
64. $88
65. 276.72
66. 52
67. $297
68. $8\frac{1}{2}$
69. 172.50
70. 189
71. $\frac{5}{16}$
72. 87° 34′ 45″ W.
73. $\frac{49}{69}$
74. $\frac{1}{4}$

Page 306.

75. $19\frac{1}{3}$
76 72
77. 3 h. $16\frac{4}{11}$ m.
78. C. $5. S. $2.
79. 42 ft. 33 ft.
80. 2 h. 40 min. P. M.
81. 3600 sq ft.
82. t : r :: 2 : 1
83. 804.2496 sq. ft.
84. 779.1168 ft.
85. $336
86. $13.86
87. $3600
88. $37.50
89. $3572.
90. $2400.
91. $\$746.66\frac{2}{3}$
92. $40.25
93. $4000.
94. $5600
95. $\$104.16\frac{2}{3}$
96. $20.
97. $8\frac{1}{3}$%
98. $38.60
99. $552.
100. $562.50
101. $1555.55+
102. $208.85

Page 308.

103. 5 yr. 6 mo. 20 da.
104. 12%
105. $\$1738.10\frac{10}{19}$
106. 225
107. $5.6227
108. $2 gain. $6\frac{1}{4}$%
109. 3631.7
110. 3.25
111. 7.07
112. A $1800. B $1500. C $1000.

APPENDIX.

Ex. Page 313.

2. 83746.84 M.
3. 29016.18 MQ.
4. 17840.048 MC.
5. 5397.04 A.
6. $647.51+
7. 2367.08+C.
8. 4213 lb. 8.667 oz.
9. 1844.375 bu.
10 476.95+ litres.
11. 266.715+ Kg.

Page 316.

2. 5 mo. 5 da. June 6, 1892.
3. March 5, 1892.
4. May 15, 1890.
5. 11 mo. 17 days.

Page 318.

2. July 7, 1892.
3. March 29, 1891.
4. Sept. 25, 1891.
5. Mar. 5, 1893.

Page 322.

1. $840 May 12, 1890
2. Sept. 6, 1889.

Page 323.

3. $10. July 9, 1885.
4. $82.85
5. $267.34
6. $99.55
7. $516.78
8. $328.80

Page 325.

2. 25¢
3. 93%
4. 19
5. 90¢

Page 326.

2. 1 of each.
3. 1; 2; 1; 6.

Page 327.

3. 80 of each.
4. $1\frac{5}{8}$ qts.
5. 200

---

1. 100 of each.
2. 3; 1; 1.

Page 330.

1. 31; 136.
2. 22; 310.
3. 3; 300.
4. 4; 140.
5. 12; 270.
6. 78
7. 1275
8. $9\frac{1}{2}$ or $\frac{1}{2}$

Page 333.

1. 128; 170
2. 96; 186
3. 3; 484
4. $\frac{1}{8}$
5. 2
6. $1\frac{1}{8}$
7. $1048575.

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